

## Review: Liping Ma (1999)

# KNOWING AND TEACHING ELEMENTARY MATHEMATICS

*Lawrence Erlbaum Associates: New Jersey.  
Anniversary Edition, 2010, Routledge: New York.*

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In 2010, the Indian Government decided to take part in an international test called PISA (Programme for International Student Assessment). About 5000 fifteen-year-old students from 200 schools in Himachal Pradesh and Tamil Nadu took tests of reading and mathematical and scientific literacy—along with over a million students in 75 other countries! The results were worse than our worst nightmares: Indian students were literally at the bottom of the list. Nearly 90% of the Indian children who took the PISA were at or below what the PISA 2012 document describes as the 'lowest levels' of literacy and numeracy:

*“...students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.”*

The Indian press were sharply critical of our government education system, but in its defence, the government said that PISA is linguistically and culturally biased against our government school children. There may be some truth to this<sup>1</sup>, and yet, there were several other developing countries that also took part in the test and fared better, which makes it difficult to shrug off the results completely.

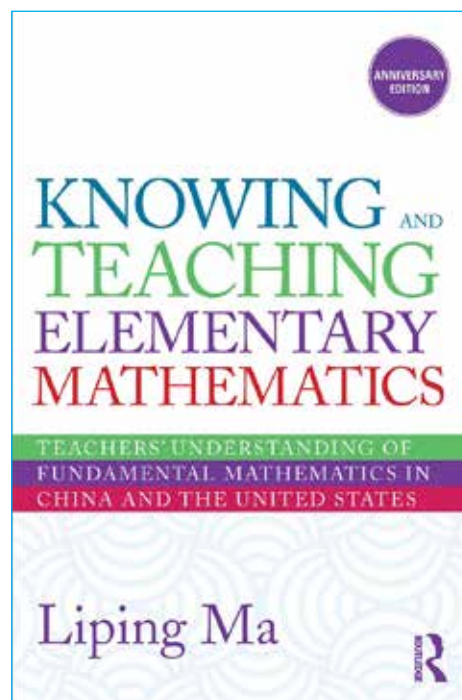
<sup>1</sup>You be the judge! Read the released 2012 mathematics test items and instructions for translation here: <http://www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf>

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Perhaps the outcome should not be surprising, since we are somewhat aware that we are failing to impart mathematical literacy to our students. Large-scale testing initiatives within India such as ASER and ASSET<sup>2</sup> tell similar stories of the poor learning levels of the majority of our country's children. The reasons range from the lack of quality in textbooks and infrastructure to inadequate teacher education. One thing is clear, though—right from a young age, children are not learning to **think mathematically**. Primary school mathematics, which is the foundation on which all further mathematics learning must rest, is itself not being taught and learned well.

Here is where Liping Ma's book **Knowing and Teaching Elementary Mathematics** comes in. It is a detailed, loving description of primary school teachers' thoughts about basic concepts in mathematics. It is also, by the way, a comparative account of Chinese and North American teachers, in which several Chinese teachers emerge as having a better understanding of the fundamentals. But Ma's aim is not to make a big deal of cross-national differences. Rather, the book is a revelation of how a really good primary school mathematics teacher *thinks* about her students' learning. Good teachers have what she calls a *Profound Understanding of Fundamental Mathematics* (PUFM), and the book provides plenty of examples to illustrate what that means. Ma uses an appealing analogy of the way people know the town they live in. Some newcomers know only their own home; some know a few places and a few fixed routes. But taxi drivers know all the roads extremely well and, "They are very flexible and confident when going from one place to another and know several alternative routes." A good teacher's PUFM is very much like the knowledge a good taxi-driver has of his town!

Ma's book is a distillation of in-depth interviews with primary school teachers in both countries.



She constructs four scenarios, each a topic in mathematics paired with a particular teaching process.

1. Subtraction with regrouping: how to approach the topic?
2. Multi-digit multiplication: how to deal with mistakes?
3. Division by a fraction: how to represent it in a story?
4. Perimeter and area: how to explore the relationship?

Ma closely analyses the teachers' responses, revealing how some ways of thinking have conceptual depth while others are more superficial, not moving beyond the procedural. You may protest—what is there beyond the procedures in primary school mathematics—it's only arithmetic after all! But, as Ma makes clear through the book, one of the most important assumptions that good teachers make is that elementary mathematics is vast, deep, and full of rich conceptual learning.

