Problems for the MIDDLE SCHOOL

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One of the scarier words in a math student's lexicon is the word *locus*! The definition (*A path traced by a point when it moves under certain condition*) seems amorphous, difficult to pin down and much too open-ended! This topic is usually introduced in high school; we are deliberately presenting problems on locus which will give students a gentler introduction to the same. By incorporating constructions, we hope to give students practice in a topic which is often taught in recipe mode; following a series of construction steps usually cooks up a figure designed to satisfy your teacher and get you the marks you need! Why do constructions work? Can one design constructions? These are the questions which lead to skill-based learning and problem solving in the mathematics class.

Problem VII-1-M.1

Construct the locus of a point which moves so that it is always at a given distance d from a given point O. Describe the locus in words.

Problem VII-1-M.2

Construct the locus of a point which moves so that it is at a fixed distance from a given straight line l. Describe the locus in words.

Problem VII-1-M.3

A and B are two fixed points. Construct the locus of a point which moves so that at every instant it is equidistant from both A and B. Describe the locus in words. Justify your construction.

Keywords: *Geometry, path, locus, constraint, constructions, area*

Problem VII-1-M.4

AB is a line segment of length 5 cm. Find the locus of a point C which moves so that it is the third vertex of triangle ABC whose area is 10 cm^2.

Problem VII-1-M.5

AB is a line segment of length 8 cm. Find the locus of a point C which moves so that it is the third vertex of parallelogram ABCD whose area is 40 cm^2.

Problem VII-1-M.6

Construct the locus of a point which moves so that it remains at equal distance from two given parallel straight lines l and m. Describe the locus in words.

Construct the locus of a point which moves so that it is a vertex of a trapezium whose base AB is on one of two parallel lines which are at a distance of 5 cm. from each other and whose area is 36 cm^2.

Problem VII-1-M.7

Given two intersecting straight lines, find the locus of the centre of the circle that touches (but does not intersect) both lines.

Note: When a straight line touches a circle, it is perpendicular to the radius of the circle at the point of contact.

Solutions

Problem VII-1-M.1

Construct the locus of a point which moves so that it is always at a given distance d from a given point O. Describe the locus in words.

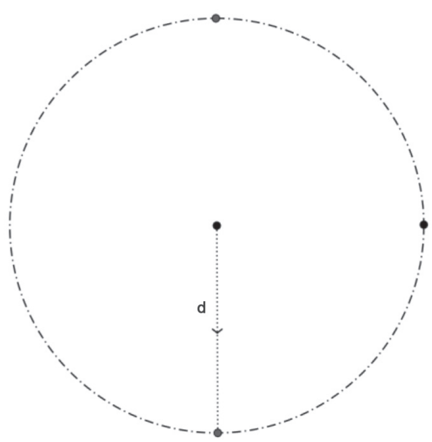

Figure 1

The locus is a circle of radius *d* centred at O. See Figure 1

Teacher's Note: This problem is deliberately set to reassure the student that the locus is nothing but an application of definitions already learnt. The very action of constructing the circle with the compass reinforces the constraint set by the locus.

Problem VII-1-M.2

Construct the locus of a point which moves so that it is at a fixed distance s from a given straight line l. Describe the locus in words.

The locus is either one of 2 straight lines parallel to *l* and at a distance of *s* units from it. See Figure 2.

