CONNECTING THE DOTS...

The art and science of interpolation and extrapolation

$\mathcal{C} \otimes \mathcal{M} \alpha \mathcal{C}$

The following is an extremely common scenario in the sciences: two variables y and x are connected by a functional relation, $y = f(x)$, but f is unknown and our task is to find it. The only actions available to us are to perform experiments and find the values of y corresponding to selected values of x. After doing these experiments, we obtain the following n pairs of values of x and y:

We may plot these points on a sheet of graph paper and get something which looks like this:

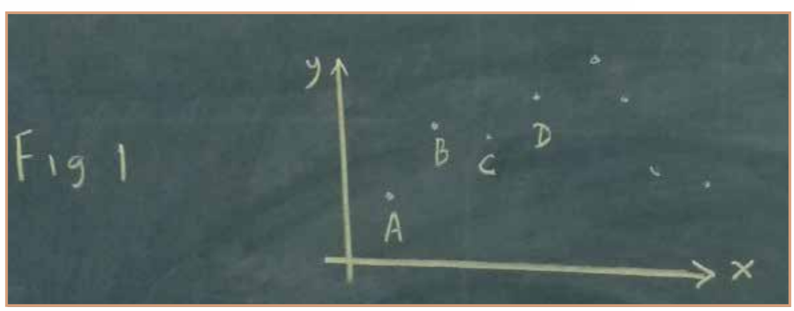

Figure 1

Armed with only these data points, can we determine the unknown function f? Another way of expressing this question is the following: *Can we fit a definite, unique curve to these data points?* Note that the curve must pass through all the points. (So this is not a problem of finding the "line of best fit" or the "curve of best fit").

A moment's reflection will tell us that the answer is **No**. The question is too broad to admit an answer when stated this way. Even if we were only interested in polynomial functions f (this is very often the case), it is still not possible for the data to yield a definite, unique answer, for there are infinitely many polynomials which will fit the given data.

Let us start with the simplest possible case: just two data points ($n = 2$). The situation now appears as shown in Figure 2:

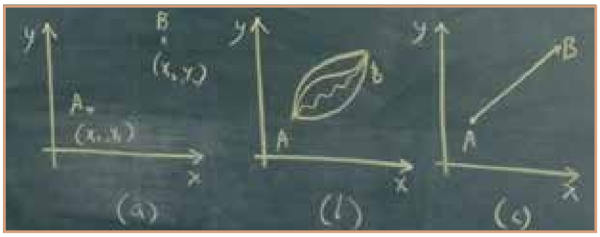

Figure 2

As Figure 2(b) suggests, many different curves may be drawn through A and B. If we are to progress, we need to impose additional conditions. The most obvious such condition is: *with the least possible degree*. In other words, we seek *the polynomial curve with the least possible degree passing through all the given points*. In the case of two points A and B, this curve is obviously the unique straight line passing through A and B, as shown in Figure 2(c). The equation of a general straight line is $y = ax + b$, with two unknown coefficients a and b. As there are also two data points, they serve to uniquely fix a and b.

In the case of three given points A, B and C (Figure 3), it may happen that the points lie in a straight line. But generally this will not be the case, and the polynomial curve with the least degree passing through the points will be a quadratic curve (upward facing or downward facing). As earlier, once we have imposed the condition of 'least possible degree' the answer becomes unique. For, the equation of a general quadratic curve is $y = ax^2 + bx + c$, with three unknown coefficients a, b and c. As there are also three data points, they serve to uniquely fix a, b and c.

In the same way, given four points A, B, C and D, the polynomial curve with least possible degree passing through these points will be a *cubic curve*. And so on.

