Vladimir Igorevich Arnold (1937–2010)

V I Arnold (referred to as Vladimir Igorevich or VI in Russian) was born into a strongly academic family, and recalls being given problems to solve at home, and later in school, from the age of five, and delighting in solving them. He entered the undergraduate class at the Department of Mathematics and Mechanics of Moscow State University in 1954, and records that it may have been possible, in spite of his Jewish mother, only because Stalin had died in 1953. His mentor, Andrei Nikolaevich Kolmogorov (hence AN), was a towering figure, and the other professors in the department such as Petrovsky and Pontryagin were of a comparable calibre. The combination of mathematics and mechanics in the title of the department would be considered unusual in most countries today, but was quite natural given the way the Russian school developed. VI describes AN giving him a problem concerning representation of functions of three variables by combining functions of two variables. When he conveyed the solution, he was informed that he had solved the thirteenth in a list of important and difficult problems proposed in 1900 by Hilbert, then an uncrowned king of the world of mathematics! Such an achievement by a nineteen-year-old student created a sensation. One parallel is John Wheeler (*Resonance*, Vol.18, No.1, 2013) in 1939, giving a twenty-year-old student, Richard Feynman (*Resonance*, Vol.16, No.9, 2011), a problem he could not solve himself, along with some ideas.

Going by VI's own account, he continued to think about related problems, representing functions on a closed curve by adding functions of the single variables x and y, and meanwhile learnt more mathematics from many people. Amazingly, this led him to the field of mechanics, where there was a long-standing unanswered question regarding motion of many particles interacting with each other. The prime example was, of course, the system of planets strongly attracted by the Sun, but also exerting gravitational forces on each other which were at least a thousand times weaker. Without the weak forces, the motion is 'multiply periodic' or 'quasiperiodic'. Quasiperiodic conveys the following idea – each planet comes back after a definite period, but the different periods may not have a common multiple, so the overall arrangement does not repeat precisely. The question was whether this behaviour could survive over billions of orbits (mathematically for infinite time), once the weak forces were put in. Kolmogorov had solved an idealised mathematical version of this problem in 1954, and encouraged VI to pursue the real case. (AN also discouraged him from publishing an application to disturbances of the heartbeat, which was rediscovered much later!) This endeavour bore fruit in the early 1960's. Quite independently, J K Moser in the United States, a student of Carl Ludwig Siegel, renowned for mathematics and celestial mechanics, had produced closely related results in 1962. The subfield which investigates if and when and how and how much multiply periodic motion survives is called KAM theory. There is no single theorem but a whole collection of them. Arnold himself has written generously of Moser's work – how it made less restrictive assumptions, which he and AN had not thought possible, and used rather different geometric methods.

At an early age, Arnold joined the faculty at the Institute of Mathematics and Mechanics, and rose to be a professor. However, he moved to the Steklov Mathematical Institute, also in Moscow, in 1986. Soon after, he was appointed a professor in the Dauphine University in Paris and he held both positions till his death. VI did not move to the West as many of his colleagues did, after liberalisation ('*perestroika*') in Russia.

History is full of examples of scientists who made early brilliant contributions but were not able to live up to them in later life. While one cannot generalise, it is often because they stay in the same field. VI did not have this problem, His energy, curiosity, extensive reading and in later life, contact with mathematicians worldwide, speed, inspiring communication in at least three languages (Russian, French, English), charismatic appeal to the brightest of students, all ensured a long, fruitful and productive career, about which one can now read in many sources on the internet (e.g., http://www.ams.org/notices/201204/rtx120400482p.pdf). The Wikipedia article (http://en.wikipedia.org/wiki/Vladimir_Arnold) lists his many awards and honours. As just one example, the citation for the prestigious 2001 Wolf Prize reads as follows:

"Professor Vladimir I. Arnold, a renaissance mathematician, has made significant contributions to an astounding number of different mathematical disciplines. His many research papers, books and lectures, plus his enormous erudition and enthusiasm, have had a profound influence on an entire generation of mathematicians.

Arnold's Ph.D. thesis contained a solution to Hilbert's 13th problem. His work on Hamiltonian dynamics, in particular as co-creator of the KAM (Kolmogorov-Arnold-Moser) theory and as the discoverer of 'Arnold's diffusion', made him world-famous at an early age. Arnold's contributions to the theory of singularities, which complements Thom's catastrophe theory, has transformed this field. He has also made innumerable and fundamental contributions to the theory of differential equations, symplectic geometry, real algebraic geometry, the calculus of variations, hydrodynamics, and magneto-hydrodynamics, often discovering links between problems in diverse areas."

One of his major preoccupations after the work on mechanics was 'singularity theory' which includes a very broad range of problems, some arising from physical applications but calling for deep mathematics. Unexpected connections between different branches of mathematics often came up in his work, and he writes of his own delight in such moments.

His knowledge of physics was wide and he wrote a fundamental paper on the existence of surfaces to which magnetic field lines are confined, under certain conditions. This is relevant to the worldwide quest for confining plasma in fusion reactors, even today. Interestingly, the first version of this paper was rejected! In VI's words,

"Mikhail Aleksandrovich (the deputy editor of the journal) explained to me that the paper could not be published in ZhÈTF due to thefollowing reasons.

1. The manuscript contained the words "theorem" and "proof" forbidden in ZhÈTF.
2. The manuscript claimed that "*A* implies *B*" while every physicist knew examples showing that *B* does not imply *A*.
3. The manuscript used the unintelligible terms "Lebesgue measure", "invariant tori", "Diophantine conditions".

Mikhail Aleksandrovich therefore proposed that I should rewrite the paper".

