Discovering the Rotation of our own Galaxy

The Astronomer as Detective

Rajaram Nityananda

In the early twentieth century, the Dutch astronomer, Jan Oort, made brilliant use of the two most basic measurements of the motions of nearby stars. These are motions away or towards the Sun based on the Doppler effect, and motion perpendicular to the line of sight, measured as a slow change in the position of the star on the sky. He was able to draw far-reaching conclusions about the rotation and the mass of the Milky Way galaxy in which our Sun is located. His arguments and calculations were based on a simple dynamical model. This model is introduced and described in this article. It can serve as an excellent exercise in an undergraduate physics course, illustrating both mechanical principles and basic astronomy.

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Introduction

Today, a great deal is known about the collection of stars of which our Sun is a member. We call this the Milky Way galaxy. This is an appropriate name for the faint fuzzy band of light which crosses the sky and was well-known to all ancient peoples. We are not so fortunate today. Our city lights and air pollution mean that many people have not seen it at all, except in pictures like *Figure* 1 below.

Mapping the Galaxy

Galileo turned his telescope to the Milky Way and realised that this was a band of stars, which looked fuzzy only because human eyes did not see them separately. By the early twentieth century, the Dutch astronomer

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Figure 1. The Milky Way galaxy as seen in Earth's sky. Photos covering the complete celestial sphere have been stitched and transformed to a panoramic image with the Milky Way as central line. The initial photos have been mostly taken from ESO observatories at La Silla and Paranal in Chile. The final panoramic image condenses 120 hours of observations, spread over several weeks.
(Image courtesy: https://commons.wikimedia.org/wiki/File:ESO_-_Milky_Way.jpg)

Kapetyn had built a quantitative model of a flattened collection of stars (somewhat like an idli) with the Sun at the centre. This was based on the observed distribution of stars in the sky as well as their apparent brightness, giving an idea of distance. But quite soon, by studying stars well away from the Milky Way, evidence arose that all was not well with this model.

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Spectroscopy [1] had given astronomers of the late nineteenth and early twentieth century a powerful tool – the Doppler effect. This is a basic property of light waves (see also the article on Doppler effect in this issue, on p.931). When the observer moves away from the source of the waves, the distance between source and observer increases. We say that the relative velocity has a radial component. This makes the observed wavelength longer, and frequency lower, than that measured at the source. Astronomers call this red-shift because a wavelength (say green at 540 nm) is shifted towards the red (say 600 nm). This terminology is not always accurate, but still used – 800 nm would be shifted to longer wavelengths, further away from 600 nm but it is still called red-shift! When the relative motion of source and observer reduces the distance between them, we see a shorter wavelength, and this is called blue-shift. The convention is that the radial velocity, denoted by v_r, is taken positive for red-shift (recession of the source) and negative for blue-shift (approach of the source).

Shapley, in the United States, carried out measurement of the radial velocities with respect to the Sun of distant stars, and clusters of stars, away from the plane of the Milky Way. He showed that there were systematic red-shifts in one half of the sky and blue-shifts in the opposite side. Further, measuring their distances by various methods made it possible to plot the three-dimensional distribution. These stars were concentrated towards one direction in the sky – the Sun was clearly not at the centre of this distribution. The centre was found to be in a direction perpendicular to this velocity (*Figure* 2). By 1920, Shapley reached the conclusion that the Sun was not at the centre of the galaxy, but was moving around a centre in the constellation of Sagittarius. He estimated the centre to be about 50,000 light years away. This was based on the assumption that on an average, the distant stars and clusters that he used were not in rotation and symmetric around the true centre of the galaxy. They could therefore be used to define a frame of reference for the Sun's motion and location with respect to the centre.

