Review

Rich Maths for Everyone: A Review of Jo Boaler's Mathematical Mindsets

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Right at the start in the Introduction to her book *Mathematical Mindsets – Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching*, Jo Boaler describes her first meeting with Carol Dweck, soon after Dweck had joined Stanford University as a professor of mathematics education. By then Carol Dweck and her team had published enough research work about mindsets and their impact on learning. Dweck categorizes the beliefs people have about how they think about their learning abilities in two kinds of mindsets – a *growth mindset and a fixed mindset*. When people have a fixed mindset (yes, people can have different mindsets in different contexts), they think of their abilities as unchangeable. In contrast, when people have a growth mindset, they think they can learn by putting in more work and also most importantly their smartness can increase with hard work. Dweck's work has shown that these mindsets change our learning behaviour and hence our learning outcomes.

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This work obviously has great significance in education since students' mindsets can affect their achievement levels. Equally important is what kind of mindset teachers have about their students, because that governs the choices teachers will make in the classrooms. While one can find people working from one of these mindset orientations in different fields of work, it perhaps would not be incorrect to assume that an unusually large proportion of students in many maths classrooms often are trapped in a fixed mindset about their abilities in the subject. So it was quite fortunate for Boaler and Dweck to have met and then to have decided to work together to find ways in which research about mindsets can enter the maths classrooms in order to change core beliefs that maths students and teachers generally hold about this subject and about their maths smartness.

This book is about developing such an open mindset about mathematics, through positive messages and creative teaching. Boaler sees the immense potential of the positive messages that teachers can convey to their students – about the brain and its learning potential as well as about maths. She also knows though that spoken words and written messages on posters on classroom walls have a limited impact if the teacher's approach and classroom practices continue to convey the opposite messages – that maths is a *performance subject,* that doing things faster is valued more than thinking deeply, that maths is all about rules and procedures, that there is little scope of discovering interconnections, that it is not possible to have a deep understanding of mathematical tasks, that boys are smarter than girls; and that once you have 'missed the bus', you cannot catch up. So, Boaler doesn't only talk about the many convincing brain studies and classroom studies which point to the need for a change in maths classrooms. She describes her own work with her team in classrooms and shares work from other teachers which shows how mindsets of students and therefore their engagement and performance in the subject can change with good teaching interventions at any stage of schooling. Though most of Boaler's work is based in the context of classrooms in the US and the UK, it is very relevant for Indian classrooms as well.

Brain plasticity

The book starts with the recent brain studies about the plasticity of the human brain and the implications of this for education. These studies have shown that changes in the brain can happen even after childhood and can happen within short spans of time that would have been considered unbelievable earlier. Based on such findings, Boaler asserts her belief that almost anybody can learn maths and can learn it well. She thinks that even if people are born with brains which function differently from the norm, the effect of those differences can easily be eclipsed with the right teaching inputs; and so ability in mathematics can be developed by everybody.

Celebrating mistakes

Another recent finding from brain studies that is mentioned is about the response of the brain to mistakes. For many, mistakes are discouraging events in a learning process or at best they are accepted as inevitable negative steps. Boaler mentions studies which show our brains 'lighting up' when mistakes are made; surprisingly they 'light up' even when we are not aware that we have made a mistake. So making mistakes is equivalent to making new connections in the brain. Further studies have linked mindsets and the response of the brain to mistakes – people with a growth mindset have their brains more active when a mistake is made, which means their brains grow more and there is a greater chance that one becomes aware of the mistake too.

The book has several examples of strategies teachers are using in their classrooms to change students' attitudes to mistakes. Some of these involve conveying growth mindset and positive brain messages; others involve welcoming mistakes and making use of them in the teaching-learning processes. Boaler's advice: "Value correct work less and mistakes more".

Mistakes are important for brain growth because they create opportunities for challenge. Boaler suggests that teachers give challenging work to students so that more such situations get created. She advocates promoting struggle and mistakes in the classroom.

