Sun, Sky and Cloud

Where Light and Matter Meet

Rajaram Nityananda

Textbooks focus first on the reflection and refraction of light, and later on individual events of emission, absorption, and scattering. In Nature, it often happens that we see light which has undergone these processes many times. sunlight passing through clouds is just one example, related phenomena occur on the visible surfaces of the Sun and stars. The intensity and angular distribution of the light we see tells us not only about the source but also the medium through which it has passed to reach us. This area of science is known as radiative transfer. Its basic features are illustrated in this article, along with some examples.

Rajaram Nityananda works at the School of Liberal Studies, Azim Premji University, Bengaluru. Earlier, he spent a decade at the National Centre for Radio Astrophysics in Pune, and more than two decades at the Raman Research Institute in Bengaluru. He has taught physics and astronomy at the Indian Institute of Science in Bengaluru and the IISERs in Pune and Mohali. He is keenly interested in optics in a broad sense, and more generally, problems, puzzles, paradoxes, analogies, and their role in teaching and understanding physics.

Introduction

The blue of the daytime sky is sunlight which has reached us after bouncing off the nitrogen and oxygen molecules of the Earth's atmosphere. We are fortunate that only about 20 percent of sunlight is diverted in this way on a clear day, the remainder reaching us directly. On the planet Venus, an observer on the surface would hardly see the Sun, although she would see light coming from all around her. One has a similar experience when descending through a cloud in an airplane, or on a hill road, or in fog – light, light, everywhere but not a source in sight! Over the years, physicists, astronomers, and atmospheric scientists have created the field of radiative transfer. The aim is to understand the propagation of light in such situations. In the sky or in a cloud, at visible wavelengths, it is mainly scattering – change of direction. But in other situations, absorption and emission are very important. Perhaps the most important for the

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Radiative transfer, scattering, clouds, Sun, spectrum.

Our atmosphere thus acts like a one-way street – energy at visible wavelengths finds it easy to get in but infrared radiation at 10 microns finds it hard to get out! The Earth will therefore keep getting hotter until it can radiate away as much as it receives.

future of mankind is absorption of the infrared radiation which emerges from the Earth's surface. The clear sky is not so clear at wavelengths around 10 microns. This is the range of wavelengths emitted by the Earth's surface which is at a temperature of roughly 300 degrees Kelvin. Molecules of carbon dioxide and methane, among others, can absorb this radiation. Our atmosphere thus acts like a one-way street – energy at visible wavelengths finds it easy to get in but infrared radiation at 10 microns finds it hard to get out! The Earth will therefore keep getting hotter until it can radiate away as much as it receives. Global warming, sea level rise, climate change are all likely consequences as the scientific and popular media tell us nowadays. Clearly, radiative transfer is of more than academic interest. In the rest of the article, we set up some simple models for radiative transfer, which enable us to understand a wide range of phenomena.

Specific Intensity and its Invariance

In radiative transfer, it is extremely useful to define a quantity I called 'specific intensity' as the energy crossing a unit area per unit time, *per unit solid angle*. This is explained in *Figure* 1. There is a good reason for including the solid angle in the definition. We normally deal with extended sources which occupy a finite solid angle. The Sun, as viewed by us from the Earth, subtends an angle of about $\frac{1}{2}$ degree, or 1/120 radian. The

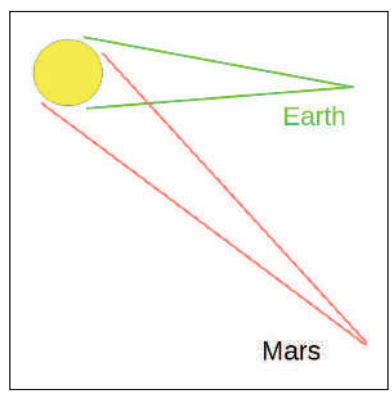

Figure 1. Energy flow per unit time per unit area per unit solid angle is the same at Earth and at Mars. A unit area receives less, but the solid angle is smaller by the same factor.

solid angle made by the Sun's rays at Earth is therefore $\pi(\frac{1}{120})^2/4 \simeq 5.5 \times 10^{-5}$ steradians. Now let us go from the Earth to Mars. The angle subtended by the Sun would go down, by a factor of 1.6, since Mars is further away from the Sun by this factor, when compared to Earth. The solid angle would go down by a factor of 1.6^2.

