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GUEST EDITORIAL

Mathematics for the billion

No child can go through school without studying mathematics, and the vast majority of professional courses need some mathematics as well. How we should, and should not, be handling the curriculum and its delivery in schools and colleges in our nation of a billion plus is surely worth examining. The title is also meant to evoke Lancelot Hogben's 1936 book *Mathematics for the Million*. The author was a Fellow of the Royal Society of London on the strength of his zoology and genetics, and indeed crossed swords with the legendary R. A. Fisher on nature versus nurture. He was also a committed socialist, and like his friend J. B. S. Haldane, believed in writing on science for a large audience, out of strong conviction that it would be of benefit to society as a whole. Given the rise of computation and data analysis, coming generations are going to confront even more mathematics in some form. The case for a wide appreciation and understanding is clear even without mantras like the 'knowledge society'.

India has the Mathematics group at the Tata Institute of Fundamental Research (TIFR) whose members have made outstanding contributions to the subject from the 1950s, many now part of advanced textbooks[1]. I have used the upper case to emphasize both the purity and the advanced nature of this endeavour. The Indian Statistical Institute (ISI) has produced world class work and people for an even longer period, and has the added distinction of running an outstanding training programme starting from the undergraduate level. The high calibre of ISI's early members certainly played an important role in setting the tone[2]. I venture that some of the vitality sprang from facing real world problems crying out for the application of statistical methods. One hopeful sign for the future of higher mathematics is that new groups, apart from the big two, are coming up in teaching institutions.

Why should a physicist editorialize about this area when there is so much proven strength? My excuse is concern is for mathematics beginning with a lower case 'm', defining this to mean what everyone goes through up to the 10th standard, plus what a very large number of students take on in the 11th and 12th standards. Let us add the mathematics needed while studying for a degree in science – even biology – engineering, and increasingly economics, commerce, finance, and even management. Anyone going through a 12th standard text book or examination paper – I did both, recently – will be left with the clear impression that the training and skills still focus on Mathematics, which is the destiny of a small fraction of students. (That it may not be ideal for pure mathematics either is another matter!) The course and evaluation certainly do not reflect the uses to which the content would be put in future by the vast majority of the students. The ability to differentiate more and more complicated functions, or integrate by ingenious substitutions, is placed higher than any appreciation of which functions one might want to differentiate or integrate in which real world context. This was so when I studied half a century ago, and it has hardly changed.

Contrast this situation with the teaching of the English language, which in my time was Milton and Shakespeare and Austen – one had to pass these to obtain a physics degree in Chennai! Today, the English language teaching community in India has recognized that language is used in many ways going beyond literature. One sees this reflected in modern curricula and textbooks which use the study of language as a route to understanding diverse cultures, and as a path to much needed communication skills. Is it too much to ask that the teaching of mathematics, as reflected in the curricula and the textbooks, and ultimately the classroom, should similarly reflect its myriad uses, its role as a means of expressing quantitative relationships in nature and in society, and even as a culture and language in its own right?

There is more than mere inertia behind this situation. Among the Athenians, Mathematics played the role of a bar which one had to cross to prove oneself witness the Platonic injunction – 'Let no one ignorant of geometry enter here'. For a small but dominant part of our educated society, speed and skill in mathematics for its own sake were a major indicator of overall academic merit – the much prized 'centum' of yesteryear. That has now been replaced by the IIT–JEE rank which hinges on the ability to jump through hoops in various subjects, mathematics included. As those who set the IIT entrance exam papers raise the stakes, so do the IIT dream merchants – the coaching classes – and the syllabi, first Central and then state, follow suit. For the majority, this overload and overemphasis on solving artificial problems simply results in half digested concepts which have to be unlearned and retaught in the first year of an undergraduate course. At the risk of sounding retrograde, I claim we would lose nothing by shrinking the calculus and linear algebra allegedly taught in great detail in the 12th standard, to the lightest of introductions. This would serve both those who will not return to the area, and those who will get the details later.