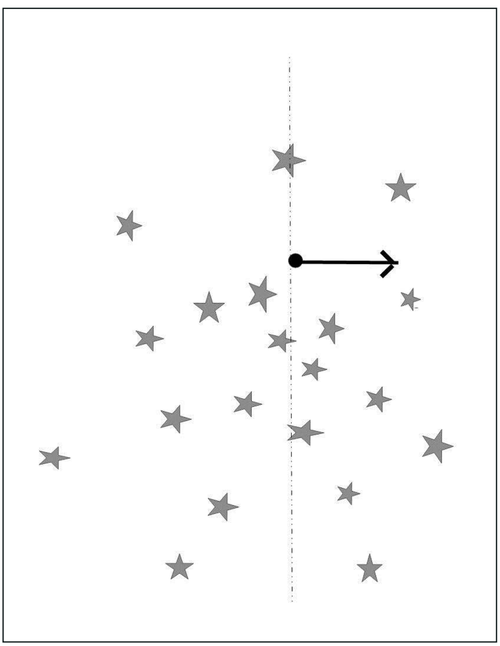

Figure 2. Shapley studied a population of objects called globular clusters, shown as five pointed coloured stars in the figure. These could be recognised and observed to great distances. Although the figure is in a plane, these were mostly above and below the plane of the Milky Way so that the view was not blocked by absorption in the plane of the Milky Way. The sun is the black dot. The vertical arrow shows the velocity vector with respect to the centre proposed by Shapley. This would explain why (a) Many more of these stars were seen on one side of the Sun (bottom of the figure) than the other and (b) The stars to the right of the dashed line showed a systematic blue-shift, on the average and those to the left a systematic red-shift.

Modeling the Rotation of our Galaxy

In 1927, Oort investigated the rotation of the Galaxy quantitatively, with a dynamical model. The opening section of this great paper is reproduced in the Classics section on p.945 in this issue of *Resonance*. He chose to analyse the motions of stars in the plane of the Milky Way less than a few thousand light years away from us. His model was that these stars, along with the Sun, were in nearly circular orbits around the faraway centre. Further the angular velocity Ω was different at different radii. In adopting this model, he was guided by the earlier work of B Lindblad in Sweden.

Given such a model, one can work out what the radial velocities of stars near the Sun would be, when viewed from a moving platform, i.e., the Sun itself. In addition to the Doppler effect, Oort used information about the transverse component of the relative velocity between the Sun and a star. This refers to the component perpendicular to the line joining them. (*Figure* 3.)

This transverse relative velocity denoted by v_t results in a change in the direction of the line joining observer and source. This can be measured using decades of repeated observations. Astronomers call this 'proper motion'. This term is used to distinguish it from effects like rising and setting, which come from the spin of the earth, or the annual parallax, which is a change in direction caused by the Earth's yearly motion around the Sun.

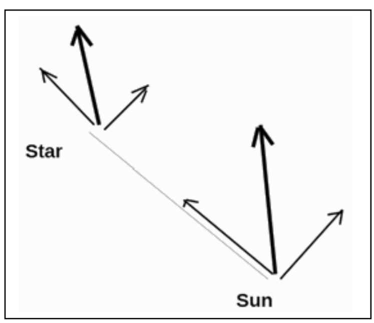

Figure 3. The geometry of radial velocity and proper motion. The thick arrows show the velocity vectors of the Sun and a star being observed from earth. The thinner arrows show the radial and transverse components of the velocities of the two objects. The difference between the two radial components would give rise to a Doppler shift in the spectrum of the star as seen from Earth. The difference in the transverse velocities, divided by the distance, would give the angular velocity of the line joining them, whch is called proper motion, usually measured in arcseconds per year. Note that anticlockwise motion is regarded as positive, even though the angular velocities shown are clockwise.

The geometry of the model is sketched in *Figure* A (see *Box* 1).

Box 1. Radial Velocity and Proper Motion as a Function of the Position of a Star Relative to the Sun

Figure A shows a star located at polar co-ordinates r, θ with respect to the Sun. The radius of the Sun's orbit is R and that of the star's orbit is $R + \delta R$. It is clear from the figure that $\delta R = r\cos(\theta)$. The angle α between the radii joining the Sun and the star to the center of the galaxy is, approximately, $\alpha = r\sin(\theta)/R$. The velocity vector of the star makes an angle α with the x-axis and has length $v + \delta v$ where $\delta v = (\mathrm{d}v/\mathrm{d}R)\delta R = (\mathrm{d}v/\mathrm{d}R) r\cos(\theta)$. We are now in a position to calculate the x and y components of the velocity of the star relative to the Sun. This is shown in the lower left of *Figure* A. (We have taken the cosine of the small angle α to be unity, so the x component is simply δv). We now take the dot product of this relative velocity $(\delta v, v\alpha)$ with the line of sight unit vector $(-\sin(\theta), \cos(\theta))$. Substituting for $\delta v, \alpha$ this gives the radial velocity, in the form

$$v_r = -\left(\frac{\mathrm{d}v}{\mathrm{d}R} + \frac{v}{R}\right) r\sin(\theta)\cos(\theta) \equiv Ar\sin(2\theta).$$

The Oort constant A is defined by the last equality, to be

$$A = \frac{1}{2}\left(-\frac{\mathrm{d}v}{\mathrm{d}R} + \frac{v}{R}\right) \equiv -R\frac{\mathrm{d}\Omega}{\mathrm{d}R},$$

where the second form, in terms of angular velocity Ω, is easily checked using $v = R\Omega$.