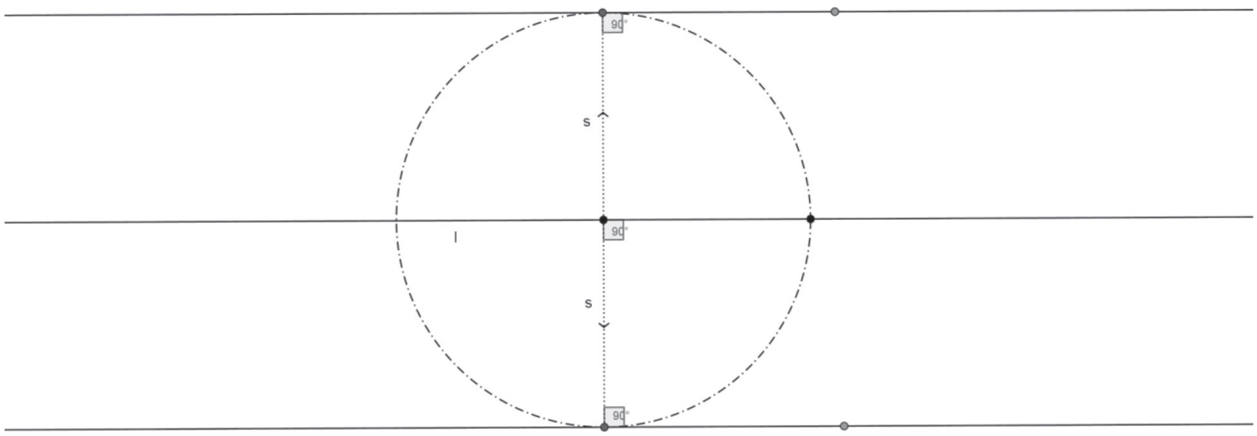

Figure 2

Teacher's Note: Again, easy enough but students get to practise the construction of two perpendiculars and also to figure out how to make the distance between the parallel lines fixed at *s*.

Problem VII-1-M.3

A and B are two fixed points. Construct the locus of a point which moves so that at every instant it is equidistant from both A and B. Describe the locus in words. Justify your construction.

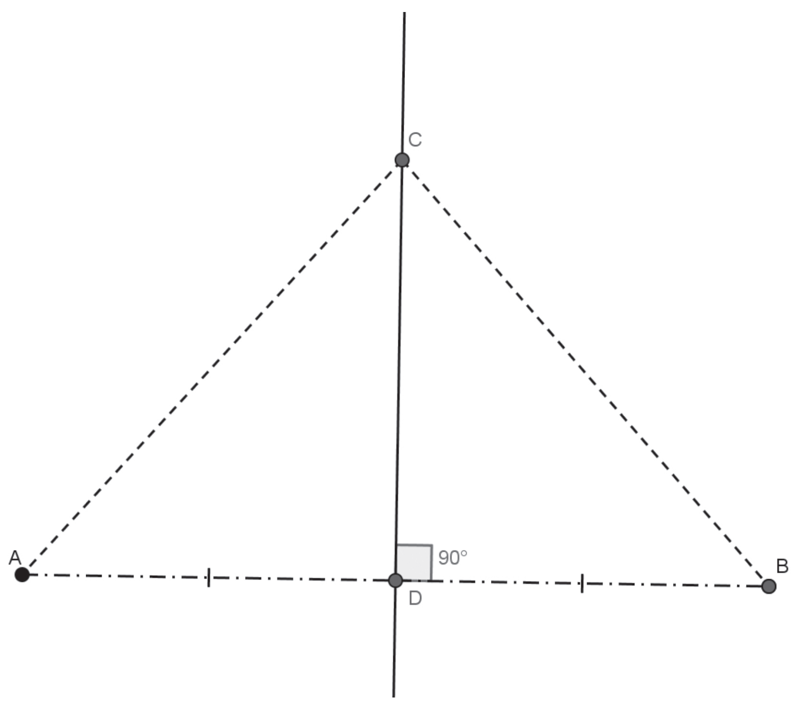

Figure 3

The locus is the perpendicular bisector of the segment joining A and B.

Teacher's Note: The construction of a perpendicular bisector is actually based on the fact that the diagonals of a rhombus bisect each other at right angles. So we start with equal sides and arrive at the perpendicular bisector. Students who have done congruency, can justify that triangles CAD and CBD are congruent by SAS and that this ensures that AC = BC. This justification starts with the perpendicular bisector and arrives at the equal sides. Talk about appreciating a view point from two angles!! Note the jump in the 'ask' of the problem, the layer of *justification* requires students to think about why their construction will work.

Problem VII-1-M.4

AB is a line segment of length 5 cm. Find the locus of a point C which moves so that it is the third vertex of triangle ABC whose area is 10 cm^2.

Since the area and the base are given, the height of the triangle is fixed and will be equal to 4 cm. Since the height has to remain constant, vertex C will have to be at a constant perpendicular distance of 4 cm. from the base. Referring to problem 2 above, we see that the vertex C will have to move on a line parallel to AB and at a distance of 4 cm. from it. There are two such lines on opposite sides of AB and these represent the required locus as shown in Figure 4.

