Introduction

We have chosen extracts from two of Oort's most famous papers for the Classics section of this issue. The earlier one, published in 1927, is on determining the rotation of stars around the centre of our galaxy, and its variation with distance. The principle of this method is explained in some detail in the article, 'Discovering the Rotation of our own Galaxy' (this issue, p. 869). You can see that Oort just writes down the corresponding formulae in his paper. What is worth learning from this paper is the rather confused situation at that time, with many ideas floating around, proposed by famous astronomers like Kapetyn, Jeans, Lindblad and Shapley. Oort is meticulous in citing his sources, but he also quickly disposes of untenable ideas, though in rather mild language. This leads him to his model, of a differentially rotating disc embedded in a large spherically-symmetric and non-rotating system – pretty much the model today! What we have not reproduced is the detailed discussion which follows the observational material analysis of errors, and fitting of the model.

The 1932 paper looks in the perpendicular direction, and determines the nature of the restoring force which keeps the stars of the disc confined to a narrow layer. The style is worth noting – it begins with an 11-part summary of the basic argument and content! (One wishes papers today were written with so much consideration for the reader). The theory, based on first-year undergraduate dynamics and gravity, is disposed of in a short section 1. You can go straight to the summary of section 11 for the bottom line: a determination of the local density of the matter responsible for this vertical component of the gravitational force of the disc. It came out two and a half times the matter accounted for by the visible stars. This remained famous for decades as the 'Oort discrepancy'. Current work has improved the data and the general conclusion is that there is no need to postulate dark matter concentrated in the plane but the method used is fundamentally the same as in this classic paper.

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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Observational evidence confirming Lindblad's hypothesis of a rotation of the galactic system, by *J. H. Oort.*

1. *Introduction.*

It is well known that the motions of the globular clusters and RR Lyrae variables differ considerably from those of the brighter stars in our neighbourhood. The former give evidence of a systematic drift of some 200 or 300 *km/sec* with respect to the bright stars, while their peculiar velocity averages about 80 *km/sec* in one component, which is nearly six times higher than the average velocity of the bright stars.

Because the globular clusters and the bright stars seem to possess rather accurately the same plane of symmetry, we are easily led to the assumption that there exists a connection between the two. But what is the nature of the connection?

It is clear that we must not arrange the hypothetical universe in such a way that it is very far from dynamical equilibrium. Following KAPTEYN *) and JEANS **) let us for a moment suppose that the bulk of the stars are arranged in an ellipsoidal space whose dimensions are small compared to those of the system of globular clusters as outlined by SHAPLEY ***). From the observed motions of the stars we can then obtain an estimate of the gravitational force and of the velocity of escape. An arrangement as supposed by KAPTEYN and JEANS, which ensures a state of dynamical equilibrium for the bright stars, implies, however, that the velocities of the clusters and RR Lyrae variables are very much too high. A majority of these would be escaping from the system. As we do not notice the consequent velocity of recession it seems that this arrangement fails to represent the facts.

As a possible way out of the difficulty we might suppose ****) that the brighter stars around us are members of a local cloud which is moving at fairly high speed inside a larger galactic system, of dimensions comparable to those of the globular cluster system. We must then postulate the existence of a number of similar clouds, in order to provide a gravitational potential which is sufficiently large to keep the globular clusters from dispersing into space too rapidly. The argument that we cannot observe these large masses outside the Kapteyn-system is not at all conclusive against the supposition. There are indications that enough dark matter exists to blot out all galactic starclouds beyond the limits of the Kapteyn-system *).

LINDBLAD **) has recently put forward an extremely suggestive hypothesis, giving a beautiful explanation of the general character of the systematic motions of the stars of high velocity. He supposes that the greater galactic system as outlined above may be divided up into sub-systems, each of which is symmetrical around the axis of symmetry of the greater system and each of which is approximately in a state of dynamical equilibrium. The sub-systems rotate ***) around their common axis, but each one has a different speed of rotation. One of these sub-systems is defined by the globular clusters for instance; this one has a very low speed of rotation. The stars of low velocity observed in our neighbourhood form part of another sub-system. As the rotational velocity of the slow moving stars is about 300 *km/sec* and the average random velocity only 30 *km/sec*, these stars can be considered as moving very nearly in circular orbits around the centre.