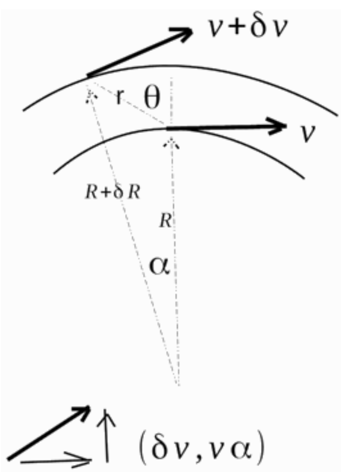

Figure A. The geometry of the simple model of galactic rotation adopted by Oort for the neighbourhood of the Sun. The velocity *v* of the Sun is taken in the positive *x* direction, the radius of its orbit is *R,* and the direction to the centre of the orbit in the negative *y* direction. A star is on a different orbit, at a radius $R + \delta R$, and its speed in its circular orbit is $v + \delta v$. The distance of the star is denoted by *r* and the angle made by the direction of the star to the outwards radial direction at the sun is denoted by θ. The angle made at the centre by the radii to the Sun and to the star is denoted by α. The bottom left of the figure shows the velocity vector of the star relative to the Sun.

Box 1. Continued...

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Coming to proper motion, we now need the component of the relative velocity transverse to the line of sight (Sun-star line). So, we now take the dot product of the relative velocity with the unit vector $(-\cos(\theta), -\sin(\theta))$ in the direction of increasing θ. The resulting formula for the transverse relative velocity is

$$v_t \equiv -r\left(\frac{\mathrm{d}v}{\mathrm{d}R}\cos^2(\theta) + \frac{v\sin^2(\theta)}{R}\right) \equiv \frac{-r}{2}\left(\frac{\mathrm{d}v}{\mathrm{d}r} + \frac{v}{R}\right) + \frac{1}{2}\left(\frac{\mathrm{d}v}{\mathrm{d}R} - \frac{v}{R}\right)\cos(2\theta).$$

This is usually expressed in terms of the proper motion, v_t/r and a second Oort constant B. Proper motion $= B + A\cos(2\theta)$ where A is our old friend and

$$B = -\frac{1}{2}\left(\frac{\mathrm{d}v}{\mathrm{d}R} + \frac{v}{R}\right) \equiv -\frac{1}{2}\left(2\Omega + R\frac{\mathrm{d}\Omega}{\mathrm{d}R}\right),$$

after using $v = R\Omega$.

We see that $A - B = v/R = \Omega$, the angular velocity of the Sun around the Galactic Centre. *Figures* 4a–4c give some geometric intuition for the two formulae.

What kind of pattern of radial velocities and proper motions is predicted by this model? The qualitative picture is as follows. The proper motion has a constant part, independent of the direction in which we look away from the Sun. But it has a part which goes through two maxima and two minima as we move in a full circle, starting by looking towards the centre of the galaxy. The extreme positive values are at $\theta = 0$ and 180^{o} and the extreme negative values are at 90^{o} and 270^{o}. The radial velocity also goes through two cycles of variation. But now the maxima are at 45^{o} and 225^{o} and the minima at 135^{o} and 315^{o}. It is an exercise in circular motion, trigonometry, and vectors, all topics which are taught towards the end of high school. *Box* 1 gives the mathematical statement of the problem and the straightforward solution.