However, the energy received per unit area per unit time also goes down by the same factor, thanks to the well-known inverse square law. (This is because the same energy per unit time from the Sun is now spread out over a sphere whose area is 1.6^2 times larger.) Thus, the specific intensity I remains the same! This invariance of specific intensity under free propagation, from Earth to Mars in our example, is very useful in radiative transfer. It means that we do not have to worry about propagation, but only about absorption, emission, and scattering when we study the changes in I. (See *Figure* 3 on p.1116 for the simple mathematics behind this argument.) The strict definition of I includes 'per unit frequency interval' and we will keep this in mind when the properties of the medium depend on the wavelength of light.

A more colloquial term for I is 'surface brightness' because it corresponds to what our eye or an optical instrument sees as brightness. You can develop an intuitive feel for the invariance of surface brightness by carrying out the following experiment. Stand at a distance of about a metre from a plain white wall and view it through a cone made of a rolled-up piece of paper, and cover the other eye. As you walk back, you will not see any change in the uniform illumination reaching your eye. In fact, you will not be able to determine the distance from the uniform part of what you see. Of course, if there are variations in the brightness of a fixed size on the wall, you will see them getting smaller in angle as you move away.

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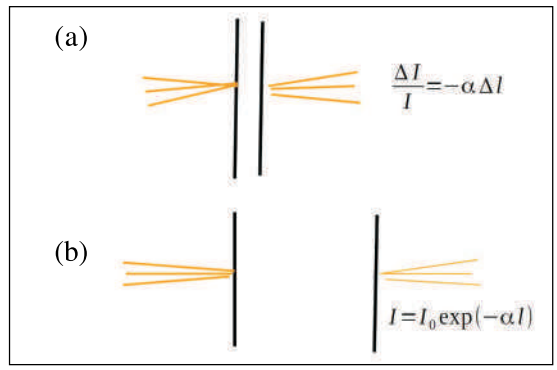

Figure 2. (a) Transmission through a thin absorbing slab. The (specific) intensity lost is proportional to the incident intensity, and the thickness. (b). Transmission through a thick slab. The emerging intensity equals the incident, multiplied by the exponential of the optical depth.

Simple Absorption

We begin with the simplest example – a slab of thickness l with radiation of specific intensity I_0 falling on it from the left as in *Figure* 2.

We are considering a narrow range of solid angle around a direction perpendicular to the slab. We can find the intensity of the radiation which emerges by subdividing the slab into thin layers of thickness Δl. The amount of radiation removed by absorption by a thin layer is proportional to the intensity of the incoming radiation. For a thin layer, it is also proportional to the thickness, because the radiation either encounters one absorber or none, and the number of absorbers is proportional to the thickness. We therefore write

$$\frac{\Delta I}{I} = -\alpha \Delta l \,. \tag{1}$$

The coefficient of absorption α has dimensions of $(\text{length})^{-1}$. The intensity is therefore reduced by a factor $(I - \Delta I)/I = (1 - \alpha \Delta l)$. The result of passing through N such slabs will be to multiply the incident intensity by the N-th power of this factor. If we write $\Delta l = l/N$, our intensity reduction factor is given by

$$\frac{I}{I_0} = (1 - \alpha l/N)^N \simeq \exp(-\alpha l) \text{ for large enough } N \,. \tag{2}$$

We have used the famous definition of $e = 2.71828.....$ as a limit. This is a nice example of how this number, which is named after Euler, crops up in many problems like radioactive decay, depreciation of an asset, compound interest (with plus sign in the exponent!), etc. In all cases, the reason is that we are carrying out repeated multiplication of numbers close to 1.

We now use a dimensionless measure of length, $\tau = \alpha l$ and look at the decrease in intensity for a small increase $\mathrm{d}\tau$. Equation (1) takes the form $\mathrm{d}I/\mathrm{d}\tau = -I$, which is the standard form for the differential equation of the exponential function. The advantage of using the dimensionless measure of thickness is that we now need not distinguish between a thick slab with low absorption and a slab half as thick with twice the absorption – both have the same value of τ, which is called the 'optical depth' or 'optical thickness' of the slab. I like to call it the 'Tau of Astrophysics', borrowing from Fritjof Kapra's book *The Tao of Physics* which was very popular when I was a student.

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Emission

We know that a pair of energy levels gives rise to absorption if the atom (or any other system) starts at the lower level, and makes a transition upwards on receiving a photon. Emission occurs if it starts at the upper level and comes down. So these two processes go hand-in-hand. Of course, if the material is at absolute zero, everything is in the ground state and there is no emission.

