Misconceptions in FRACTIONS

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Introduction

Mathematics is notorious for being a difficult subject. Algebra, as a whole, is feared and despised; so also is a topic like Fractions. There are many reasons that make mathematics a difficult subject. One of them is its already earned reputation. Teachers, parents and children (as a result of the other two) start the process of teaching and learning mathematics with a pre-conceived notion of the subject being difficult.

There are other reasons too, rooted in the nature of the subject, that make it difficult to grasp and grapple with, unlike other school subjects. It is a highly abstract area of study, based on assumptions and logical derivations. Because of its logically derived subject content, it is hierarchical in nature. So the knowledge of previous concepts is essential for further study in the subject. For example, to understand the concept of multiplication, a learner needs to understand and be comfortable with the concept of addition.

One of the most abstract concepts introduced in primary classes is Fractions. Unlike other number sets introduced until now (natural numbers and whole numbers), fractional numbers are not used for counting. They basically denote a proportion. There is much research and writing around difficulties in learning of fractions and also about its pedagogy. In this article, we shall focus only on some of the misconceptions related to fractions that children develop.

Misconceptions or errors

The words misconceptions, errors, mistakes, alternative frameworks, etc., are often used interchangeably. Here, I make

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two categories to understand them better. The words 'misconceptions' and 'alternative frameworks' are close to each other, as are the words 'errors' and 'mistakes.' Though there may be small differences in words which have been placed together, for the purpose of this paper, I will not engage with that.

An error is a result of carelessness, misrepresentation of symbols, a lack of knowledge of that particular area, or a task that is far too demanding of the child's current level. A misconception, on the other hand, implies that the learner's conception of a particular idea or topic, of a rule or algorithm, is in conflict with its accepted meaning and understanding in mathematics (Barmby et al., 2009). It could be a wrong application of a rule, an over- or under-generalization, or an alternative conception of the situation. For example, the rule that "a number with three digits is bigger than a number with two digits" works only in some cases. When you compare 35 and 358, the rule gives the correct answer, but not when you compare 35 and 3.58.

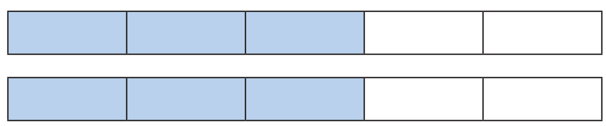
Misconceptions, unlike errors, are a sign of what the child knows, or a sign of the child's present level of understanding. Thus, from a teacher's perspective, an attempt to uncover the misconceptions of students is a very productive activity, as it is a guide to the future teaching-learning process.

Some misconceptions in Fractions

Fractions have often been considered one of the significant culprits in scaring people away from mathematics. As discussed above, the abstractness of fractions as numbers is difficult to grasp and, to top this, the introduction of the algorithms of operations widens the understanding gap. At a time when children require more experience in visualizing fractions as numbers, complicated procedures are introduced to carry out operations. As a result, children often make efforts to remember the procedure without understanding why it is being done, and experience of teacher training indicates that often the teachers do the same. Since children often remember the procedures and not the reasons, they end up applying them in incorrect situations. Example: Both students and teachers know that to add fractions, you need to take the LCM of the denominators. Is it correct? Yes. But is it enough? No. A child who knows this as a rule does not understand why the LCM is needed to add and subtract fractions. As a result, it is common to see children extrapolate this 'rule' to multiplication as well.

- **Missing the importance of equal parts:** The denominator in a fraction not just represents the parts into which the whole has been divided, it also implies that the whole has been divided into as many equal parts. A common misconception is to focus only on the number of parts, and not at their being equal. This misconception is often propagated by the colloquial usage of phrases like a 'half-glass of milk' or a 'half-roti.' In such usages, it is not essential for the glass to be exactly half-full or empty, but just less than full. This misconception shows itself when students are asked for the pictorial representation of a fraction.

- **Representation of a situation in fractions and presenting a fraction:** A common misconception about fractions is that a fraction represents part of a whole. This comes out when asked to pictorially represent an improper fraction. For example, represent 3/5 and 7/4:



A person who is able to represent 3/5 knows that the denominator indicates the number of parts into which the whole has been divided, but when it comes to 7/4, the person reverses the understanding. Why? Perhaps because it is impossible to show 7 out of 4. So it must be 4 out of 7.

In a test given to 9th class students, the following question was asked, "A watermelon is cut into 16 parts and of those, I ate 7 parts and my friend ate 4 parts. How much watermelon was eaten by both of us? Represent as a fraction." In a class of more than 20 students, only 2 got the answer and the rest made a variety of errors, some of which indicate misconceptions about fractions.

