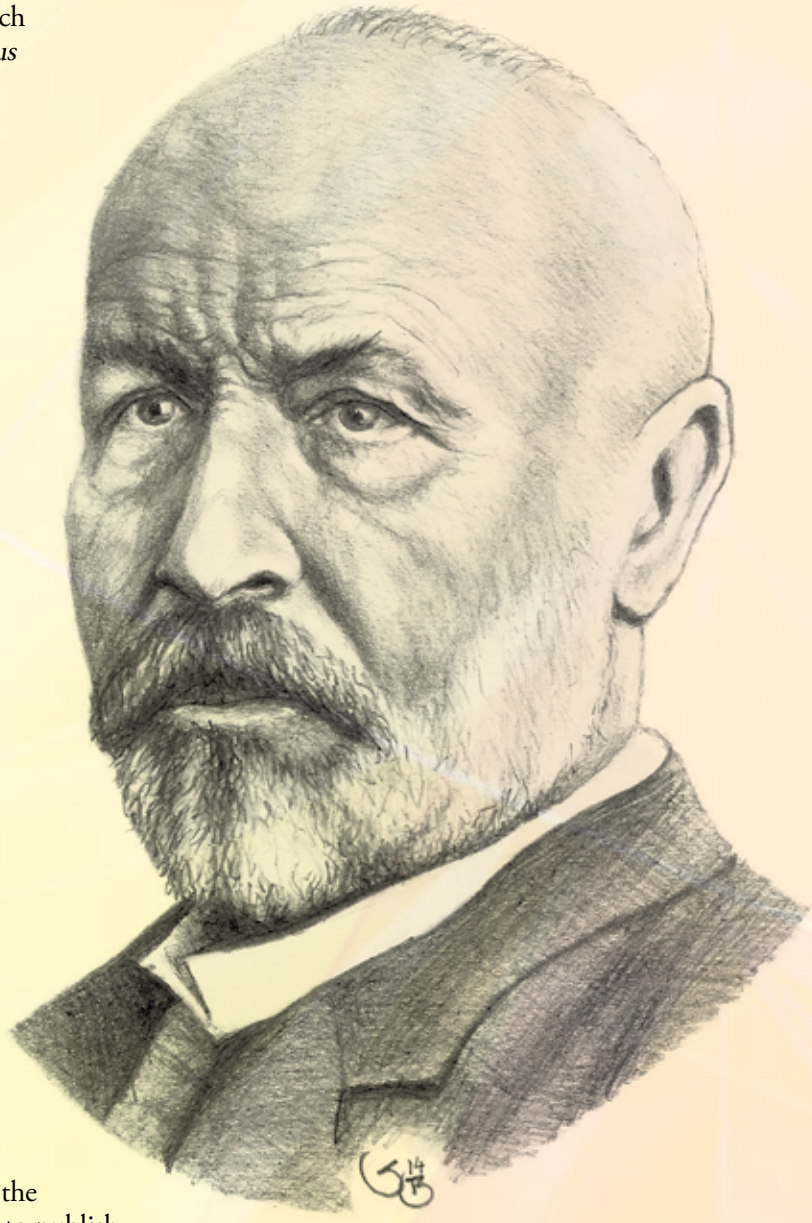
GEORG CANTOR

Georg Cantor was born in 1845 in St Petersburg, Russia, and when he was eleven he moved with his family to Germany. Cantor's father was a devout Christian and had a strong influence on his son. In a letter to 15-year-old Cantor, he wrote: *". . .How often the most promising individuals are defeated after a tenuous, weak resistance in their first struggle following their entry into practical affairs. Their courage broken, they atrophy completely. . . "* He goes on to say that such people lacked *"that steady heart"* and the *"truly religious spirit"* which one obtained by a *"humble feeling of the most reverence for God."* Cantor's father seemed to have an uncanny prescience for the enormous difficulties that Cantor would face. This strong religious influence would shape Cantor's world-view, and perhaps explains how he dealt with the severe criticism he experienced in his later life as a mathematician.



Cantor had an urge to study mathematics from a very early age. In 1862 he wrote to his father: *". . . I hope that you will still be proud of me one day, dear Father, for my soul, my entire being lives in my calling. . . "* In 1866, Cantor completed his study in the University of Berlin, studying with some of the great mathematicians of the time: Weierstrass, Kummer and Kronecker. In 1867, he began working at the University of Halle, living there till his death in 1916.