It is also useful to see how emission changes the specific intensity. Let us take a layer, of thickness Δl. *Figure* 3 shows a detector of area A_d at a distance r from the layer looking into a small solid angle $\Delta\Omega_d$, and recording the number of photons per unit time. These will originate from a volume $r^2\Delta\Omega_d\Delta l$. The fraction of radiation from each atom which enters the detector equals $A_d/4\pi r^2$.

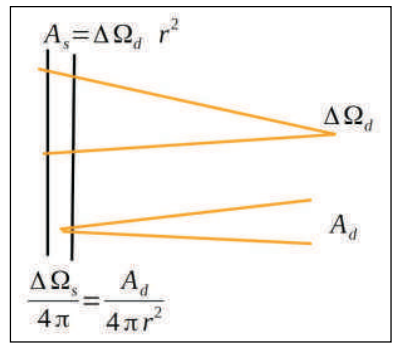

Figure 3. The contribution of emission from a thin slab to the specific intensity I is proportional to (a) the thickness, (b) the area A_s at the source which contributes and (c) the fraction of the solid angle which the detector subtends at the source, $\Delta\Omega_s/4\pi$. Notice that the product of (b) and (c) does not depend on the distance r and is proportional to the product of the solid angle and area at the detector, $\Delta\Omega_d$ A_d which appears in the definition of I. The emission contribution to I is therefore independent of distance.

As before, we see this crucial cancellation of the two r^2 factors. What is received from each atom goes down by r^2, but the number of atoms contained in the solid angle goes up by r^2. No matter where the observer is located, the contribution of the layer to the specific intenity is proportional to the layer thickness Δl (*Figure* 3). We therefore write $\Delta I = \epsilon \Delta l$. The emission coefficient ε depends on the properties of the atoms in the layer, and the number per unit volume.

Combining Absorption and Emission

We are now in a position to analyse what happens to radiation when it passes through a finite layer when both emission and absorption occur. Our basic equation now reads

$\Delta I = -\alpha I \Delta l + \epsilon \Delta l$ or in differential form, after dividing by α

$$\frac{\mathrm{d}I}{\alpha \mathrm{d}l} \equiv \frac{\mathrm{d}I}{\mathrm{d}\tau} = -I + \frac{\epsilon}{\alpha} . \qquad (3)$$

We see that our old friend, the optical depth τ, has made its appearance. And now, in addition to the absorption term, we have a term $s \equiv \epsilon/\alpha$ called the 'source function' on the right-hand side. Nothing in the discussion so far has assumed that the properties of the slab are constant – so the source function and the absorption can depend on position. Any first course on differential equations tells us how to solve this (*Box* 1). Here, we just pursue the simpler case when the source function is a constant.

Box 1. The Differential Equations of Radiative Transfer

For a slab extending from $x = 0$ to $x = L$, the absorption is $\alpha(x)$ and the emission $\epsilon(x)$. The specific intensity $I(x)$ obeys $\mathrm{d}I/\mathrm{d}x = -\alpha(x)I(x) + \epsilon(x)$. This is the famous Bernoulli-type ordinary differential equation which he (Jakob, one should add – the Bernoullis were a whole family of mathematicians!) solved by moving the $\alpha(x)I(x)$ to the left-hand side and multiplying both sides by $\exp(\int_0^x \alpha(y)\mathrm{d}y) \equiv \exp(\tau(0,x))$. This expression is called the 'integrating factor' but notice that it has a physical meaning – it contains the optical depth from zero to x. Mathematically, the point is that the derivative of this expression with respect to x equals $\alpha(x)$ times itself. The equation then becomes $\mathrm{d}/\mathrm{d}x(I(x)\exp(-\tau(x))) = \epsilon(x)$. Integrating both sides, we get $I(x)\exp(\tau(0,x)) = \int_0^x \epsilon(y)\exp(\tau(0,y))\mathrm{d}y + I_0$. We therefore get the following expression for the specific intensity at x.

$$I(x) = I_0 \exp(-\tau(0,x)) + \int_0^x \epsilon(y)\exp(-\tau(0,x) + \tau(0,y))\mathrm{d}y\,.$$