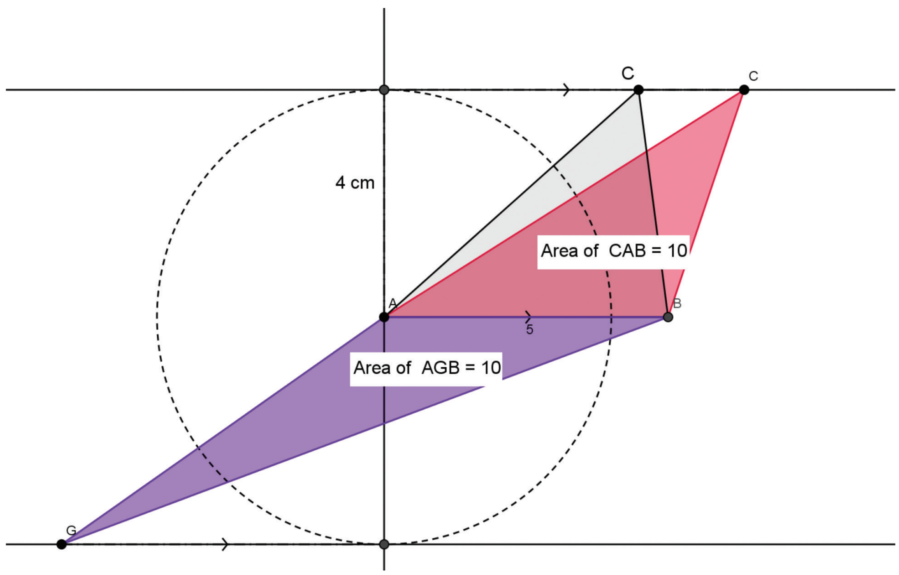

Figure 4

Teacher's Note: In this problem, there is an interesting juxtaposition of the formulae learnt in mensuration with constructions in geometry. Do give students the time to mull over the problem. An additional point - The idea of two possibilities for the locus is something that may not occur to students.

Problem VII-1-M.5

AB is a line segment of length 8 cm. Find the locus of a point C which moves so that it is the third vertex of parallelogram ABCD whose area is 40 cm^2.

Teacher's Note: The difference in this problem is that only 2 points of the parallelogram are given. Two more points are required to be found: this, however, is not a difficult task as, finding just one of the points will immediately fix the other also. Since the area of the parallelogram is fixed and so is the base, the locus of C is one of the lines parallel to the base and at a perpendicular distance of 5 cm. from AB. We show only one of the possible loci, to avoid visual clutter.

An interesting twist on this problem would be: find the locus of vertex D. (The answer is that D travels on exactly the same line as vertex C; so its locus is identical to the locus of C.)

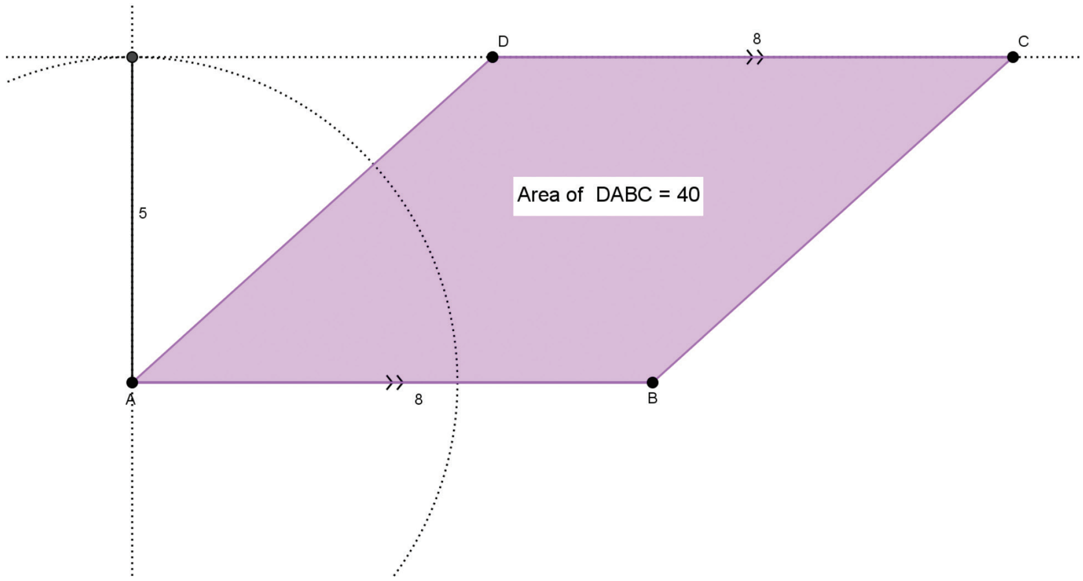

Figure 5

We leave it as an interesting variation for the reader to get the locus of the vertex of the trapezium.

