An easy construction for the HARMONIC MEAN

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We are given two positive numbers a and b. We wish to show how to construct a length corresponding to the harmonic mean of a and b, namely, the quantity

$$\frac{2ab}{a+b}$$

Construct $\triangle ABC$ in which $BC = a$ and $AC = b$. The third side c (alternatively, the included $\measuredangle ACB$) can be chosen arbitrarily; see Figure 1. Next, draw the internal bisector of $\measuredangle ACB$. Let it intersect side AB at D. Draw DE parallel to side BC, with E on side AC. Let x be the length of DE. Then:

$$x = \frac{ab}{a+b} = \text{half the harmonic mean of } a \text{ and } b$$

This can be checked using the applet https://www.geogebra.org/m/etMhyzaE. For the proof, please examine Figure 1.

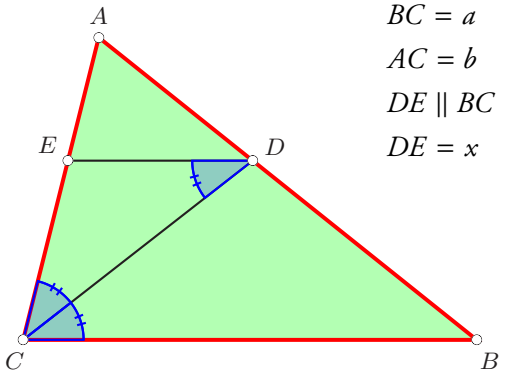

Figure 1

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