Creative Mathematics

Doing a lot of routine, procedure-based problems for "practice" is however not a learning challenge. If we want to provide opportunities to our students to learn mathematics, we will have to take a hard look at the mathematical tasks we want the students to be engaged in. Everybody knows that mathematicians don't spend their time doing calculations, that mathematics is not about rules and number facts. Even employers are not looking for people who can calculate fast and accurately; machines are always going to be far ahead of us in that area. Boaler suggests that students get to do creative maths: asking questions, using maths for modelling, visualising problems, discovering connections between different representations and pathways, explaining and challenging each other.

Mathematical Mindsets

Which practices in maths classrooms reinforce fixed mindsets among students and convey a narrow view of mathematics to them? Boaler answers the age-old questions about the necessity of memorising maths facts, e.g. multiplication tables, importance of "practice", place of homework and timed tests.

She argues that developing *number sense* is more important than memorising number facts. Developing number sense involves visual and intuitive thinking, using multiple representations, using new information or concepts in different contexts, all of which makes learning more powerful. Her examples of strategies to make all this possible reminds me of Eleanor Duckworth (a former teacher and now-retired teacher educator) and her focus on providing opportunity to children to have their own ideas:

The having of wonderful ideas is what I consider the essence of intellectual development. And I consider it the essence of pedagogy to give Kevin the occasion to have his wonderful ideas and to let him feel good about himself for having them.

["The Having of Wonderful Ideas" by Eleanor Duckworth]

How often does the school curriculum give such occasions to children to feel good about having their own wonderful ideas?

Since maths so often is taught as a *performance subject*, curriculum tends to simplify the matter by dividing the content neatly into water-tight compartments, providing bland problems placed in pseudo-real contexts. It definitely helps some students to succeed, especially those who can memorise and repeat lots of procedures without seeking any connections among them. Others, however, find maths full of isolated procedures devoid of any meaning outside the "exercises" and the tests. Boaler sees this as one of the key reasons behind the narrow idea of maths that people generally have:

The oversimplification of mathematics and the practice of methods through isolated simplified procedures is part of the reason we have widespread failure in the United States and the United Kingdom. It is also part of the reason that students do not develop mathematical mindsets; they do not see their role as thinking and sense making; rather, they see it as taking methods and repeating them. Students are led to think there is no place for thinking in math class.

Equity in Maths Classrooms

There seems to be enough evidence to suggest that girls are more affected by this narrow treatment of maths than boys. It seems girls seek more connections in what they learn, and so maths 'ticks them off'. They find it difficult to move through the labyrinth of disconnected concepts and procedures and soon give up on maths.

There is another reason why girls don't do as well as boys in maths. Some recent studies have shown that the more people in any particular field think that ability in their field cannot be developed by hard work, the fewer women one finds in that field. In other words, the idea of 'effortless giftedness' in maths seems somehow to affect girls more than the boys.

Another way in which the idea of giftedness and hence the idea of a fixed mindset gets promoted

is by having ability grouping in schools, called tracking in the US.

The strong messages associated with tracking are harmful to students whether they go into the lowest or highest groups. ... Students most negatively hit by the fixed messages they received when moving into tracks were those going into the top track.

These observations are the findings of several classroom-based research studies.

Seen closely, it shouldn't be surprising that students growing with fixed mindsets will gradually grow averse to taking risks, making mistakes and challenging themselves, because they begin to see any intellectual struggle as a threat to their smartness.

Boaler doesn't underestimate the challenges for a teacher responsible for teaching a heterogeneous group. After describing ways in which maths tasks can be adapted for a heterogeneous group, she shares in great detail a case study of an urban school in California, where teachers decided to stop tracking and instead started teaching maths using what is called *complex instruction*. Besides the examples of some actual maths work that the students in this school did as part of the complex instruction, the case study also describes in detail very efficient ways in which teachers in this school got the students to engage in *group work*, by becoming responsible for each other's learning. There is a wealth of very good ideas here.