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Box 1 and *Figure* 4 give us both analytical and geometric understanding of the two basic results for dependence of the two measured quantities on the distance r and direction θ of a star, as observed from the Sun. These are:

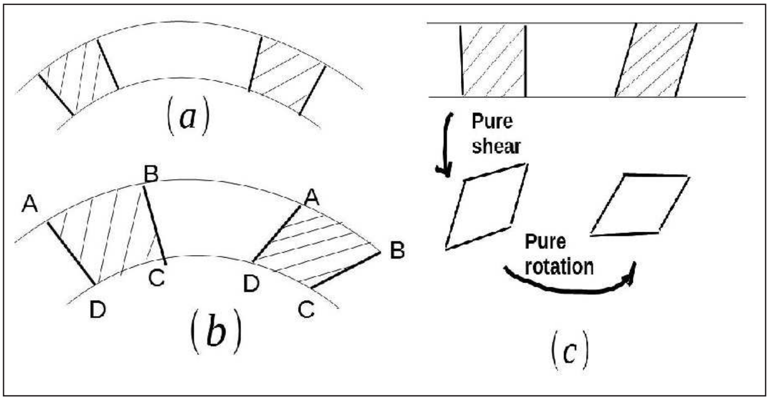

Figure 4. (**a**) If the angular velocity Ω was constant, then a given region of the galaxy would retain the same shape and only change in orientation. In this case, there would be no change in the distance between the Sun and any star. Hence there would be no Doppler shift. However, there would be proper motion since the direction of the Sun-star line would rotate at Ω. This contributes a term independent of θ to the proper motion.

(**b**) The effect of 'differential rotation'. This term refers to the variation of the angular velocity with $d\Omega/dR$ radius. The region ABCD now changes in shape as well as in orientation. The diagonal AC has shrunk and the diagonal BD has expanded. This means that differential rotation causes a Doppler red-shift which is maximum at $\theta = \pi/4$, $5\pi/4$, (the AC diagonal) and a blue-shift which is maximum at $\theta = 3\pi/4$, $7\pi/4$ (the BD diagonal). This is consistent with the general formula $\sin(2\theta)$ derived in *Box* 1.

(**c**) Illustrating the effect of differential rotation on proper motion. The pure rotation of *Figure* 4a has been removed by straightening the orbits. The transformation of the shape occuring in *Figure* 4b is broken up into two steps (i) A pure shear, i.e., stretch along one diagonal and compression along the other. Note that this actually turns the base of the rectangle. Examining this figure shows that this contributes a term proportional to $\cos(2\theta)$ to the transverse velocity, and hence to the proper motion, as shown in *Box* 1. This effect is zero where the radial velocity is maximum, and maximum where the radial velocity is zero, (ii) A pure rigid rotation which brings the base back parallel to the orbit.

Radial velocity: $v_r = Ar\sin(2\theta)$

Proper motion = Transverse velocity / $r = B + A\cos(2\theta)$

The two constants A and B are related to the variation of the orbital velocity, or the angular velocity, with distance from the centre of the Galaxy. We have

$$A = \frac{(v/R - \mathrm{d}v/\mathrm{d}R)}{2}, B = \frac{-(v/R + \mathrm{d}v/\mathrm{d}R)}{2}.$$

Expressions in terms of the angular velocity and its radial derivative $\Omega, R\mathrm{d}\Omega/\mathrm{d}R$ are given in *Box* 1.

One can carry out simple consistency checks on the formulae. For example, at $\theta = 0$, we expect no radial velocity and also expect the proper motion to depend only on $\mathrm{d}v/\mathrm{d}R$. Indeed, v cancels when we form $A+B$. Likewise, we get maximum radial velocity where $sin(2\theta)$ is a maximum, along the diagonals of the rectangle. A final point of convention – astronomers usually use polar co-ordinates with a zero pointing to the galactic centre. Our angle differs by π but since we always double the angle, it only adds 2π and makes no difference to the sines and cosines.

Fitting the Model to the Observations

Since there are only two unknowns, A and B, it would seem that the measurement of the radial velocity and proper motion of just one star with known distance r and direction θ would do the job! In fact, Oort's task was just beginning with the derivation of these formulae. First of all, how are we sure that the Sun is on a circular orbit? In fact, it is not. It would be more accurate to make a statistical statement. We expect the average position of a large enough group of nearby stars surrounding the Sun to move in a circle around the centre of the galaxy. Studies of the radial velocities of these very-nearby stars showed a random component but also a systematic component. This could be explained by a velocity vector of the Sun of 20 km/s in a specific direction, with respect to the average of the group. So this velocity had to be used to correct all measurements of Doppler effect and proper motion. The goal is to refer the motions of other stars to a 'local standard of rest' (LSR) and not to the Sun which is only one wayward member of the neighbourhood. 'Rest' is a misnomer – the LSR is actually moving around the centre of the galaxy as Shapley had already determined! Of

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course, the velocity vector of the Earth around the Sun also contributes to the measured Doppler shifts – this is well-known and easily corrected.