Problem VII-1-M.6

Given two intersecting straight lines, find the locus of the centre of the circle that touches (but does not intersect) both lines.

The required locus is either one of the angular bisectors of the two angles between the lines.

Teacher's Note: The note is given for the benefit of middle school students, who may not know this fact. This question is slightly more difficult and will require careful facilitation by the teacher, who may start by asking students to sketch a line on which the centre will lie. Next, students may be asked to fill in all the constraints imposed by the problem. A sample figure which may emerge is given below.

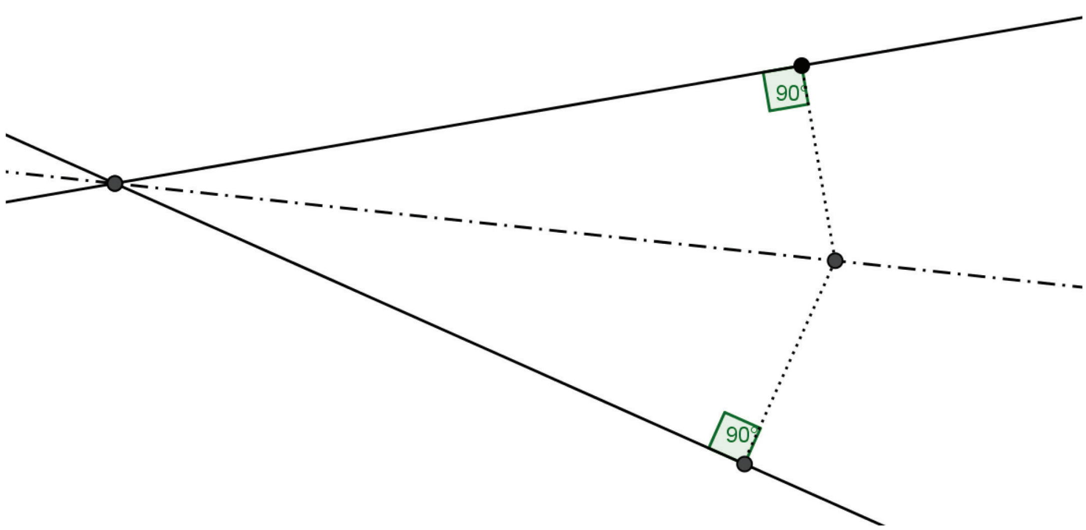

Figure 6a

Next, the teacher may ask students to shade in the two triangles that have emerged.