The quantity multiplying $\epsilon(y)$ inside the integral sign is nothing but $\exp(-\tau(y,x))$ which makes complete physical sense. Material at y emits $\epsilon(y)\mathrm{d}y$ and that is attenuated by the exponential of minus the optical depth from y to x. If we put the absorption and emission coefficients equal to constants, and put x equal to L, the length of the slab, we can easily do the integral and get back the expression in the main text:

$$I(L) = I_0 \exp(-\alpha L) + \epsilon(1 - \exp(-\alpha L))/\alpha\,.$$

If we have α negative but ϵ positive, it describes a device in which intensity is added proportional to the incident intensity. This is called an optical amplifier, and is used to build high power laser systems. Nature has already done this in our galaxy and others. Molecules such as H_2O in interstellar space show this kind of behaviour. (See the article on p.1079). How about when both α and ϵ are both negative? A little thought will show that this describes the financial situation many people find themselves in nowadays. They have taken a loan and I stands for their total debt, which would grow exponentially due to compound interest (negative α). They repay it at a steady rate ('EMI' standing for equated monthly instalments) which would make I decrease linearly with time (negative ϵ). Clearly the repayment has to be large enough, greater than the initial interest outflow αI_0, so that the debt can reach zero in a finite time, usually three to five years.

We now come to pure scattering. In the simple one-dimensional model which we introduced, let a layer of thickness $\mathrm{d}x$ scatter a fraction $\alpha_s\mathrm{d}x \equiv \mathrm{d}\tau$ of the radiation falling on it. We have defined an optical depth τ, now for scattering. In the equation below, F is the specific intensity in the positive x direction and B in the negative x direction. This fraction $\alpha_s\mathrm{d}x$ is removed from the incident beam, that is the first term in the equation below. The second term tells us that the scattered part is sent equally in the forward

Box 1. Continued...

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and reverse direction, and this is true for both F and B. This process is described by the pair of equations,

$$\mathrm{d}F/\mathrm{d}x = -\alpha_s F + \alpha_s(F+B)/2, \mathrm{d}B/\mathrm{d}(-x) = -\alpha_s B + \alpha_s(F+B)/2\,.$$

Notice that in the second equation, we have been careful to take into account the fact that B represents radiation travelling along $-x$, and thus the derivative is taken with respect to $-x$. Collecting the different terms, we have $\mathrm{d}F/\mathrm{d}x = -\alpha_s(F-B)/2$ and $dB/dx = -\alpha_s(F-B)/2$. This tells us straightaway that both F and B vary in the same way with x, the difference remains constant. The fact that $F-B$ is a constant is physically clear – it represents the net flux in the x-direction. Since there is no true absorption, energy conservation demands that the amount of radiation crossing from left to right at x should be the same as the amount at any other point, that is, independent of x. Let us call this net flux $\phi = F - B$ since we have already used up F for 'forward'. The equations now read $\mathrm{d}F/\mathrm{d}x = \mathrm{d}B/\mathrm{d}x = -\alpha_s\phi/2$. Since ϕ is a constant, we see that F decreases linearly with distance, and the same is true for B.

This is the same result which we got from our more elementary model in the main text. We can also rewrite the earlier equation as

$$\phi = -\left(\frac{1}{\alpha_s}\right)\mathrm{d}\frac{(B+F)}{\mathrm{d}x} \equiv -\left(\frac{1}{\alpha_s}\right)\frac{\mathrm{d}U}{\mathrm{d}x} \equiv -\frac{\mathrm{d}U}{\alpha_s \mathrm{d}x} \equiv -\frac{\mathrm{d}U}{\mathrm{d}\tau}.$$

We define the scattering optical depth of the full layer of thickness l as $\tau_s = \alpha_s l$. We have introduced the symbol $U = F + B$ which is proportional to the total energy at x regardless of direction. For a flux $\phi = 1$ and $B = 0$ at $\tau = \tau_s$, the exit of the slab, we have the solution $F = 1 + \tau_s - \tau,\ B = \tau_s - \tau$. Since $F = \tau_s + 1$ at the entrance $x = 0$, the transmission is $\phi/F = 1/(1+\tau_s)$. This is very similar to the model in the main text of this article, where the transmission $1/(1+L)$.

The form $\phi = -\left(\frac{1}{\alpha_s}\right)\frac{\mathrm{d}U}{\mathrm{d}x}$, or in words, flux proportional to the space derivative of the density, is called the diffusion equation. This is familiar as Fourier's law of heat conduction, and Fick's law of diffusion. While this form of the law is exact in our model, it is not so in a more realistic model which takes into account radiation travelling in all directions, not just forward and backward. However, it is still true that the transmission goes as the reciprocal of the optical depth for large τ_s. Combining pure absorption and scattering is a very interesting problem. Apart from optics, this theory is needed for neutrons in a reactor, which undergo scattering, absorption, but also multiplication by the chain reaction. No wonder mathematicians like Wiener and Hopf, astrophysicists like Schwarzschild, Eddington, Ambartsumian, and Chandrasekhar, and nuclear physicists like Fermi and Wigner have worked on many aspects of radiative transfer.