Cantor's early work was in number theory. In Halle he studied trigonometric series, which led him to explore the real numbers (the continuum) very deeply. He worked on the construction of irrational numbers from an infinite sequence of rational numbers, and this might have made him begin to suspect that there are more irrational than rational numbers. In an 1873 letter to Dedekind, he described the notion of using one-to-one correspondence to compare two infinite sets. Beginning with an 1874 paper in which he established the nondenumerability of the continuum, Cantor went on to publish ground-breaking results in set theory and the theory of transfinite numbers for the next 25 years.

From the outset, Cantor's work on infinities invited the ire of mathematicians, philosophers and theologians. Cantor had ventured into the theory of the infinite which was the realm of God, and hence was by its very definition not the business of man to comprehend. There was a tradition (perhaps more in the west than in the east) that one must not deal with infinities as a whole, because the concept of a 'completed infinity' would lead to strange paradoxes. Gauss for example writes to a friend: *"... I protest above all against the use*

of an infinite quantity as a completed one, which in mathematics is not allowed. The infinite is only a fason de parler (form of speech), in which one properly speaks of limits." From the time of Aristotle, thinkers had realized that infinities lead to strange results. For example, they 'annihilate numbers,' because $\infty + a = \infty$, for any positive finite number *a*, which contradicts properties of positive numbers. Galileo had stumbled upon the question of comparing the number of points in circles of different radii. He also discovered that the set of natural numbers can be put into a one-to-one correspondence with their squares and hence have the same cardinality (this is now known as Galileo's paradox). Thinkers before Cantor had decided that the best way to avoid these paradoxes was to avoid treating infinities as a whole. Cantor's response to these was that one could not impose the properties of finite numbers on infinite numbers. He expresses this eloquently: *". . . the infinite numbers, if they are to be considered in any form at all, must (in contrast to the finite numbers) constitute an entirely new kind of number, whose nature is entirely dependent upon the nature of things and is an object of research, but not of our arbitrariness or prejudices."* Cantor deeply believed that the essence of mathematics was its freedom.

However, some mathematicians, Kronecker in particular, were 'against' the infinite and even irrational numbers! They felt that only entities constructed by finite processes should be allowed into mathematics. Kronecker's ideology and personality were so strong that he seems to have worked very hard to belittle Cantor's work. He made sure that Cantor did not get a position in the more prestigious institutions in Germany, and tried to prevent the publication of his work. In contrast, Poincaré, who also believed quite strongly that set theory was a "disease from which mathematics should be freed", seems to have been on good terms with Cantor.

It was not just Cantor's infinite quantities and irrational numbers that created a stir. His 'naive' definition of sets as *any collection into a whole of definite and separate objects of our intuition or our thought* would later lead to the famous paradoxes in set theory, which in turn unleashed a whole crisis in the foundation of mathematics and engaged the best minds in mathematics for nearly half a century.

In spite of the repeated doubts expressed about his work, Cantor himself had no doubts. In 1888, he said: *"My theory stands firm as a rock; every arrow directed against it will return quickly to its archer. How do I know this? Because I have studied it from all sides for many years; because I have examined all objections which have ever been made against the infinite numbers; and above all, because I have followed its roots, so to speak, to the first infallible cause of all created things."* Cantor deeply believed that he had discovered the transfinite numbers with the help of God, and that they would lead to the *"one true infinity"* which was incomprehensible by man.

No note on Cantor can fail to mention his struggle with mental health. Cantor's first breakdown came in 1884, and here we find some possible misrepresentation. One story propagated by many historians is that Kronecker's vicious attacks drove Cantor to madness; another story is that Cantor's dabbling with abstruse concepts like transfinite numbers did it. Examination of his medical records suggests that he suffered from manic depression. Perhaps no matter what career Cantor had pursued, he would have suffered from mental illness. Certainly the hostility that he encountered did nothing to help.

But it is also true that Cantor had many friends and supporters throughout his life. He was awarded the Sylvester medal in 1904, the highest honour of the Royal Society of London. His work also generated positive interest among mathematicians in his lifetime. Minkowski referred to him as one of the deepest mathematicians of the time, and Bertrand Russell believed that he was one of the greatest intellects of the 19th century. No greater compliment can be paid to his work than Hilbert's proclamation *"No one will drive us out of this paradise that Cantor has created for us!"*

References

i. Duaben, Joseph Warren. *Georg Cantor: His Mathematics and Philosophy of the Infinite*. Princeton University Press, 1979.

ii. Reid, Constance. *Hilbert*. Copernicus,1996.