We may now apply an analysis similar to that

*) *Astrophysical Journal*, 55, 302, 1922; *Mt Wilson Contr.* N°. 230.

**) *Monthly Notices R.A.S.*, 82, 122, 1922.

***) *Astrophysical Journal*, 48, 154; 49, 311 and 50, 107, 1919; *Mt Wilson Contributions* Nos 152, 157 and 161.

****) OORT, *Groningen Publications* N°. 40, pag. 63, 1926.

*) *Cf. Hemel en Dampkring*, Jan. and Feb. 1927.

) *Arkiv. f. Mat., Astr. o. Fysik*, Bd **19 A, Nos. 21, 27, 35 and Bd **19 B**, N°. 7 (*Upsala Meddelanden* Nos 3, 4, 6 and 13); also: *Vierteljahrschrift*, 61ter Jahrgang, p. 265.

***) Of course the rotation considered is not generally one of constant angular velocity throughout the sub-system. In the following comparisons between the speeds of rotation these speeds are taken for stars at the same distance from the axis.

used by JEANS in his discussion of the motions of the stars in a "Kapteyn-universe" *), the only difference being that in the present analysis we do not introduce a second system rotating in the opposite direction. Adopting some probable formula for the gravitational potential we can derive the rotational velocities for each of the sub-systems from our knowledge of the distribution of the peculiar velocities (defined as the velocities remaining after correction for the effects of rotation). The higher the average peculiar velocity in a certain sub-system the slower its rotation will be, and the less flattened it will appear in a direction perpendicular to the galactic plane. If we refer our motions to the centre of the slow moving stars in our neighbourhood, the members of a sub-system with higher internal velocities will appear to lag behind, and LINDBLAD has shown that in this way we can arrive at a connection between average peculiar velocity and systematic motion of the same form as that computed from observation. **)

LINDBLAD's hypothesis conforms beautifully with the well-established fact that the average direction of the systematic motion of the high velocity stars is perpendicular to the direction in which the globular clusters are concentrated (galactic longitude 325°, latitude 0°). At first sight it might be hard to imagine how such a mixture of sub-systems of different angular speeds could ever come into existence; but the *possibility* cannot be denied, as is apparent from a comparison with spiral nebulae. ***)

If somewhere there existed a rapidly rotating system of stars and by some cause the internal velocities in this star-system were increased, an asymmetry in the stellar motions would necessarily result in the long run. It must be admitted, however, that the part played by the globular clusters cannot be so easily understood.

The following paper is an attempt to verify in a direct way the fundamental hypothesis underlying LINDBLAD's theory, namely that of the rotation of the galactic system around a point near the centre of the system of globular clusters. In a subsequent paper I hope to be able to make a more detailed comparison of the theory with the observational facts concerning the stars of high velocity collected in *Groningen Publications* N°. 40.

2. *Theoretical effects of the rotation.*

In the present discussion I shall altogether disregard the idea of a number of separate galactic clouds and take into consideration only the forces arising from the greater galactic system as a whole. The gravitational force, K, is consequently directed to the centre of this system and is only a function of the distance, R, from this centre.

Let us now consider a group of stars at a distance r from the sun and let us suppose that r/R is so small that all terms of second or higher order in r/R can be neglected, then it is easily seen that the residual velocity caused by the rotation is equal to

$$rA \sin 2(l - l_0)$$

in radial direction, and to

$$rA \cos 2(l - l_0) + rB$$

in transverse direction, if l_0 represents the galactic longitude of the centre (about 325°), l the longitude of the stars considered, R the distance of the sun from the centre,

$$V = \sqrt{RK}$$

the circular velocity near the sun,

$$A = \frac{V}{4R}\left(1 - \frac{R}{K}\frac{\partial K}{\partial R}\right)$$

and

$$B = A - \frac{V}{R} \quad .$$

The rotation is supposed to take place in right-hand direction as observed from a point North of the galactic plane.

If, as LINDBLAD tentatively supposed, the principal part of the greater galactic system is formed by an ellipsoid of constant density, the force K will be proportional to R. In this case

$$A = 0 \text{ and } B = -\frac{V}{R} \, ,$$

the system rotates as a solid body and we shall not find any indications of rotation in the radial velocities, but the proper motions in galactic longitude should be systematically negative for stars in all longitudes.