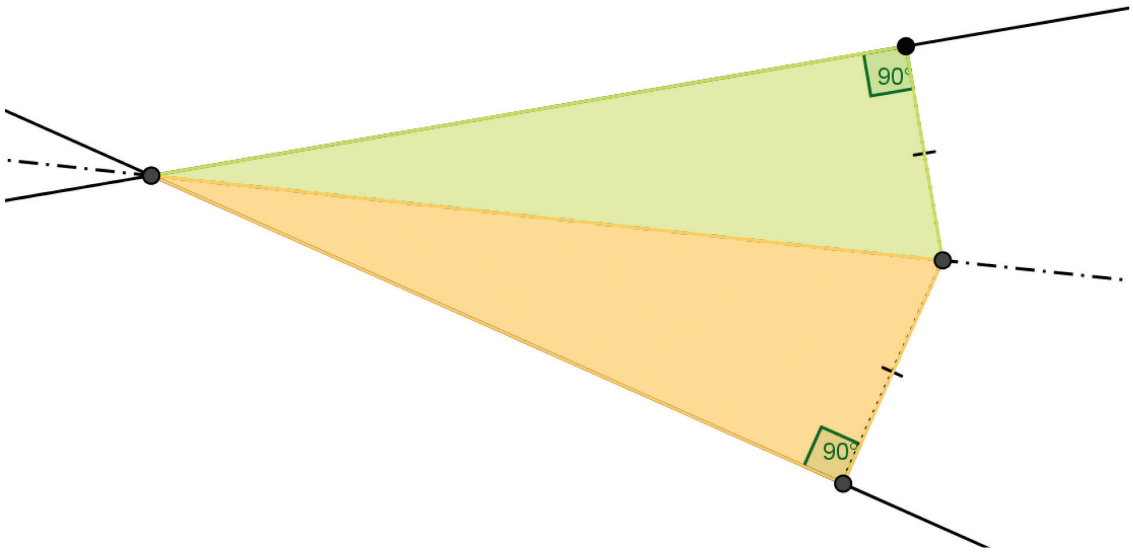

Figure 6b

Careful questioning will allow students to realise that the two triangles should, in fact, be identical- the suggestion of folding the paper along the line on which the centre lies may help this realization. From here, the idea that the locus is the angular bisector of one of the angles formed by the two lines should be a logical step. Some students may point out that there is a second locus, that is, the bisector of the other two angles at the vertex.

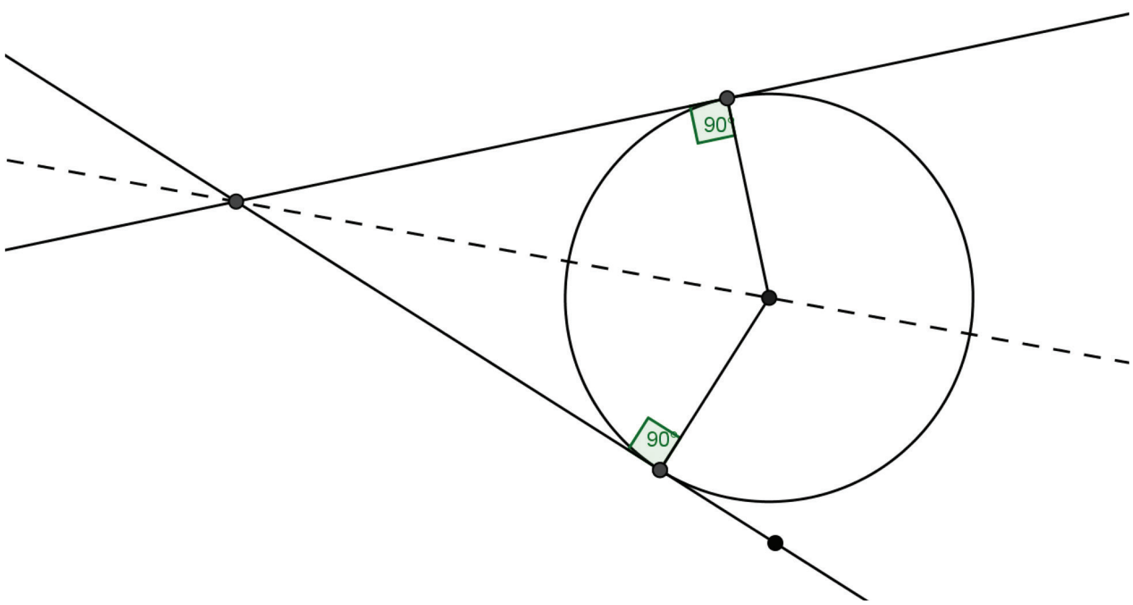

Figure 6c

Pedagogy: Problem solving strategies that have been imparted with this problem set are visualization, representation, trial and error, and logical reasoning. These exercises will develop the skills of mathematical communication, justification and proof, and give students plenty of scope for drill and practice in simple constructions in an interesting manner.

Some locus problems for students who have done circle properties are given below, their solutions have been uploaded https://www.geogebra.org/m/hzMBVtnv, https://www.geogebra.org/m/FwmeaSKV.

Exercises:

1. A is a fixed point on a given circle, and B is a variable point on the circle; it travels around the circle. M is the midpoint of AB. What is the locus of M?

2. A ladder leans against a vertical wall. A cat is seated on its central rung (i.e., at the midpoint of the ladder). The ladder, for some reason, starts to slip away from the wall. The cat stays on the central rung. What is the locus described by the cat?

3. The same question as Q2, but this time the cat is not seated at the midpoint of the ladder. What is the locus described by the cat in this situation?