A simple trick works, viz to change the variable to $J \equiv I - s$. We now have simply $\frac{\mathrm{d}J}{\mathrm{d}\tau} = -J$.

The difference between I and s decays exponentially! We learn that repeated emission and absorption drives the specific intensity to a value s, depending only on the ratio of emission to absorption. We have just rediscovered Kirchoff's law, stated in textbooks of heat. Kirchoff was interested in the case when the body was in thermal equilibrium, and proposed that the source function would depend only on temperature. Today, we understand why this should be so – the emission depends on the number of atoms in the upper state, and the absorption on the difference between upper- and lower-state populations. In thermal equilibrium, population ratios depend only on temperature. It is satisfying to see that as radiation passes through a large optical depth, it reaches equilibrium with matter – this is the basis for what is called black body radiation[1].

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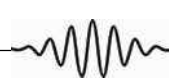
[1] *Resonance*, Vol.13, No.2, 2008.

With a finite slab, $J = I - s = C\exp(-\tau), I = s + C\exp(-\tau)$. The constant C is determined using the value $I = I_0$ at the entrance of the slab, $\tau = 0$. We get $C = I_0 - s$. Substituting, we then get our basic formula,

$$I = I_0 \exp(-\tau) + s(1 - \exp(-\tau)) \,. \qquad (4)$$

Equation (4) can be interpreted quite nicely in physical terms. The first term we have seen before – it is simply the decrease, due to absorption, of any radiation incident on the slab. The second term describes the emission from the slab, and is best understood in terms of two limits. When $\tau \ll 1$, it is simply $s\tau = (\epsilon/\alpha)\alpha l = \epsilon l$. It is proportional to the emission coefficient and the thickness of the slab, i.e., the volume of emitting atoms. Because the slab is thin, whatever they emit is able to escape. In the opposite limit, $\tau \gg 1$ we have the emission term equal to $s = \epsilon/\alpha$. It is as if we are seeing emission from a layer of thickness $1/\alpha$, i.e., a layer upto $\tau = 1$. This agrees with our intuition that radiation



The Sun is a hot ball of gas, with energy flowing outwards from the nuclear reactions (conversion of hydrogen to helium) in the centre. The density and temperature of the gas keep decreasing as we move outwards. The light we receive comes from the neighbourhood of a layer of optical depth unity, which is naturally called the 'photosphere'.

from deeper layers, $\tau \gg 1$ is more strongly absorbed and does not contribute at the exit face of the slab. The general solution of *Box* 1 shows that the radiation emerging from the slab mainly originates from a layer where the optical depth is of order 1, even when the properties of the medium depend on thickness.

The Solar Atmosphere

Our first application of equation (4) is to the outer layers of the Sun. This is a hot ball of gas, with energy flowing outwards from the nuclear reactions (conversion of hydrogen to helium) in the centre. The density and temperature of the gas keep decreasing as we move outwards. The light we receive comes from the neighbourhood of a layer of optical depth unity, which is naturally called the 'photosphere'. Using the known absorption properties of matter near the surface of the Sun, astrophysicists can calculate that this layer occurs for visible light, where the temperature is about 5700 K. This is what we 'see' as the temperature of the Sun's surface. However, this is not the whole story. The absorption is much greater at the frequencies corresponding to energy-level spacings of various atoms (bound electrons). We see a different layer when we look – for example – at wavelengths of 589 nm, at which neutral Na atoms absorb light. This layer lies closer to us than the photosphere, since we can reach optical depth of order unity in a shorter distance if the absorption coefficient is more (*Figure* 4). At wavelengths between these lines, the absorption is smaller and we a deeper layer. It occurs by electrons being ejected from atoms, or electrons already free but colliding with ions – this part of the spectrum

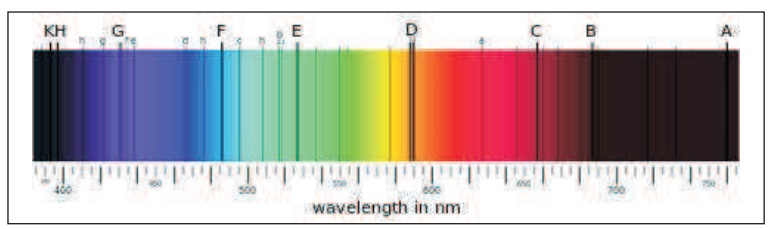


Figure 4. Dark lines in the spectrum of the Sun. At these wavelengths, the absorption is more and we see a higher, cooler, layer of the Sun's atmosphere which explains the darkness compared to neighbouring wavelengths, which 'see' deeper layers. Courtesy: Reproduced from https://en.wikipedia.org/wiki/Fraunhofer_lines.

is called the continuum.