As another extreme case we might suppose that the whole mass is concentrated in the centre and that K is inversely proportional to the square of R. We get

$$A = +\frac{3}{4}\frac{V}{R} \qquad B = -\frac{1}{4}\frac{V}{R}.$$

We shall see below that observations seem to prove that the second alternative is nearly correct. We shall then have to expect a systematic effect in the radial velocities showing maxima at 10° and 190°

*) *Monthly Notices R. A. S.*, 82, 122.
**) STRÖMBERG, *Astrophysical Journal*, 59, 228, 1924; *Mt Wilson Contr.* N°. 275.
***) LINDBLAD, *Upsala Meddel.* N°. 13.

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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

The force exerted by the stellar system in the direction perpendicular to the galactic plane and some related problems, by *J. H. Oort.*

Notations.

z	distance from the galactic plane,
Z	velocity component perpendicular to the galactic plane,
Z_o	the value of Z for $z = 0$,
l	modulus of a Gaussian component of the distribution of Z (formula (5), p. 253),
$K(z)$	the acceleration in the direction of z,
Δ	the star-density,
ρ	the distance of a star from the sun,
$\Phi(M)$	the number of stars per cubic parsec between $M - \frac{1}{2}$ and $M + \frac{1}{2}$,
$A(m)$	the number of stars per square degree between $m - \frac{1}{2}$ and $m + \frac{1}{2}$,
b	galactic latitude,
ϖ	distance to the axis of rotation of the galactic system,
δ	$\partial \log \Delta / \partial \varpi$.

Summary of the different sections.

1 and **2.** In these sections a short discussion is given of KAPTEYN's previous investigation on the subject and of the reasons why the problem has been treated anew. In the second section the formulae are given which show the connection between $K(z)$, $\Delta(z)$ and the velocity distribution (formulae (5) and (6)).

3. The distribution of Z and its dependence upon spectral type and visual and photographic absolute magnitude is studied in some detail. The adopted results are in Tables 7 (spectral types), 9 (visual absolute magnitudes) and 11 (photographic absolute magnitudes). The average velocities of giants and dwarfs of the same spectrum appear to be practically identical in the z direction. On account of their irregular distribution the B0 — B9 stars have been excluded in forming the velocity laws for the different groups of absolute magnitude.

It is shown that stars at various distances north and south of the galactic plane indicate no signs of systematic motions in the z-direction (Table 12).

4. From VAN RHIJN's tables in *Groningen Publication* No. 38 the density distribution $\Delta(z)$ has been computed for four intervals of visual absolute magnitude (Table 13 and Figure 1). Figures 2 and 3 show $\log \Delta(z)$ for A stars and yellow giants, as derived by LINDBLAD and PETERSSON.

5. With the aid of the data contained in the two preceding sections I have computed the acceleration $K(z)$ between $z = 0$ and $z = 600$. The computations were made by successive approximations; the B stars were eliminated first. The results are in Table 14 and Figure 4, $K'(z)$ giving the values finally adopted. The good agreement between the practically independent values of $K(z)$ derived from the separate absolute magnitude groups is a strong argument in favour of the approximate correctness of the data up to $z = 400$. The result may be summarized by stating that the absolute value of $K(z)$ increases proportionally with z from $z = 0$ to $z = 200$; between $z = 200$ and $z = 500$ it remains practically constant and equal to $3{\cdot}8.10^{-9}$ cm/sec^2.

6. In this section the different spectral classes are investigated separately. A comparison of numbers computed with the aid of $K(z)$, with direct counts in high galactic latitude revealed a great discrepancy for the K stars, probably due to an error in the adopted luminosity law (compare *B. A. N.* No. 239). A slight correction to the average velocity of the A stars was also indicated. Both corrections have been applied throughout the greater part of the present investigation.

For comparison with future observations of fainter stars the computed numbers of each spectral type and visual apparent magnitude are given in Table 17, for 20°, 40° and 80° galactic latitude. The table also shows the relative numbers of giants and dwarfs to be expected for each magnitude. Finally, Table 18 shows the corresponding average colour indices and the mean square deviations from the average. No great accuracy can be claimed for these values.

7. From the best sources available mean values of $\log A(m)$ were computed for visual as well as photographic magnitudes and for latitudes ± 50° and ± 80°. The results, referring to stars later than B9, are in Figures 5 and 6. Table 21 gives the corrections necessary to reduce the visual counts to SEARES' photovisual scale.

