Review

# BEAUTIFUL, SIMPLE, EXACT, CRAZY

## By Apoorva Khare and Anna Lachowska

*Reviewed By Jishnu Biswas and Rema Krishnaswamy*

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This is a review of the book "Beautiful, Simple, Exact, Crazy" written by Apoorva Khare and Anna Lachowska. The authors write in the preface that this book arose out of an introductory course called *Mathematics in the Real World* which they co-designed (and taught at Stanford and Yale University, respectively). The target audience of that course consisted mainly of undergraduates of humanities and social sciences – students whose principal interests lay outside of mathematics and the sciences. The preface explains the choice of topics for the course and the book: the mathematics should be simple to explain, and it should "generate a wide range of practical, amusing or impressive real world applications."

Guided by these principles, the authors have put together a dazzling collection of mathematical ideas and their uses in the real (and abstract) world, and they have achieved this assuming very little (and in particular, without using any calculus). Not only is the mathematics interesting and inspiring but the style in which it is presented – with connections in almost every page to literature, history, art, music, physics, biology, medicine, geology, archaeology and many other subjects – is absolutely compelling. We feel that the material in the book will be of interest not only to the intended audience but to anybody who has even a passing interest in mathematics, from high school students to mathematicians and mathematics teachers.

*Keywords: Fibonacci numbers, Golden ratio, Phyllotaxis, Mathematics in the Real World, Monty Hall problem, Seven Messengers.*

After reading the book, we also did a search on the two writers. One of them, Apoorva Khare, is now a faculty member of the Indian Institute of Science, Bangalore. The other, Anna Lachowska, is now at the Ecole Polytechnique Federale de Lausanne in Switzerland (her homepage – the link [AL] given below – contains some more instances of mathematics applied to the real world). One of us (RK) also used this book as one of the sources to teach a pre-calculus course for first year undergraduate students at the Azim Premji University (during July-November, 2017).

Here is a brief discussion of the topics covered in the book.

The initial chapters deal with applications of the Newtonian laws of motion, rational and irrational numbers, the notions of simple and compound interests. Logarithms are introduced and real life situations where they arise are discussed: the brightness levels of stars, the Richter scale of earthquakes, the decibel level which measures the noise level of sound and the equal temperament scale in music. The exponential growth and decay models are defined, and the reader is introduced to several examples like population growth models, the Fibonacci sequence, the process of radioactive decay and radiocarbon dating (which leads to a very interesting account of the Voynich manuscript). Finite geometric series are used to calculate mortgage rates. Infinite geometric series lead to fractals and their dimensions and their uses (a very recent and interesting application of fractals to archaeology is discussed). Modular arithmetic is introduced with many examples (the final one is the Diffie-Hoffman public key exchange) and an interesting historical discussion on the modern (Gregorian) calendar, and the earlier (Julien) calendar. The last part of the book deals with counting and some probability (many applications of Bayes law) and statistics (how to analyse data and fit curves using the least squares method).

Here are a few of our favorite examples from the book in some detail.

The Seven Messengers, a novella written in the 1940's by Dino Buzzati, is the story of a prince who sets out with seven knights to explore his father's kingdom. The prince keeps moving in one fixed direction at a constant speed while the seven knights keep oscillating (at different points in space and at a constant speed, greater than that of the prince) between the prince and the capital, as the prince moves further and further away. The prince (who is also the narrator) makes several numerical statements in the story, which Khare and Lachowska explain and verify. But they go far beyond, and give a geographical explanation of how the prince could have travelled for so long in a fixed direction, given the earth is round. They introduce the International Date Line to explain the notion of losing or gaining a day of a traveler going round the globe. Finally, they consider the abstract concept underlying the story of an object moving away from an observer and messengers oscillating between them, and discuss the similarities with a space probe like the Voyager (the messengers are photon particles in this case). This wonderful example contains interesting mathematics, and alongside there is also literature, history, geography, physics and philosophy. This is mathematical story-telling at its best.