This higher layer happens to be cooler, and hence we see radiation corresponding to a lower temperature, at wavelengths corresponding to absorption by atoms of different chemical elements, and even some simple molecules which survive in this furnace! The Sun looks darker at these special wavelengths. This is the explanation of the dark lines which cross the spectrum of the Sun, first discovered by Fraunhofer in 1814.

Some 45 years after Fraunhofer, Bunsen and Kirchoff gave the explanation we have outlined above for the origin of absorption lines – cold gas in front of a hotter layer. Notice that the total optical depth along the path we are looking is large in this situation. In the laboratory, when sodium is introduced into a flame (a Bunsen burner!), we see emission lines, because there is no hotter layer behind the flame. The same situation occurs when the upper layers of the Sun's atmosphere are viewed during a solar eclipse. Because one is looking tangentially (*Figure* 5), there is no hotter layer behind the cooler layer, and one sees emission lines. This is the so-called 'flash spectrum' of the Sun. It is present even when the Sun is not eclipsed, but is overwhelmed by the absorption spectrum when we let in even a tiny amount of light from the photosphere.

The fact that the layer which we 'see' depends on wavelength was brought home to me many years ago by colleagues who were studying the Sun at long radio waves,

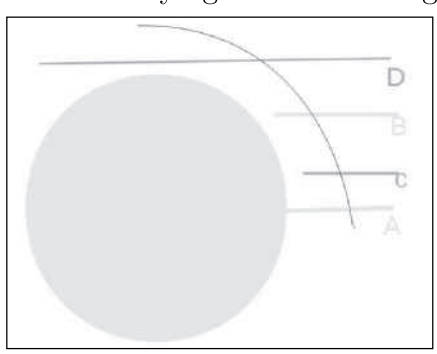


Figure 5. Looking at the Sun at different wavelengths and different points. A, in yellow, is at the centre of the Sun, at a wavelength in the continuum. It reaches optical depth unity at the edge of the yellow disc, which is the photosphere. The thin dark line shows a lower density, lower temperature layer above the photosphere, for reference. B is a ray at the same wavelength, but away from the centre of the disc. It reaches optical depth unity at a higher, cooler layer because it travels at an angle. Hence 'limb darkening'. C in orange, is at the wavelength of absorption by sodium – hence we see a higher layer, even at the centre of the disc. D is a ray at the same wavelength, but looking tangentially during a solar eclipse. Now one sees emission lines the 'flash spectrum'.

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Scattering

Our account of absorption of light by matter assumed that the excited atoms lost the energy by some other process, say collisions, so that the energy was simply removed from the beam, not in the form of light. Emission was assumed to be from an upper level populated by collisions, not by light. These processes were therefore treated as separate and independent. This may not always be a good assumption. For sunlight encountering the molecules of the Earth's atmosphere, the radiation is re-emitted almost instantly with a change in direction. This process is called scattering. Light from the Sun travelling vertically down has only a small probability of being scattered, around 20% in the middle of the visible spectrum, increasing to 40% on the blue side and decreasing to about 10% on the red side. But when we look at the setting Sun on the horizon, we are looking through 300 km of air rather than 10 km. So even red light is reduced by a factor of more than 10, and blue light is essentially cut off (*Figure* 6). This explains the red colour of the setting Sun.

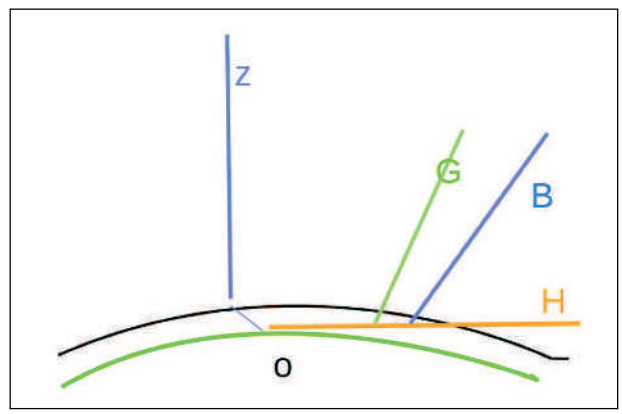

Figure 6. Z is a ray of sunlight coming from the zenith. It is scattered by the Earth's atmosphere to the observer at O, looking away from the Sun, who sees a blue sky because blue light is more strongly scattered. H is a ray travelling horizontally to an observer viewing the sunset. Note that it traverses a much longer path of atmosphere. G and B represent green and blue light being removed from the beam by scattering. So, we have mainly red light reaching the observer at O.

