in the classroom

Student Learning and Progress

How About A Math Portfolio?

Maintaining records of math work

A not-so-common method by which the student and teacher can trace how the student's mathematical understanding develops. The article describes what a math portfolio is and how it can be used, what kind of mathematical skills it fosters, the kind of problems that can go into it and the logistical details of setting up this system.

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"*What are the main goals of mathematics education in schools? Simply stated, there is one main goal — the mathematisation of the child's thought processes. In the words of David Wheeler, it is 'more useful to know how to mathematise than to know a lot of mathematics'.*" (Goals of Mathematics Education Math, NCF 2005)

A math portfolio is an excellent way to trace the mathematisation of a child's thought processes. It is a collection of memorabilia that traces a student's growing understanding of the subject and of herself as a student of the subject. The mathematics portfolio is one of the ways by which a teacher can encourage children "to learn to enjoy mathematics", "to use abstractions to perceive relationships and structure", to realize that "Mathematics (is) a part of children's life experience which they talk about" and "to engage every child in class" – all worthwhile outcomes mentioned in NCF 2005.

Wikipedia gives the literal meaning of a portfolio as "a case for carrying loose papers" and this definition works very well for a student math portfolio. Look at the skills and attitudes developed by giving a student such a portfolio to maintain - 'a case': *organization*; 'for carrying': *enthusiasm, attachment to the subject*; 'loose papers': *spontaneity, ability to mathematise real life situations*.

Running a math portfolio project is, however, not so much a play on words as thought given to the task assigned to the student.

The math portfolio is intended to be a collection of the student's work and her reflections on mathematics over a period of time. While some teachers have chosen to use the portfolio to highlight the student's best work, I feel that the portfolio is better used as a record of progress. The work put in the portfolio need not necessarily be all 'correct' or 'perfect' – rather, it is more like a work in progress giving insights (to both teacher and student) into the development of the student's mathematical knowledge and problem solving skills. While it is maintained by the student, the teacher plays a major role in defining the student's understanding - both of the subject as well as of the level of engagement with it. Regular appraisal and review is therefore, an important part of the project.

I have found that the math portfolio is a wonderful way to acquaint myself with students at the beginning of a course and so the first task I usually set is an essay which encourages students to share their experiences in mathematics and their attitude to it. One such essay was based on the 2001 movie 'A Beautiful Mind'.

Arrange the logistical details

- Where will the portfolios be stored?
- Can the students take the work home?
- Are there any restrictions on quality of printing/publishing?
- Are e-portfolios an option?

I used the quote 'There has to be a mathematical explanation for how bad that tie is' and asked students "Do you agree with John Nash's underlying sentiment that mathematics is all pervasive? Describe some of your encounters with mathematics in unexpected situations." When a homesick international student spoke of being in a minority and how his emotions were related to numbers, I knew that the math portfolio had done what no counselor could do – get an adolescent boy to speak about his feelings!

Nothing encourages a student like success; therefore, in the initial stages, it is better to start with problems that the students find interesting and easy. Consequently, it is important to ensure that the problems are open to interpretation and exercise several skills and competencies. For example, a well-known problem states: "A person starts from home at 6.00 a.m. and reaches her destination after a journey in which she made several stops and travelled at varying speeds. If she stays at her destination overnight and starts the return along the very same route at the same time (i.e. 6.00 a.m.) the next morning, would she at any point on the return journey be at exactly the same spot at the same time as the previous day?"

I have posed this question to several groups of students and have always been struck by the diverse ways in which students attempt to understand and solve this problem. One student – who incidentally, has a learning disability – stunned me by restating the problem so as to have two people starting at the same time from both ends of the journey; the question, she said, thereby became: will they meet at any point? The more visual thinkers tried to solve the problem using a graph. And those of my students with a more theoretical bent of mind attempted a formal mathematical solution using continuous functions. I found that, in attempting this question, students are engaging with a mathematical question that exercises several skills, including that of making a logical mathematical argument. It can be attempted by students who process data in different ways and it is not so difficult that students get discouraged by its complexity. It can be used by the teacher to set the stage for teaching a variety of topics from plotting graphs to using numerical methods to solve equations. More importantly, using this problem as a portfolio problem gives students time to mull over the problem and the freedom to interpret and solve it by the method that makes most sense to them. Instead of mining a rich diversity of thoughtful solutions from students who are set this as a portfolio problem, such problems are often set as classroom exercises where they suffer from the 'fastest hand first' method of solution.

Cryptarithms (see Fun Problems in this issue) are perfectly suited to be portfolio problems, being appropriate for students who know basic multiplication, some divisibility rules and who are interested in exercising systematic logical reasoning. The teacher may even provide some scaffolding to the student in order to begin the solution. This may seem like giving away the solution, but as the explorations in a portfolio can be completely individualized, this should present no difficulty; the teacher can decide, based on her own understanding of the children, the degree of scaffolding which is appropriate for each individual child. Providing such support will make it a more meaningful exercise for students who simply say that they are 'bad at doing this kind of problem'; it may shed some light on how to start the problem and solve something that they thought they could not.

That being said, the point of the portfolio is that students move from the 'exercise paradigm' to 'landscapes of investigation'. The next problem is from a Japanese source and is reproduced in the book on "The Open-Ended Approach" by Jerry P. Becker & Shigeru Shimada. It was part of a presentation on open-ended questions at the NCTM Annual Meeting.

A transparent flask in the shape of a right rectangular prism is partially filled with water. When the flask is placed on a table and tilted, with one edge of its base being fixed, geometric shapes of various sizes are formed by the cuboid's faces and the surface of the water. The shapes and sizes may vary according to the degree of tilt or inclination. Try to discover as many shapes and sizes as possible and classify these shapes according to their properties. Write down all your findings.