<sup>2</sup>Annual Status of Education Report (<http://www.asercentre.org/>) and Assessment of Scholastic Skills through Educational Testing (<http://www.ei-india.com/introduction/>)

The word *elementary* is not taken to mean *simple*, but rather *at the foundation of all further learning*. These teachers use what Ma calls **arithmetic with a reasoning system**, which takes the four operations (+, -, x and ÷) and all their associated algorithms, from the practical into the theoretical realm, from ‘how’ to ‘why’.

For example, instead of using terms like carrying and borrowing, good teachers prefer to use the terms composing and decomposing bundles of ten units. One of them says:

*“The term ‘borrowing’ does not mean the composing-decomposing process at all. ‘Borrowing one unit and turning it into 10’ sounds arbitrary. My students may ask me how we can borrow from the tens. If we borrow something, we should return it later on. How and what are we going to return?”*

Before going into multi-digit multiplication, these teachers first make sure that students have understood place value and the distributive rule. Only then will the ‘staircase’ procedure of long multiplication make sense. For example, one teacher says that to multiply  $123 \times 645$ :

*“I’d review place value and show them that those partial products you can separate them out, just multiply 123 times 5 and then 123 times 40 and then 123 times 600 and then add them all up.”*

They also said that they would first teach students how to multiply any number by 10, 100 and so on. This would be necessary to understand the way a long multiplication sum is written out.

Good teachers thus also think about the skills necessary before attempting a new lesson. When asked how they would approach subtraction with regrouping, they told Ma that they would first have to teach the children subtraction within 20:

*“Given that my students do not have a firm grasp of problems within 20, how could they solve problems like  $37-18=?$  and  $52-37=?$  Whenever they follow the algorithm, they will face problems like  $17-8=?$  and*

*$12-7=?$  Are we going to rely on counting sticks all the time? All the subtraction procedures in problems with bigger numbers, after all, are transformed into subtraction within 10 and within 20.”*

Misconceptions are generally avoided by Ma’s good teachers. They don’t tell their students that a larger number cannot be taken from a smaller number, because this is not true. They don’t tell them that the smaller number can borrow from its neighbour, because that implies that the two are independent numbers, rather than two parts of one number.

Ma’s good teachers also demonstrated multiple ways of solving the same problem. Alternative ways were often faster than using standard procedures. Remember the taxi driver who can choose flexibly among several alternative routes? Ma writes, “The reason that one problem can be solved in multiple ways is that mathematics does not consist of isolated rules, but connected ideas.” Subtraction with decomposing was connected with addition with composing; division was connected with multiplication. According to her, teachers with PUFM have transcended algorithms to reach the essence of an operation.

In contrast, her interviews with teachers who did not have this kind of understanding either showed a lack of confidence, or a misplaced confidence! Here are two such responses to the question of how to divide  $1\frac{3}{4}$  by  $\frac{1}{2}$ .

*“For some reason it is in the back of my mind that you invert one of the fractions. Like, you know, either  $\frac{7}{4}$  becomes  $\frac{4}{7}$ , or  $\frac{1}{2}$  becomes 2. I am not sure.”*

*“You could be using pie, a whole pie, one, and then you have three fourths of another pie and you have two people, how will you make sure that this gets divided evenly, so that each person gets an equal share.”*

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Ma's own story is quite unusual, and moving. She was sent away from the city to rural China as part of the Cultural Revolution when she was just thirteen years old. The local school needed a teacher and she was asked to step in. For seven years she taught everything to the children of that village! Later she returned to Shanghai and studied education formally, ending up in the US for a PhD in mathematics education. In the anniversary edition of her book, published in 2010, she says, "Deep in my memory is the image of eyes—the bright eyes of my students in the rural area of South China where, as a teenager, I became a teacher. Whether they belong to Chinese or U.S. children, young students' eyes revealing a desire for learning have set the direction of my work."