Radiative transfer with pure scattering (no emission or absorption) is quite different from what we have discussed so far. The source function now is not given beforehand, but depends on the intensity of the radiation at that location. We build up our understanding making two simplifying assumptions: (a) We have no true absorption or emission, but only scattering. (b) We consider only two directions of radiation, forward and backward. The treatment of this problem using differential equations is given in *Box* 1.

Here, we treat an even simpler model, in which the medium is divided into a finite number of layers. *Figure* 7 explains the geometry and the notation. Let us assume that each layer scatters all the radiation which falls on it, which is divided equally between the forward and backward directions. We can say that half of the incident radiation is 'reflected' and half is 'transmitted'. I have put these in quotes because this is not the same as the usual meaning of these terms. We are just simplifying a situation in which half the radiation is scattered by angles less than 90 degrees and half by angles greater than 90 degrees (you dont see a true reflected image of your face in a vessel of milk!). If we have radiation of intensity F_N going in the 'forward' (positive z) direction falling on layer number N, from the left, and B_N (B for backward) falling on the same layer from the right, we will be able to write down $B_{N-1} = (B_N + F_N)/2$ as illustrated in *Figure* 7.

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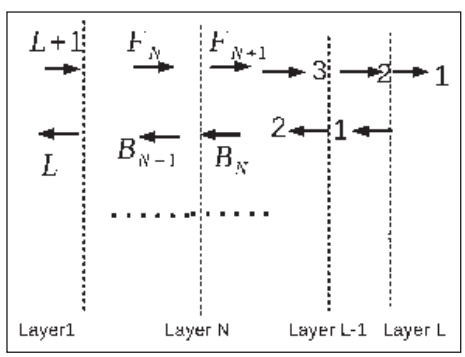

Figure 7. A simple one-dimensional model for the attenuation of a beam by scattering. The medium is divided into *L* layers. The *N*th layer has specific intensity F_N falling on it from the left (forward direction) and B_N from the right (backward direction). Each layer transmits half and reflects half of what falls on it, from either direction. The last two layers *L–1* and *L* are shown as well. The final outgoing intensity is taken to be 1. At the first layer, the incident intensity is *L+*1 and the reflected intensity is *L*. The dots stand for layers which are not shown.

In words, the radiation falling on layer $N-1$ from the right is made up of the reflected part of F_N and the transmitted part of B_N. At the last layer, with number L, we say that B_L (for last) is zero. There is no layer $L+1$ to send radiation back to layer L – we only have transmitted radiation of strength $F_{L+1} = F_L/2$. We can now solve the problem working backwards from the last layer. This is described in words below, and also depicted in *Figure* 7. For simplicity, let us put the transmitted intensity $F_{L+1} = 1$. We then get $F_L = 2, B_{L-1} = 1$, since incident intensity of 2 on layer L is needed to produce transmitted intensity of 1, and that automatically produces a reflected intensity of 1, which goes and falls on layer $L-1$. We can now figure out what is happening at layer $L-1$. Since 0.5 of $F_L = 2$ is accounted for by the reflected part of $B_{L-1} = 1$, the remaining 1.5 must be accounted for by transmission of F_{L-1}, which must therefore be equal to 3. Now, clearly, 1.5 of B_{L-2} is accounted for by reflection of $F_{L-1} = 3$, and 0.5 by transmission of $B_{L-1} = 1$, so $B_{L-2} = 2$.

The pattern is clear. The difference between the forward and backward intensities sitting between any two layers is 1. It should be, since this difference is the net current or flux of energy moving to the right. The forward and backward intensities both rise linearly as we traverse more layers starting from the exit at the L-th layer. We conclude that at the first layer, we have an incident intensity of $F_1 = L+1$, and a reflected intensity of $B_1 = L$. The transmission fraction of the entire stack is thus equal to $1/(L+1)$. Satisfyingly, this formula gives 1 when L=0 and 0.5 when L=1. Notice that the transmitted intensity *does not* fall off exponentially with thickness! The physical reason is that once intensity is removed from the forward direction by scattering, it can be sent forward again by another scattering. The more mathematical treatment, given in *Box* 1, shows that the transmission equals $1/(1+\tau_s)$ where τ_s is the optical

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depth for scattering.

