feature

Proof Without Words

Insight into a Math Mind: Prof. A R Rao

Elegance, they say, cannot be defined, merely demonstrated. Mathematics — and mathematicians — can have incredible style. Read on to find out how.

Shailesh Shirali

It is always a pleasure to read a beautifully crafted proof; and when the proof is of the kind which uses a minimum of words — 'Proofs Without Words' as they are called — the pleasure is doubled. Here is one such proof, from the late Professor A R Rao, about whom we shall say more at the end of this article. It is a solution to the following problem: *Show that the perimeter of a quadrilateral inscribed in a rectangle is not less than twice the diagonal of the rectangle.*

Thus, for the rectangle $ABCD$ in Figure 1 and its inscribed quadrilateral $PQRS$, we must show that $PQ + QR + RS + SP \geq 2AC$. Prof Rao's solution makes extensive use of reflections. Figure 2 shows the constructions used.

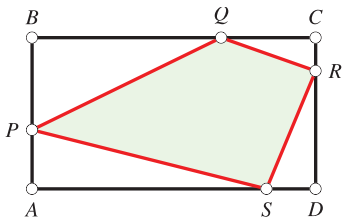

Fig. 1, Quadrilateral in a rectangle

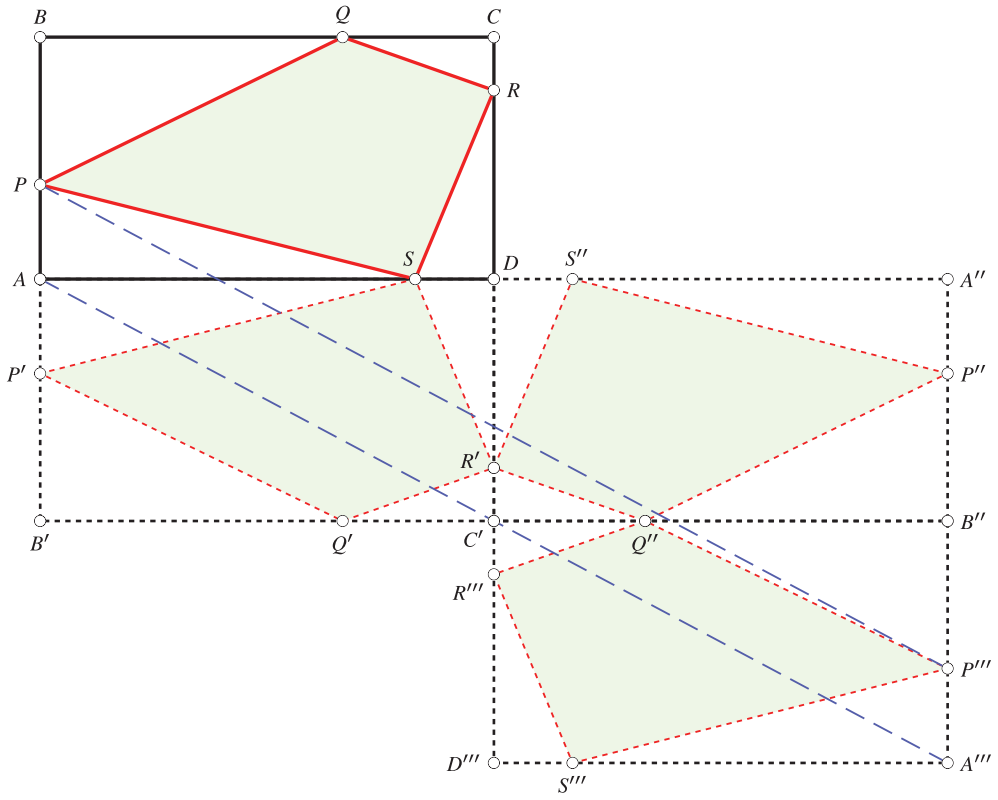

Fig. 2, Prof. A R Rao's proof - Can a diagram say a thousand words?

We first reflect the entire figure in line AD; the resulting rectangle is $AB'C'D$, and the resulting quadrilateral is $P'Q'R'S$. Then we reflect the new figure in line CD; the resulting rectangle is $A''B''C'D$, and the resulting quadrilateral is $P''Q''R'S''$. One final reflection is needed: we reflect the new figure in line $B''C'$. The resulting rectangle is $A'''B''C'D'''$, and the image of the quadrilateral is $P'''Q''R'''S'''$.

Since the length of a segment is unchanged by a reflection, the perimeter of $PQRS$ is the same as the length of the path P–S–R'–Q''–P''' (for: $SR = SR'$, $RQ = R'Q''$ and so on). The endpoints of this path are P and P'''. Since the shortest path joining two points is simply the segment which joins them, we can be sure of the following:

$$PS + SR' + R'Q'' + Q''P''' \geq PP'''.$$

Therefore, the perimeter of $PQRS$ is not less than PP'''.

Since reflection preserves length, $AP = P'''A'''$; and as the two segments are parallel to each other, figure $APP'''A'''$ is a parallelogram; thus, $PP''' = AA'''$. So, the perimeter of $PQRS$ is not less than AA'''.

But AA''' is simply twice the diagonal of $ABCD$! It follows that the perimeter of $PQRS$ is not less than twice the diagonal of $ABCD$. And that's the proof!

Professor A R Rao (1908–2011), Teacher Extraodinaire . . .

Fig. 3, Source of photo: http://www.vascsc.org

Prof A R Rao, 23 September 1908 to 11 April, 2011. This is a picture from his 98th birthday celebrations. Naturally, it is a maths lecture! Study the 4 × 4 magic square shown (Prof Rao seems to be telling us just that!) and work out how it has been formed.

Professor A R Rao was an extraordinary teacher and professor of mathematics whose life spanned an entire century. He taught in various colleges in Gujarat, and then worked as Professor Emeritus for the last three decades of his life at the Vikram A. Sarabhai Community Science Centre in Ahmedabad. He established a Mathematics Laboratory at this Centre, filled with all kinds of puzzles, games, mathematical models and teaching aids. He was amazingly creative with his hands, and just as creative in his ability to come up with beautiful solutions to problems in geometry. He had a deep love for Euclidean and projective geometry, and also for combinatorics and number theory. He was the recipient of numerous awards for popularizing mathematics. He was deeply revered and loved by large numbers of mathematics teachers and students across the country.

For a moving account of his life and career, reference (1) is highly recommended. Reference (2) is from the website of the Vikram Sarabhai centre. Reference (3) is about a remarkable theorem in geometry (the 'Pizza Theorem') for which Professor Rao found a beautiful proof using only concepts from high school geometry.

References

1. C R Pranesachar, Professor A R Rao. Can be downloaded from: http://www.ramanujanmathsociety.org/mnl/BackIssues/mnl-v21-jun11-i1.pdf
2. http://www.vascsc.org/about_arr.html
3. Shailesh Shirali, *A Pizza Saga*. Can be downloaded from: http://www.ias.ac.in/resonance/May2011/p437-445.pdf

SHAILESH SHIRALI is Head of the Community Mathematics Centre which is housed in Rishi Valley School (AP). He has been in the field of mathematics education for three decades, and has been closely involved with the Math Olympiad movement in India. He is the author of many mathematics books addressed to high school students, and serves as an editor for the undergraduate science magazine Resonance. He is currently engaged in outreach projects in teacher education in his locality. He is also a keen nature enthusiast and enjoys trekking and looking after animals. He may be contacted at shailesh.shirali@gmail.com.