8. As a check on the computations of the 5th section numbers of stars of each visual magnitude were computed with the aid of the acceleration $K(z)$ derived in that section. The necessary formulae are shown on page 263 (formulae (10) and (11)). As was to be expected the agreement with the counts according to the Harvard scale is perfect (Table 23). For SEARES' photovisual scale the accordance is not as good. This would indicate a somewhat steeper rise of $K(z)$ between $z = 0$ and $z = 150$, but even then the agreement cannot be made very satisfactory. The difference may be due to inaccuracies of $\Phi(M)$.

With the aid of the corrected K star luminosity law new photographic and visual luminosity laws for all spectra together (except B stars) were computed and tabulated in the 2nd and 4th columns of Table 22.

For the computation of $A\ (m_{pg})$ for fainter magnitudes we need an extrapolation of $K\ (z)$ to heights of about 5000 ps. Two different extrapolations were tried, based on different assumptions as to the main attracting mass of the galactic system. These have been denoted by K_a and K_b (Figure 7). K_a is very probably nearest to the truth and has been adopted for the computations in the last two sections (the first part being taken in accordance with $K'(z)$ in Table 14). The comparison with the observed numbers of faint stars (shown in Table 25) is difficult for two reasons. Firstly there are indications of a considerable error in the photographic luminosity law and secondly our knowledge of the frequency and distribution of high velocities appears to be altogether insufficient. A more extensive knowledge of the latter is indispensable if we want to obtain a somewhat trustworthy picture of the density distribution and of the luminosity law at heights above 1000 or 1500 ps. In order to illustrate this the computations of $A\ (m)$ in Table 25 were made with four different assumptions as to the distribution of high velocities.

A general impression of the distribution of stars at 80° latitude and of photographic magnitudes 11, 14, 17 and 18 over different absolute magnitudes and distances may be obtained from Figures 8 and 9. The 18th magnitude stars appear to be mostly high velocity dwarfs of K and M types. It may be noted that formulae (10) and (11) (p. 263) may be used to compute the approximate distribution in distance for stars of any magnitude and in any region of the sky above 15° latitude if the density gradients derived in the 9th section are taken into account.

9. In Table 26 and Figure 10 a comparison is given between mean counts of stars at different latitudes with those computed on the assumption that the layers of equal density are parallel to the galactic plane. The close parallelism of the observed and computed values indicates that the influence of absorption or of an eventual local system only becomes sensible below 15° latitude. The average absorption at 20° latitude can hardly be larger than $0^{m}\cdot3$. At latitudes above 20° this unknown which renders density determinations in the galactic plane almost impossible appears to vanish. The present result seems in remarkable contrast with the average absorption of more than a magnitude indicated by the extragalactic nebulae in this zone.

The star counts between 20° and the poles have now been analysed according to galactic longitude. There is distinct evidence of a density increase in the direction of the centre of the large system (Figure 11) and it is shown that very satisfactory determinations of the density gradient can be obtained (Table 28). The density gradient found is of the same order as that found theoretically from the distribution of high velocities (*B. A. N.* No. 159). Some data are also given for the change of this gradient with z and ϖ. Star counts between 30° and 60° latitude appear to be best suited for determining the *regular* features of the galactic system (compare Figure 11).

The equidensity surfaces computed with these empirical numbers make angles of about 9° with the galactic plane. For $\Delta = \cdot04$ and $\Delta = \cdot01$ they are shown as dots and crosses in Figure 12. They can be closely represented by ellipsoids symmetrical about the axis of the great galactic system and with axial ratios of 9·3 and 6·4 respectively. It should be noted that only part of the *outer* envelopes of the system (beyond $z = 500$) have been derived in this way.

10. In this section the density distribution in the direction perpendicular to the galactic plane has been computed for different types of stars as well as for the integrated light and the mass (Table 29). Half of the photographic light is situated between $z = -166$ and $z = +166$ ps. From a comparison of this number with the distance from the sun to the centre (which is of the order of 10000 ps) we may infer that the denser part of the galactic system must have an exceedingly flat shape. Because of this extreme flatness it seems impossible to investigate the density distribution in the galactic plane, even from areas at very low latitude, without taking into account the decrease of star density with z.