Because of random velocities, the formulae derived only apply to an average of a group of stars at a given r and θ. Further, one should sample a range of these two variables to make sure that the formulae really describe the observations. Painstaking sifting of a large volume of data, removal of systematic effects, averaging of random effects by using large numbers, and validation of the underlying model, are the bread and butter of observational astronomy. Oort was heir to the great Dutch tradition in this field, and passed it on to later generations. The Classics section gives some feel for this style.

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What are the units in which we should measure these constants? From the defining equations, it is clear that they have units of velocity/distance, or angular velocity (from the proper motion equation) or simply, 1/second.

But astronomers like to be different. The appropriate scale for velocities is kilometres per second, (km/s) and the scale for distance is a kiloparsec (One kpc, equals 1000 parsecs, approximately 3260 light years, or 3.1×10^{19} m).

So following this tradition, Oort's values were 30 km/s/kpc for A and -10 km/s/kpc for B, with rather large uncertainties. This unit equals $3.2 \times 10^{-17} s^{-1}$. To get a feel for what this number means, recollect from *Box* 1 that $A - B = \Omega$, the angular velocity of the Sun around the centre. This gives $\Omega = 1.28 \times 10^{-17}$. The orbital period $T = 2\pi/\Omega = 155$ million years (the modern measurement is 228 million years). Modern values of the Oort constants differ considerably from his early estimates. Based on measurements made from the *Hipparcos* satellite built specifically to explore parallax and proper motion, the currently accepted values are $A = 14.8, B = -12.4$ in the same units, km/s/kpc. Notice that $A + B$ is nearly

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zero, which implies that the gradient of the velocity, $(\mathrm{d}v/\mathrm{d}R)$ is also close to zero. This fact has a deep dynamical significance, which has been discussed elsewhere in *Resonance* [2].

The Modern View of Galactic Rotation

Much later work – the pioneer being Vera Rubin in the United States – from the late 1970s onwards showed that in fact $(\mathrm{d}v/\mathrm{d}R)$ is close to zero over a large range of R for most galaxies which have flattened discs, like our Milky Way! This is quite different from the solar system in which the orbital speeds of the planets fall off inversely proportional to the square root of the distance. Since the circumference is proportional to the orbital radius, these two factors conspire to make the period proportional to the 3/2th power of the radius – Kepler's law. But in galaxies, Kepler's law is overthrown, simply because, unlike the solar system, the mass is not concentrated at the centre. In fact, a simple calculation, again based on elementary circular motion, tells us that the inward force, which Oort denotes by K, equals v^2/R. A constant v then means that K varies as $1/R$, instead of $1/R^2$ for a concentrated mass which is the case in the solar system. This can be explained if the mass within the radius R itself grows proportional to R. However, the stars are not distributed in this way – their mass is strongly concentrated towards the centre. Astronomers were forced to accept that there was another form of matter, invisible to all their telescopes, which dominated the outer parts of most galaxies – 'dark matter'. The best estimates say that this exceeds our normal form of matter – made of protons, neutrons, and electrons – by a factor of around 5. Its nature remains mysterious but particle physicists are hoping that the LHC will discover a new kind of weakly interacting particle which could be the source of dark matter. Another mystery which may be harder to solve is why the discovery of a new form of matter has not yet won a Nobel Prize!

It is fascinating that the motions of stars in the plane of our galaxy, within a small neighbourhood of the Sun were able to reveal so much to the right detective. Our Classics section also includes the first three pages of Oort's 1932 paper, which again used the simple physics of oscillation of stars perpendicular to the plane of the galaxy, to infer the mass density in the neighbourhood of the Sun. Oort raised the question of dark matter in this paper! But that is another story in itself.

Suggested Reading

[1] J C Bhattacharyya, Astonomical Spectroscopy, *Resonance,* Vol.3, No.5 and No.6, 1998.

[2] Bikram Phookun and Biman Nath, Dark matter – what you see ain't what you got, *Resonance,* Vol.4, No.9, 1999.

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