Rich Maths

Hopefully, having convinced her readers about the need to revolutionise maths classrooms, Boaler spends five chapters and sixty pages of appendices to show how this can actually be done by teachers. She gives examples from her own classroom research work and she shares ideas from her classroom observations of other teachers at work.

The fifth chapter of the book titled *Rich Mathematical Tasks* is a collection of a few case studies of exciting maths experiences. One of them is from Boaler's interaction with the members of the Udacity team in Silicon Valley; the others are from the classrooms. My favourite one is the one where a "growing shapes" problem (seeing the pattern in a series of growing shapes and finding the nth shape) asks, after giving three figures numbered Fig. 2, Fig. 3 and Fig. 4:

What would Figure 100 look like?

And then: *Imagine you could continue your pattern backward. How many tiles would there be in Figure –1?* (That's figure negative one, whatever that means!)

Boaler, after describing these case studies, presents the questions teachers can ask themselves to make the maths tasks in their classrooms mathematically rich by design and sometimes by adaptation:

Can you open the task to encourage multiple methods, pathways, and representations?

Can you make it an inquiry task?

Can you ask the problem before teaching the method?

Can you add a visual component?

Can you make it low floor high ceiling?

Can you add the requirement to convince and reason?

Assessment for Growth Mindset

Timed tests continue to remain the primary tools of assessment of learning in most classrooms, even more so in maths classrooms. The problem is not just the public exams in India or standardised testing in the US as Boaler points out, "math teachers are led to believe they should use classroom tests that mimic low-quality standardized tests, even when they know the tests assess narrow mathematics."

Frequent testing in maths reinforces the fixed mindsets among students about their abilities in the subject. Grading following these tests creates further problems:

When students are given a percentage or grade, they can do little else besides compare it to others around them, with half or more deciding that they are not as good as others. This is known as "ego feedback," a form of feedback that has been found to damage learning. Sadly, when students are given frequent tests scores and grades, they start to see themselves as those scores and grades. They do not regard the scores as an

indicator of their learning or of what they need to do to achieve; they see them as indicators of who they are as people.

Challenging the view that grades motivate students and help learning, Boaler mentions studies that have shown that grading reduces achievement of students. Removing grades from the assessment/ feedback process has been shown to even improve the achievement gap between male and female students.

So, what is the alternative? The book lists nine different ways of assessing students' learning. Some of these involve students reflecting on their own learning and using a self-assessment rubric. Most of these are designed to also develop greater self-awareness and responsibility. Though these alternative assessment practices are general and can be used for any subject, Boaler has gone to the trouble of giving specific examples of how these strategies can be used by maths teachers.

Conclusion

Boaler has written this book in simple and accessible language. Though the book refers to several studies and research articles (the list of references run into eight pages!), one doesn't get a sense of reading technical writing. Many of the mathematical tasks and ideas for the classrooms have been put together separately in the form of a long appendix so that teachers can easily access them for use in their classrooms. In my view, the message of the book gains strength not so much from the mention of brain research and classroom studies but more from the detailed descriptions of actual work done by maths teachers in their classrooms.

It is a book that maths teachers should keep on top of their book pile, even after finishing reading it, to get back to for ideas and inspiration. It does a marvellous job of reinstating faith in all that is beautiful and possible in maths classrooms in the times when success has come to be equated with fast performance, when there is a relentless pressure on teachers to adopt dehumanizing practices of a system that is designed to sort, to eliminate and to select a few.

Jo Boaler starts the fifth chapter of this book with: I am passionate about equity. The chapter ends with:

I want to live in a world where everyone can learn and enjoy math, and where everyone receives encouragement regardless of the color of their skin, their gender, their income, their sexuality, or any other characteristic.

This book will, I hope, be a good step towards that world.

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