I have also computed the change of the luminosity law with z, which change is very considerable (Table 30; B stars are included).

Table 31 shows average distances from the galactic plane corresponding to different average velocities, assuming Gaussian distributions of the latter. Some examples are added to show how these can be utilized in computing average parallaxes of distant stars or average velocities perpendicular to the galactic plane. A remarkable difference of about 8 times between the average radial velocity and the average value of $|Z|$ is noted for the O stars.

11. It is found that the total density of matter near the sun is equal to $6{\cdot}3.10^{-24}$ g/cm³ or ·092 solar masses per cubic parsec. The observed total mass of the stars down to + 13·5 visual absolute magnitude is found to be ·038 solar masses per ps³ (Table 34). It is probable that this value would still be greatly increased if we could have taken the next 5 absolute magnitudes into account, so that the total mass of meteors and nebular material is probably small in comparison with that of the stars. There is an indication that the invisible mass is more strongly concentrated to the galactic plane than that of the visible stars (Table 33).

Integrating over a column perpendicular to the galactic plane I find that an average unit of photographic light corresponds to a mass of 1·8 (if both are expressed in the sun as unit), approximately agreeing with the proportion found in the central region of the Andromeda nebula, the only available case where a comparison is possible.

In a note on VAN MAANEN's star the probability of a large mass for this star has been discussed.

1. *Introductory; previous investigations.*

It cannot be doubted that in many respects the galactic system is very irregular. It is important to find out whatever regularities there are and to utilize these to obtain a deeper knowledge of the general characteristics of the system.

To some extent the distribution of the later type stars in a direction perpendicular to the galactic plane may be classed among the regular features, as is evident, for example, from the great resemblance between the distribution on both sides of the galactic plane. It is evident, also, from the results of the present article in which it will be shown that the assumption of a well-mixed condition, together with a knowledge of the force, $K(z)$, exerted in this direction by the stellar system, enables us to represent all known features about stellar distribution perpendicular to the galactic plane.

There is good reason to suppose that the gravitational force is approximately of such a size as to keep the stars together during times which are as short as 10^8 or 10^9 years. It would seem unreasonable to assert that the extreme flatness of the galactic system is a peculiarity of this particular moment and that, after the course of 10^8 years, it would have undergone such a radical change in appearance as, say, a ten-fold increase of its thickness. The steady state hypothesis is more directly supported by the fact that the stars at different distances on both sides of the galactic plane do not show an observable systematic motion towards or away from the galactic plane (compare Table 12).

The supposition that $K(z)$ is such that it will approximately maintain the present density distribution is the working hypothesis of the following investigation, in which I have tried to derive the magnitude of this acceleration at different distances from the galactic plane from a comparison of velocity and density distributions in this direction without making any supposition about the density distribution *in* this plane. Earlier investigations, mainly by JEANS [1]), have proved that a rigorous application of the steady state conditions lead to the result that the distribution of the velocity components perpendicular to the galactic plane would ultimately be the same as that of the components directed to the axis of rotation. However, the steady state implied by this condition seems to be of a quite different class from the simple state of being well-mixed which is presupposed for the present investigation. It seems indeed probable that in a system like the one we are investigating the average peculiar motions of the stars in directions perpendicular and parallel to the galactic plane will, at least for small motions, remain practically independent of each other during times comparable with the time of development of the galactic system. The empirical fact that for most types of stars the average velocity components perpendicular to the galactic plane are considerably different from those directed towards the centre of the system points in the same direction.

A second aim has been to derive new values for the density distribution perpendicular to the galactic plane for various absolute magnitudes, as a good knowledge of this function is a prerequisite for all investigations of star-density, also at low latitudes. With the aid of these densities the density arrangement of the great galactic system in the entire region between the poles and 20° latitude has been investigated.

A third purpose was the derivation of an accurate value for the total amount of mass, including dark matter, corresponding to a unit of luminosity in the surroundings of the sun.

So far as I know the first significant numerical investigation along these lines is contained in KAPTEYN's well-known "First attempt at a theory of the arrangement and motion of the sidereal system" [2]). His formula

[1]) *Problems of Cosmogony*, Chapter X; *Astronomy and Cosmogony*, Chapter XIV.

[2]) *Astrophysical Journal*, **55**, 302, 1922; *Mt. Wilson Contr.* No. 230.