Another example deals with the golden ratio phi which is intimately connected with the arrangement of leaves, florets, and seeds in many plants. A hexagonal scale on the surface of a pineapple is part of three spirals (Figure 1A). The numbers of such spirals on most pineapples are five, eight, thirteen or twenty one, which are Fibonacci numbers. The florets in a sunflower (Figure 1B) form clockwise and anticlockwise spirals, the numbers of these two types of spirals appear in the ratios of consecutive Fibonacci numbers (34 to 55; 55 to 89; 89 to 144; depending on the size of the sunflower). The limit of the ratio of two consecutive Fibonacci numbers is the golden ratio phi. A mathematical reason for the golden ratio to appear in the formation of leaves and florets is that it is 'more irrational' (in a certain sense) than other irrational numbers. The leaves along a twig or the stems along a branch prefer to appear at irrational angles to each other in order to maximize exposure to sunlight, air and moisture. The angle between the two directions made by joining two successive leaves to the stem, known as the divergence angle, splits 360 degrees in the ratio 1 to phi. The book [ML] (referred to by Khare and Lachowska) also provides a physical explanation for the spiral patterns from an interesting experiment in physics where magnetic drops (that act as tiny magnetic dipoles or bar magnets) at the silicon oil-air interface settle into a spiral pattern to minimize their repulsive energy.

A

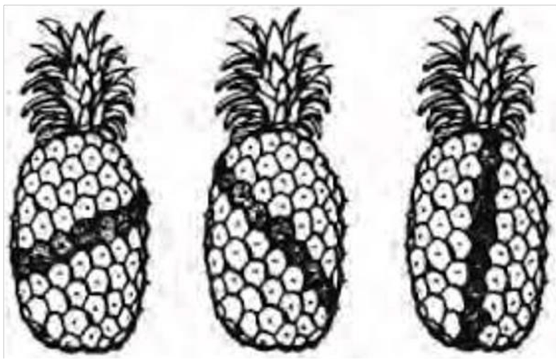

Figure 1A: Hexagonal scales on a pineapple form a part of three different spirals, *http://moziru.com/explore/Drawn%20pineapple%20golden%20ratio/#gal_post_8118_drawn-pineapple-golden-ratio-5.jpg.*


Figure 1B: Spiral patterns formed by the florets of a sunflower, *https://www.hxbenefit.com/are-sunflower-seeds-good-for-you.html.*

A third example is the famous Monty Hall problem. Here is a brief description, if you have not heard of it before. You appear on a television show and the host shows you three closed doors (named A, B, C), one of which conceals a car while the other two have goats behind them. You can win a car by choosing the right door, and you choose door A. The host then opens door B and shows you a goat behind it. He then gives you a chance to choose again. Should you choose C, or remain with your original choice of door A? The history of this very interesting problem is discussed, along with the (not at all obvious) answer. A minor modification of this problem, which completely changes the answer, is also discussed. In the problems at the end of this chapter, two generalizations of the Monty Hall problem are discussed.

All chapters, except the first, consist of two parts, a first part of a few pages containing the mathematics, and a much longer second part consisting of the applications of the math done initially. Each chapter also contains problems which are solved and practice problems with just the answers given. There is a list of problems at the end of each chapter and the solutions to odd numbered problems are provided. Six practice tests are given; the solutions of all the tests are given too. The two authors are not just wonderful expositors; they are sympathetic teachers as well! They have made sure that an interested person can read and learn this book completely on his or her own! At the end, the authors have written a small section ("The bigger story") in which connections between concepts appearing in different chapters are noted and a few books on different branches of interesting mathematics (Knot Theory, Graph Theory, Topology) are suggested for the reader who wants to do some further reading.

We have a few (very) minor quibbles about the book. It might have helped if the authors had included a short section on functions and their graphs, as these notions are completely basic and are used quite a few times in the book. The phyllotaxis part seemed a bit harder for us to read than the rest of the book, it might have been nicer to include a few more photos and pictures, and perhaps give a bit more explanation. We also found a total of three very minor typographical errors in the book.

In the preface, Khare and Lachowska write, "Our main goal is to convince our reader that mathematics can be easy, its applications are real and widespread, and it can be amusing and inspiring.” There is no doubt in our minds that they have succeeded brilliantly. If you know a high school student who is even mildly interested in mathematics, please do give him or her a copy of this book as a gift!

## References


1. [ML] Mario Livio, The Golden Ratio: The Story of Phi, the World’s Most Astonishing Number, Broadway Books, 2002
2. [AL] http://dedis.cs.yale.edu/~anouchka/book/index.html



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