A child while attempting to answer this question wrote as follows:

- Watermelon: 16
- Pieces: 7
- Friend: 4
- Total: 52

Another child wrote: 16 + 7 + 4 = 27

These responses indicate problems at two levels: the first is about how children read and comprehend/unpack a word problem and the second is about the conception of number sets, which, in these cases seem to be limited to integers or perhaps only to whole numbers.

The responses indicate that some children while reading a question are only trying to ascertain given numeric data and then applying some operation to this. The randomness of the operation to be applied is evident in this case. It seems that these children have an understanding that everything else written in the question is either distraction or meaningless frills. They do not seem to infer from the question which operation needs to be applied.

The second misconception that seems evident from the above responses is about the conception of number sets. The number system for many children is restricted to positive integers or whole numbers. The double-decker numbers (fractions) are not seen as a part of the number system. Thus children with such misconceptions would attempt to work with fractional numbers, when presented as fractional numbers (i.e., double-decker form), but would not be able to represent situations or pictures in p/q form. Another example of this misconception was found in a response where the child wrote 11 watermelons (which is the sum of 7 and 4) instead of 11/16 which represents the portion of watermelon consumed.

Some responses reveal that the understanding of fractions emerges in stages. So the first stage as discussed above is where there is no familiarity with the p/q form. And the second stage is when there is familiarity but the child is still unable to represent the situation correctly in the fractional form. Two responses to the above question, such as 16/5 and 7/4, indicate this particular misconception. In the first response the child has written 16 (the total number of pieces) as the numerator (instead of denominator) and in the second case, the two numbers, which indicate parts of watermelon have been written in the p/q form.

- Believing that in fractions, numerators and denominators can be treated as separate whole numbers.

  It is common to see children add or subtract fractions by treating numerator and denominator as separate whole numbers. In the above mentioned question itself, a child who could correctly represent the two fractions involved as 7/16 + 4/16, wrote the final answer as 11/32.

- **Misconceptions related to simplifying:** Dividing top and bottom by common factors is often loosely referred to as canceling common factors or numbers which then leads to responses as follows:

$$\frac{7}{16}+\frac{4}{16}=\frac{7+4}{16}=\frac{7+1}{4}=\frac{8}{4}=2.$$

  Another issue highlighted by the response is that following algorithms and getting an answer has no connection with the question at hand. As a result, the absurdity of the response does not bother the child.

- **Failing to find a common denominator when adding or subtracting fractions with unlike denominators:** Students often do not understand why it is important to make the fractions like before adding or subtracting them. For example,

$$\frac{2}{3}+\frac{4}{7}=\frac{6}{10} \quad \text{or} \quad \frac{1}{6}+\frac{2}{3}=\frac{3}{9}.$$

Without understanding the reason or the need, procedural knowledge of taking the LCM and proceeding poses its own misconceptions as seen in the following example:

$$\frac{7}{16}+\frac{4}{16}=\frac{1+7+1+4}{16}=\frac{8+5}{16}=\frac{13}{16}.$$

The learner here took the LCM, which is 16 and then instead of multiplying with 1 (obtained by dividing LCM with the denominator) has added it to both the numerators, thus obtaining an incorrect answer.

- **Leaving the denominator unchanged in multiplication of like fractions:** an overgeneralization of the addition rule to multiplication leads to responses as follows:

$$\frac{2}{5}\times\frac{1}{5}=\frac{2}{5}.$$

This misconception is basically an overgeneralization of the algorithm for adding like fractions.

- **Failing to understand the invert-and-multiply procedure for solving fraction division problems:** The procedure for division of fraction numbers seems to cause many problems to children. The following response indicates one such:

$$\frac{7}{4}\div\frac{3}{2}=\frac{7}{4}\times\frac{2}{3}=\frac{21\times 8}{12}=\frac{168}{12}.$$

The child here did invert the divisor, but after that, instead of simply multiplying them, did a cross multiplication, followed by another multiplication between the two numbers in the numerator.

Another division related misconception, which is perhaps an overgeneralization of the multiplication procedure, is to cancel before inverting. For example,

$$\frac{2}{3}\div\frac{6}{7}=\frac{2}{1}\div\frac{2}{7}=\frac{2}{1}\times\frac{7}{2}=\frac{1}{1}\times\frac{7}{1}=\frac{7}{1}.$$

Implication for teachers

A study of misconceptions is both productive and interesting. It reveals patterns of learning of a particular child but, in most likelihood, is not unique to him/her, and thus it becomes a window to understand how children learn.

A discussion on identified misconceptions will not only help the particular child but all the children. An acceptance of misconceptions as a process of evolving understanding would help us in developing learners who are confident about their learning and process of learning. This would obviously require teachers to develop a culture of discussion and analysis of errors and not just a discussion of the correct process of solving a problem.

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