One interesting consequence is that if we take two layers, each of thickness L, placed one after the other, the transmission is $1/(2L+1)$. This is *not* equal to the product of the transmission of each layer taken separately, which would have been $1/(L+1)^2$. Contrast this to the case of pure absorption, where it is true that $\exp(-2\tau) = \exp(-\tau)\ \exp(-\tau)$.

A cloud is a good example of a layer in which there is mainly scattering by droplets of water, though there would be some absorption. A cloud of optical depth 10 would send back 90% of the radiation, and transmit only 10%. Contrast this with an absorbing cloud of optical depth 10 which would transmit only $\exp(-10) \simeq 0.5 \times 10^{-4}$. We should be grateful for the reduction of light in a cloud being a reciprocal rather than exponential of the optical depth! We often see the bottom of clouds dark, even when the top is lit by the Sun.

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Random Walk and Escape Time Interpretation

A physical interpretation of the equation $\tau = \alpha l$ is that optical depth counts the number of 'mean free paths' of the radiation contained in the length l which the radiation has to traverse. We are interpreting the length $1/\alpha$ as a kind of 'mean free path' as in the kinetic theory of gases. In the case of pure absorption, we can say that each mean free path attenuates the beam by a factor of e, so the final attenuation is $\exp(-\tau)$. In the case of pure scattering, we can think of the radiation as undergoing a random walk with a step length of approximately $1/\alpha_s$. We use a result from the theory of random walks which has been extensively covered in *Resonance* (August 2005, page 49; December 2005, page 106; July 2009, page 638). If the number of steps, each of length $1/\alpha_s$ is S in random directions, then the distance travelled, again in a random direction, is typically of the order of $\sqrt{(S)}(1/\alpha_s)$. Equating this to the layer

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size R gives the number of steps needed to cross the layer. Without scattering, the distance travelled would have been $S(1/\alpha)$, in whatever direction the radiation started.

It is as if the speed of the radiation has been reduced by a factor $\sqrt{S} = R\alpha_s$, which is nothing but the optical depth. You are warned that this is an 'effective speed' which depends on the size R of the layer! If the radiation travelled straight out to the surface, the time taken would be R/c. But with scattering, it is reduced by a factor of τ. For the Sun, the repeated absorptions and re-emissions through the multiple layers give rise to an optical depth of about 10^{12} from the core of the Sun to the surface. So instead of taking $R/c = 2.33$ seconds to reach the surface, the time is approximately 2.33×10^{12}s, i.e., of the order of one lakh years.

More Applications of Radiative Transfer

We have mentioned the trapping of solar energy by the 'greenhouse gases' CO_2 and CH_4 as a crucial application of radiative transfer. Astrophysicists apply radiative transfer in constructing models of the outer layers of stars – the field of stellar atmospheres. This means accounting for thousands of lines in a cool star. The reward is a huge amount of information – the chemical abundances, the gravity at the surface (which determines how rapidly the density falls), temperature, chemical abundance, turbulence, magnetic field are still an incomplete list.

In a very hot star, many elements are ionised, so there may not be many lines to worry about. Now a new effect shows itself. Remember that a beam of radiation carries momentum as well as energy. When scattering of the outgoing radiation occurs, the average momentum of the photons decreases. This is because they carry outward momentum before the scattering, but are sent in all directions after. This momentum is transferred to

the atoms or free electrons which do the scattering. The outward force due to radiation pressure can drive material outwards – this is called a 'radiation-driven stellar wind'. Very hot stars can lose a good fraction of their mass during their lifetime in this way. Modeling this is a major challenge. The material is in motion, so in addition to the force, one has to account for the Doppler shift of wavelengths, and the cooling due to expansion. In fact, there is a limit to how much radiation can come out of a hot star before the outer layers get pushed off in spite of gravity. The most massive stars live close to this limit. The explosion which ends their lives again has an important role for radiation. Such problems call for both powerful computers and elaborate programmes to take into account all processes simultaneously. There is much more to this journey than the few simple steps taken in this article.

Address for Correspondence
Rajaram Nityananda
Azim Premji University
PES Institute of Technology Campus
Pixel Park, B Block
Electronics City, Hosur Road
(Beside NICE Road)
Bengaluru 560 100
Email: rajaram.nityananda@gmail.com

Suggested Reading

[1] Arnab Rai Choudhuri, *Astrophysics for Physicists*, Cambridge, 2010, Chapter on radiation.