The link http://mste.illinois.edu/users/aki/open_ended/flask_problem.html includes an interactivity which allows students to understand and visualize the problem before attempting the solution. Here, it is important that the teacher clearly states his expectations from the student. So it will be necessary to elaborate on the problem (which only asks for a classification based on the observed properties of the shapes). The teacher will first want the student to develop the skills of observation and abstraction and should therefore ask for diagrams. Then, the teacher will want the student to exercise recall of concepts taught in geometry and he should therefore ask that the diagrams be classified according to shape. After this, there should be a process by which a student can recognize the flow from one shape to another and the teacher could ask for a short paragraph describing the process. Since the dimensions of the flask are given in the link, there can be questions on the sizes of the shapes too.

In the words of Young (1992), the communication pattern in a traditional mathematics classroom is "Guess What the Teacher Thinks". This pattern tends to classify answers as 'right' or 'wrong' in absolute terms and consists of trying to avoid making mistakes and moving towards a predetermined outcome. Unfortunately, this means that the huge learning opportunities afforded by mistakes are not exploited by teachers or students. Neither can the student stumble upon discoveries that, on investigation, provide fresh insight into familiar concepts.

For example, in the activity suggested by Figure 1, the teacher should expect a thoughtful essay backed by calculations on whether the student would go for such a deal on not. For example, whether spending ₹5000 in a single month on clothes would sit well in the family budget (use of a pi chart). Whether 36 clothes bought at an average price of ₹138/- (calculation and concept of average) would ensure quality. Drawing up several deals (can this deal include a most desirable pair of jeans, if so what would the remaining 35 clothes be like, would you end up spending ₹5000 for that pair of jeans plus 35 useless clothes) and so on.

- A possible portfolio problem based on this is:
- Will you be saying "Bye Bye to Big shopping bills" if you go in for this deal?
- Do you agree with the statement in this advertisement?

Present your argument in a short essay, showing all the calculations you would base your reasoning on. Remember that there is no 'right' or 'wrong' decision, your argument should be based on your family's current monthly spend on clothes and on quantity vs. quality

Fig, 1 shows an advertisement in a bus that I travelled in recently.

Skills Tested

1. Understanding of mathematics concepts used.
2. Fluency in the language of mathematics.
3. Ability to
    i. Perform correct calculations.
    ii. Understand and evaluate answers obtained in order to take decisions
    iii. Use charts to communicate
    iv. Place this problem in the larger context of a monthly budget and evaluate its significance.
    v. Draw a logical conclusion based on calculations made.

As students gain a certain level of comfort with the portfolio project, the problems can increase in complexity and cross traditional boundaries between geometry, algebra, arithmetic and so on. It is useful for the teacher to have a collection of problems at different levels. Students could even be told that the portfolio must consist of 'at least one problem' from each level.

Tips on Building Student Portfolios

- Clarify how the portfolio system works. Explain the logistics of storage, submission for assessment and maintenance.
- Issue the portfolio calendar with dates for publishing and submission of problems.
- Explain the system of assessment- with a rubric that gives credit to recording and cognition of learning rather than just solutions to problems.
- Ensure that they understand that this is neither a design competition nor a collection of stories of success but a record of stories of their learning.
- Encourage them to record their process of problem solving (in words, diagrams, calculations), to give reasons for procedures chosen or rejected and to analyse the implications of answers obtained.
- Explain copyright issues and the dangers of plagiarism and the need to cite all sources.
- Set basic standards of neatness and organization - an index, record of dates and so on.

Along with the content, an assessment rubric that has both a problem specific as well as a general component needs to be created by the teacher. The former would assess logical reasoning and accuracy without stressing on a particular method. The general component on the other hand, would record and give credit to innovative problem solving, conjecture and structured investigation as well as clarity of presentation. Of course, students would need some benchmarks to guide them in this direction.

It is important to understand that student specific and not uniform standards of excellence should be the goal here. A component of self-assessment can also elicit student understanding of the lack of a particular skill and the teacher and student can assess whether there is improvement in that skill as more problems are added to the portfolio.

Much has been written about math portfolios but how practical is it for students who are studying the Indian curriculum to maintain portfolios? It may not be feasible for a student to spend more than an hour and a half a week on the portfolio. Since this time also must include components of work, presentation and reflection, it may not be practical to issue more than 4 portfolio problems a term. While many students and even parents may complain about additional work, particularly in the senior classes, the freedom as well as the responsibility of maintaining a portfolio has long term benefits on the student's independent study skills, problem solving skills, interest in the subject and knowledge of mathematics that may not be curtailed by the syllabus.

Set up the System

1. Portfolio calendar- dates/days on which problems are published, submission dates, etc.
2. Are the portfolios going to be shared with other students/parents, etc. If so, when?
3. How is this portfolio going to feed into the overall assessment along with tests, class quizzes, lab activities?
4. Will students be encouraged to create problems? If so, what are the guidelines to be given?
5. Will there be an end-of-portfolio activity- such as a summarizing of learning, a selection of 'good' problems or solutions and so on?

References

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4. Helle Alro and Ole Skovsmose: Dialogue and Learning in Mathematics Education

SNEHA TITUS, a teacher of mathematics for the last twenty years has resigned from her full time teaching job in order to pursue her career goal of inculcating in students of all ages, a love of learning the logic and relevance of Mathematics. She works as a retainer at the Azim Premji Foundation and also mentors mathematics teachers from rural and city schools. Sneha uses small teaching modules incorporating current technology, relevant resources from the media as well as games, puzzles and stories which will equip and motivate both teachers and students. She may be contacted on sneha.titus@azimpremjifoundation.org