The emphasis on the word 'least' needs to be commented upon. It is common in the

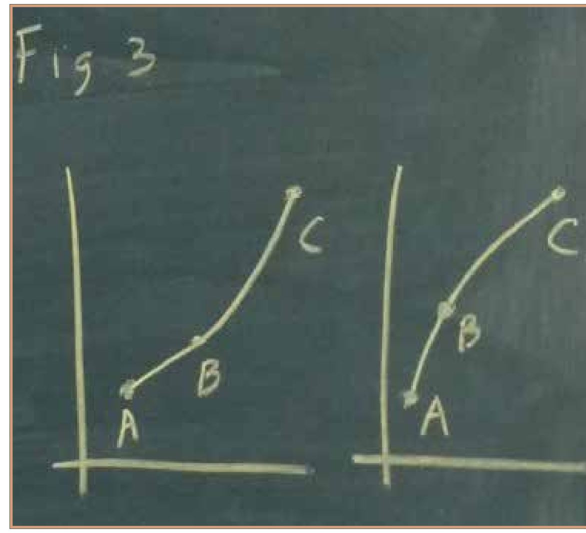

Figure 3

sciences, when we are attempting to explain some phenomenon, that a number of competing theories are available. Other things being equal, the general principle used to choose among the theories is that of *simplicity: choose the simplest theory available*. Here the word 'simplest' could mean: *that which makes the fewest assumptions*. In our curve-finding context, it could mean: *that polynomial curve with the least possible degree*. This principle is often referred to as **Occam's razor** (see [1]). It is regarded as an extremely important principle in the philosophy of science.

Note: We need to point out the difference between the two terms 'interpolation' and 'extrapolation'. Suppose we are given n data points, and we have been able to find a curve corresponding to a polynomial f of degree n which passes through all the n points. We may now want to use this knowledge to estimate the y-value corresponding to some x-value lying within the range of the given data. All we need to do now is to substitute this value of x into the function f, and we get the desired estimate. This process is known as ***interpolation***. What happens if the x-value lies outside the range of the given data? We may *assume* now that the same functional relationship holds between y and x even if such is the case. Therefore, to get the desired estimate, all we do (as earlier) is to substitute this value of x into the function f. This process is known as *extrapolation*.

Interpolation and extrapolation are vitally important elements in numerical analysis and in the application of mathematics to the sciences. The interested reader could refer to web links [2] and [3] for more details on interpolation and extrapolation respectively.

[1] Wikipedia, Occam's razor, https://en.wikipedia.org/wiki/Occam%27s_razor

[2] Wikipedia, Interpolation, https://en.wikipedia.org/wiki/Interpolation

[3] Wikipedia, Extrapolation, https://en.wikipedia.org/wiki/Extrapolation

Finding the Square of an Integer: A Simple & Quick Method

We know that finding the square of a large number is a difficult job. Here we give an easy method to find the square of any number whose units digit is 9. Also we have an algorithm to find the square of any two digit number just by doing a few calculations.

The square of a two digit number $n = 10a + b$ as given by the following procedure.

Add b to the given number; then multiply by $10a$.
Add b^2 to the number obtained in Step 1.

The resulting number is the square of n.

Proof: Given number is $n = 10a + b$. Add b; we get $10a + 2b$. Multiplying by $10a$, we get $(10a)^2 + 20ab$. Lastly, add b^2; we get $(10a)^2 + 20ab + b^2$, which is the square of $10a + b$.

Illustration: $(87)^2$

Old method: $87 \times 87 = 7569$

New method: $87 + 7 = 94$
$94 \times 80 = 7520$
$7520 + 49 = 7569$

Here's another shortcut for squaring a number n whose units digit is 9:
n^2 can be calculated by using the formula $n^2 = (n - 1)(n + 1) + 1$.
Example: $39^2 = (39 - 1)(39 + 1) + 1 = (38 \times 40) + 1 = 1520 + 1 = 1521$.

The formula is easily verified. Its advantage is that $n + 1$ is a multiple of 10.

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Vikram A. Ghule

Ekalavya Shikshan Sanstha's, Sant Tukaram Vidyalaya Ghatshiras Tal. Pathardi, Dist.
Ahmednagar 414106 (India) **Email:** vikramghule2016@gmail.com