The foreword to Ma's book is written by her PhD advisor, educational psychologist Lee Shulman. His praise for the book is glowing, and one of the main reasons is worth going into a little bit. Shulman's own work focussed on what he believed was a **false content-pedagogy divide**. In a 1986 classic paper, he explains how teacher education in the US has shifted emphasis from content to pedagogy. He gives the example of a typical test item in an 1875 teacher licensing examination for elementary school certification: *Divide 88 into two such parts that shall be to each other as 2/3 is to 4/5*. Not a trivial question! Today, teacher education is much more heavily tilted towards *"how teachers manage their classrooms, organize activities, allocate time and turns, structure assignments, ascribe praise and blame, formulate the levels of their questions, plan lessons, and judge general student understanding..."*

Current teacher education programmes in India assume that content knowledge has been covered in whatever basic education the student-teacher has gained. But surely it is extremely important to deepen her knowledge of mathematics *in order to be a better teacher?* Otherwise, as Shulman asks, "...how does he or she employ content expertise

to generate new explanations, representations, or clarifications? What are the sources of analogies, metaphors, examples, demonstrations, and rephrasings? How does the novice teacher (or even the seasoned veteran) draw on expertise in the subject matter in the process of teaching?"

In his paper, Shulman quotes two great thinkers on what defines a teacher. One is Bernard Shaw: "He who can, does. He who cannot, teaches." The other is Aristotle: "What distinguishes the man who knows from the ignorant man is an ability to teach."

When great minds disagree so fundamentally, we are forced to think for ourselves! Is content more important, or pedagogy? Ma might actually refuse to answer this question; in fact she doesn't even use the word pedagogy in her book. She seems to see content and pedagogy as inextricable parts of the 'whole' of being a great teacher. Chinese elementary school teachers have studied arithmetic with a reasoning system while at school themselves. When they become teachers, their classrooms do not look pedagogically 'cutting-edge'. Students sit in rows, the textbook is very much in use, and the teacher sets the agenda for class. However, one also sees in these classrooms a clear focus on conceptual understanding, and enthusiastic student participation. In progressive classrooms, where students are working in small groups and using concrete materials, there is no guarantee that conceptual learning is going on. As Ma says, *"The real mathematical thinking going on in a classroom, in fact, depends heavily on the teacher's understanding of mathematics."*

Ma's good teachers keep that learning frame of mind strongly throughout their teaching career. When she asks how they build upon their understanding, they describe four key things:

- **Lesson planning:** *"I always spend more time on preparing a class than on teaching, sometimes three, even four times the latter."*



- **Learning from colleagues:** *“I am the eldest one and have taught for the longest time, yet...my young colleagues...are usually more open-minded than I am in their ways of solving problems.”*
- **Learning from students:** *“The little ones have surprised me so many times...I had never thought that [the problem] could be solved in so many different ways.”*
- **Learning by doing math:** *“To improve myself, I first of all did in advance all the problems which I asked my students to do.”*

These certainly inspire me to re-examine my 20-year-old teaching practice!

The final chapter is devoted to a clear and insightful analysis of teacher education, taking examples from the US and China. We in India can find ourselves in several of her descriptions. A strong message from this section is that if a textbook is excellent, it serves as a script for the teacher, because *“in China, teaching a course is considered to be like acting in a play.”* The teacher is not expected to write or rewrite the script, but can and *must* explore creativity in the enactment of it. But of course, we need good playwrights as well as good actors—textbook writing then becomes a supremely important activity.

Within a few years of its publication in 1999, Ma’s book became such a runaway bestseller that in 2010 an anniversary edition was brought out, including a few additional sections that frame the educational context of the book in more recent terms. The original was a collector’s item; this edition is a must for any educational library. Ma’s examples come from the primary school, but mathematics teachers *at all levels* can learn the principles of how we must approach the fundamental and interconnected mathematical understanding inherent in what we teach. It also stands as a counter-example to a common assumption: ‘If a book has a great deal of conceptual depth, it must be difficult to read!’ The language and clear organisation of chapters allow the reader to move quickly while understanding a great deal.

For several years now, my primary interest has been in educational psychology, but I have also taught mathematics and statistics from primary to high school. This book is, for me, the perfect confluence of these two loves. The strong message I hope any reader will take away from it is that of humility as a teacher of this beautiful subject. As one of Ma’s teachers says: “It is easy to be an elementary school teacher, but it is difficult to be a good elementary school teacher.” QED.

## References

1. Shulman, L S (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, Vol. 15, No. 2, pp. 4-14.