The paper was rewritten and became cited widely by the plasma community. We mention this as just one example among many of his work relating to physics. It also illustrates the then attitude of theoretical physicists in Russia, particularly the famous Landau school, to advanced mathematics. Things have greatly changed, of course, since then.

After liberalisation, Arnold travelled and lectured widely. The scientific community of Bangalore had a unique opportunity to interact with him, face-to-face, during a visit to the Indian Institute of Science in 1991. His office was in the maths building, and he gave three talks on widely varying themes, including the large-scale structure of the universe, and 'quasicrystals', i.e., objects showing five-fold symmetry. The rest of the time, he was available for discussions, and I was fortunate that two areas I was working on, gravitational lensing and galaxies, were close to his interests in dynamics and singularity theory. I got a chance to spend a few hours (in bits and pieces) with him. He would instantly take up what was described to him and give some relevant comments, references, open problems, pointers to other workers, or anecdotes, all of which were very illuminating. More than once, he explained some mathematical result in a few minutes, informally but insightfully. This was no 'professional' carefully guarding his time, choosing those he would talk to, and about what, and optimising his benefit/cost ratio -- a type of scientist not uncommon these days, even among the best. I could only marvel at his giving his time to educating a total stranger he would never meet again. Surely, this came both from his joy in the subject and his commitment to values as a teacher and scholar, and I am sure this was the experience of all those who spent time with him during this visit.

To bring out something of VI's non-mathematical personality, we reproduce below a letter that he wrote after a trip to Italy, just two years before he died. This brings out many of his interests and qualities. We reproduce it here with the kind permission of VI's colleague, the topologist Professor Dmitry Fuchs who was one of the recipients, and who translated it. Some explanatory notes are in italics.

Paris, March 26, 2008

Dear Mitya, *(short form for Dmitry, in Russian)*

I have recently returned to Paris from Italy where I wandered, for three months, in karstic mountains working at ICTP (the International Center for Theoretical Physics) at Miramare, the estate of the Austrian prince Maximilian who was persuaded by Napoleon III to become the Emperor of Mexico (for which he was shot around 1867 as shown in the famous and blood-drenched picture of Edouard Manet).

I lived in the village of Sistiana, some 10 kilometers from Miramare in the direction of Venice. It was founded by the pope Sixtus, the same one who gave names to both the chapel and Madonna. Passing the POKOPALIŠCE (the cemetery) some 3 versts (*VERSTA is an old Russian measure of length* ≈1.1 *km*) to the North, I reached a deer path in a mountain pine grove. These deer do not pay much attention to a small tin sign, DERŽAVNAYA MEŽA (the state border). After that it is Slovenia to which I ran, following the deer. But at the next sign, PERICOLO, the deer refused to go any farther. The local people (whose language is closer to Russian than Ukrainian or Bulgarian) explained to me that the sign is a warning that the nearby caves have not been demined. And they were mined during the FIRST world war when my deer path was called SENTIERA DIGUERRA and was a front line (described by Hemingway in "A Farewell to Arms").

I did not go down to these particular caves, but every day I visited tens of them, of which some (but not all) were shown on a map (where they were called YAMA, GROTTA, CAVA, CAVERNA, ABISSA, depending on the difficulty of the descent). All these caves look pretty much the same (a colorful scheme is provided): there is a hole on the mountain, a meter in size, and down go walls, of not even vertical but rather a negative slope. The depth of the mine is usually around 10 or 20 meters (but I descended to YAMA FIGOVICHEVA with the officially declared depth of 24 meters and to the half of the height, or rather the depth, of GROTTA TERNOVIZZA whose depth is marked as 32 meters and to which one cannot descend without a rope). At the bottom of the YAMA a diverging labyrinth of passages starts, of the lengths on the order of 100 meters. They go to lakes, stalactites, etc. Sometimes there is even a descent to the Timavo river (which flows about 50 kilometers at the depth 100 or 200 meters, depending on the height of hills above). Before this 50 kilometers it is a forest river resembling Moscow River at Nikolina Gora (*a village some 30 kilometers from Moscow where many remarkable Russian people* (*including Dima*) *used to spend their vacations*) with a charming Roman name of REKA. Both words belong to old Russian (*Russian YAMA means a gap, REKA is the Russian for a river*).

This was a part of Jason's expedition (with argonauts). On his way back from Colchis (with the golden fleece) he sailed his ship Argo upstream Ister (Danube) and its tributaries to the Croatian peninsula named Capudistria (which is visible from my window at Sistiana), then they dragged the ship to REKA and, following Timavo, they reached to northernmost point of the Adriatic, where the Roman city of Aquileia was later built.

Near Aquileia, I discovered a goddess Methe, new to me, but this is a separate story. (She saves any drinker of drunkenness, however much he drank. Allegedly, she was the mother of Athena, and Jupiter ate her, since he was afraid that she would give birth to a son, and that this son would dethrone him, precisely as he himself had dethroned his father.) Aquileia is a Roman port of the first century, preserved as well as Pompeii, without any Vesuvius: simply Attila who destroyed the city left the port intact, including the canals, ships (which survive to our time), quays, knechts and basilicas (which became Christian in the IV century) with mosaics of 50 m × 100 m in size, and absolutely everything as in Pompeii. No room to describe everything, I am just sending my best (Easter) wishes.

On June 3, I go to Moscow, there will be a conference dedicated to the centenary of LSP (*Lev Semenovich Pontryagin, Russian mathematician*).

Dima (*A short form of Vladimir)*

There are some widely held beliefs regarding mathematicians – that they are absent minded, not very physically active, shy, withdrawn, narrow in their interests outside mathematics, etc. The writer of such a letter, overflowing with physical vitality, knowledge and curiosity about history, geography, art, geology, mythology, and more, was clearly the exact opposite of the stereotype!

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