The situation in the 11th and 12th standards is made worse, ironically, by the very well thought out efforts of the National Curriculum Framework and the NCERT textbooks based on this for the earlier classes which are now gaining currency more widely in the Central School system. These do indeed focus on motivation, real world examples, and on being student friendly. The discontinuity in going from the 10th to the 11th is a fossil of the now defunct New Education Policy of the 90s, which naively assumed that most students would not go on to the 11th, leaving higher education for a breed apart.

It is interesting in this context to look at the history and geography of maths education. The (in)famous 'New Math' experiment of the sixties in the US, and to a lesser extent in Europe, was widely recognized to be a failure. Interestingly the reform in Russia at about the same time, led by A. N. Kolmogorov, seems to have been more successful. The three volume collection *Mathematics, its methods, content, and meaning* edited by Kolmogorov, Alexandrov and Lavrente'ev (translation: Dover, 1999) is an eye opener to the thinking of this school. On the one hand, the authors had impeccable Mathematical credentials. But each article is full of motivation, history, uses, connections. Kolmogorov himself was an outstanding example. He started with very pure set theory in his early career and his axioms for modern probability theory are graven in stone. He then went on to build a major school and, interestingly, made fundamental contributions to turbulence, to celestial mechanics, and to the foundations of complexity and entropy, which would all be regarded as theoretical physics today.

One of Kolmogorov's most distinguished students, Vladimir Arnold, visited the Indian Institute of Science (IISc) for a week in 1991 – (and incidentally contributed to the July issue of *Current Science*[3] of that year). He engaged with mathematicians and physicists with equal ease, giving four lectures on widely different topics and discussing with many individuals. He possibly tired his hosts and was seen dissipating his excess energies in the IISc swimming pool! Arnold had the reputation of poking fun at his fellow mathematicians, particularly the very abstract style preferred by the French – according to him a French child asked what two plus three is would reply 'three plus two'[4].

There is no question that the serious pursuit of Mathematics allows no compromises. It is founded on rigour, abstraction, and generality as necessary but not sufficient conditions – I hear from practitioners that there is an aesthetic which goes beyond these. For those few who are cut out for this, talent spotting programmes and specialized nurture programmes exist. But what of the others? I am not advocating wrong mathematics which has to be unlearnt, or excessively popular and descriptive material. The argument is for a scheme of instruction occupying the middle ground where the results are not necessarily the most general, the axioms not the most sparse, the level of abstraction lowered, but with more motivation, more help to intuition via examples and analogies, and more connection to real world situations brought out. There is a concern, not unjustified, that some of those cut out for the subject in its purest form might find the baggage demotivating. This is part of a broader concern that any teacher faces, of how to deal with a heterogenous classroom. A practical approach is to target the central group and make special provision for the outliers. Especially in higher education, there is provision for electives, for those with advanced inclinations. Further, at least some Mathematicians have drawn inspiration from real world problems, in addition to purely mathematical issues. For Kolmogorov and Arnold just mentioned, and for Poincaré before them, it was the stability of the solar system among other things. Such connections were much more frequent in the nineteenth century and died out by the mid-twentieth, but are seeing a major revival in the late twentieth and present centuries. Overall, the kind of changes suggested should not weaken, and may even strengthen, Mathematics.

The optimistic scenario is that practitioners and stakeholders across the spectrum come together to understand and respect each others needs and viewpoints and recognize that the current situation needs to change. They can then work on the challenges of delivering mathematics to an audience of unprecedented scale and diversity. Basically, one has to cater much more to multiple goals, with multiple paths even to each goal.

The pessimistic scenario was best formulated by David Hilbert, a Mathematician, if ever there was one. He reassured an audience of applied mathematicians that there was no possible conflict between their discipline and his. After all, '*Sie haben nichts miteinander zu tun*' – they have nothing to do with each other! I have used upper and lower case, above, as a rhetorical device. But I personally believe in the continuum and hope the binary split can be reversed to the ultimate benefit of mathematicians of either case, and the students they teach.

1. Ramadas, T. R., *Curr. Sci.*, 2014, **107**, 694–701.
2. Prakasa Rao, B. L. S., *Curr. Sci.*, 2014, **107**, 895–901.
3. Arnol'd, V. I., *Curr. Sci.*, 1991, **61**(2), 69–70.
4. Nityananda, R., *Resonance*, 2014, **19**, 787–796.

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