FRACTIONS

A PAPER FOLDING APPROACH

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NOTE TO THE TEACHER

These activities comprise a possible approach that can be used at the Upper Primary level to **revisit** the concept of fraction, and deepen their understanding of the various rules used in the arithmetic of fractions.

The activities suggested must be repeated with many examples before patterns in results are observed, and an attempt can be made towards generalization.

At Right Angles
A Resource for School Mathematics

Azim Premji University

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FRACTIONS

A PAPER FOLDING APPROACH

SOME CAUTIONS THAT THE TEACHER NEEDS TO OBSERVE

1. **Allow children to experiment with paper freely even if there is some wastage.** They need to figure out things for themselves. Step in only when they seem to be stuck.
2. While holding any activity it is important to do a **consolidation at the end in written form** – the teacher writing on the board, and the children recording in the notebook. If possible, all the materials created should be preserved in some form, maybe pasted in the notebook, or drawn.
3. Often at the initial stage itself teachers refer to to $\frac{1}{2}$ as '1 by 2' instead of 'half'; $\frac{1}{3}$ as '1 by 3' instead of 'one third'; $\frac{1}{4}$ as '1 by 4' instead of 'one fourth'; and so on. **It is best that in the early stages teachers say 'one half', 'one third', 'one fourth', 'two thirds', 'three quarters', 'seven eighths' and so on.** This is important at the initial stage, till the children gain a proper understanding of fractions.

ACTIVITY ONE

Purpose:
To consolidate the idea of fraction and its relationship to the whole.

Materials required:
Six paper strips of equal size - either cut from ordinary ruled notebook paper or from A-4 paper or plain colour paper rolls (available in gift shops), crayons or coloured pencils, scissors

Step 1: Each strip can be considered as 1 whole.

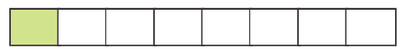
Step 2: Fold the second strip into 2 equal parts. Colour one part to show half. Show how $\frac{1}{2}$ is written. Point out: the denominator represents the total number of parts in the whole, and the numerator is the number of parts we have taken or coloured. Point out that 2 halves make 1 whole.

Step 3: Fold the third strip into 3 equal parts. Colour the first part. (Children will need guidance in folding a paper into 3 equal parts. They should not crease it till they can see 3 equal parts.) Point out that each part is called a third and that 3 thirds make 1 whole. Show that $\frac{1}{3}$ and $\frac{1}{3}$ make $\frac{2}{3}$, and that $\frac{2}{3}$ and $\frac{1}{3}$ make $\frac{3}{3}$. You can also discuss: "What is $1-\frac{1}{3}$?", "What is $1-\frac{2}{3}$?" At each step, let them record the answer in fraction form.

Step 4: Fold the fourth strip into 4 equal parts. Colour the first part. Point out that each part is called a fourth, that 4 fourths make 1 whole, that one-fourth and one-fourth makes two-fourths, and so on. Discuss: "What is $1-\frac{1}{4}$?", "What is $1-\frac{2}{4}$?", "What is $1-\frac{3}{4}$?"

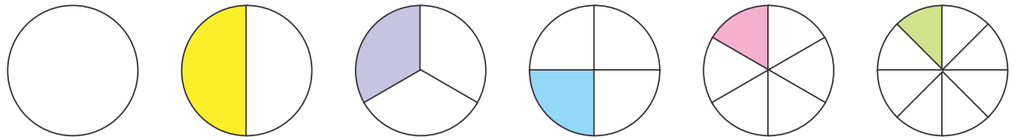
Step 5: Fold the fifth strip into 6 equal parts. (Children will again need guidance in folding a paper into 6 equal parts. Discuss with them till someone points out that they will have to first make 3 equal parts and then halve them, or first make 2 equal parts and then make 3 parts of each one.) Colour the first part. Point out that each part is called a sixth. Ask them: "How many sixths make 1 whole?" Show that that one-sixth and one-sixth make two-sixths, and so on. Discuss: "What is $1-\frac{1}{6}$?", "What is $1-\frac{2}{6}$?", "What is $1-\frac{3}{6}$?", etc.

Step 6: Fold the sixth strip into 8 equal parts. Colour the first part. Ask: "What is each part called?" and "How many eighths make 1 whole?" Show that one-eighth and one-eighth make two-eighths, and so on. Discuss: "What is $1-\frac{1}{8}$?", "What is $1-\frac{2}{8}$?", "What is $1-\frac{3}{8}$?" etc.



Step 7: Extend the activity by asking: "How many ninths make a whole?", "How many twelfths make a whole?", and so on.

VARIATION: The same activity can be done using paper plates (6 plates are needed). They can cut the parts and arrange them one over the other to understand ordering. To divide a paper plate into three equal parts, children will need to use a protractor. If they do not have the necessary skills to measure 120 degrees, they can be given a cutout for outlining. (Some children may know how to use a compass to divide a circle into six equal parts.)



ACTIVITY TWO

Purpose:
To understand the relationship between unit fractions.

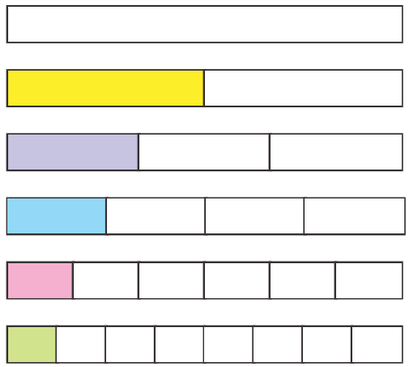
Materials required:
Use the paper strips prepared for activity 1.

Step 1: Ask the children to arrange the strips one below the other.

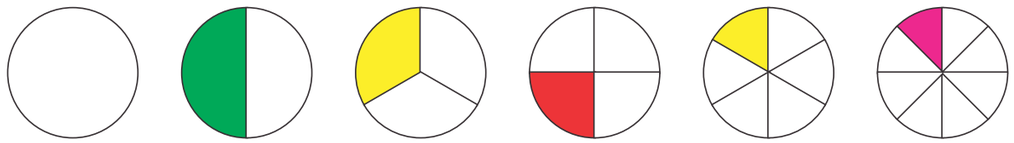
Step 2: Observe what they notice. As the denominator increases, the part size decreases.

Step 3: Children record the result using standard mathematical symbols, either in a record book for pasting paper strips, or a notebook: $1/2 > 1/3 > 1/4 \ldots$.

Step 4: Extend the activity by asking them further questions: "What is less than $\frac{1}{9}$ but greater than $\frac{1}{11}$?"



VARIATION: The same activity can be done using paper plates (6 plates are needed)



ACTIVITY THREE

Purpose:
To understand equivalent fractions.

Materials required:
Five paper strips of equal size; crayons or coloured pencils; scissors

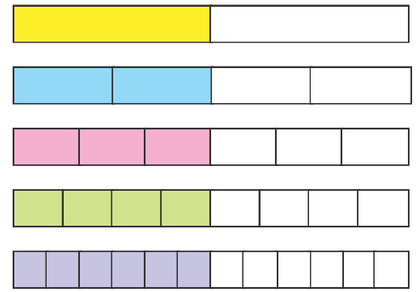
Step 1: Fold the first strip into 2 equal parts. Colour one part.

Step 2: Fold the second strip into 4 equal parts. Colour the first 2 parts.

Step 3: Fold the third strip into 6 equal parts. Colour the first 3 parts.

Step 4: Fold the fourth strip into 8 equal parts. Colour the first 4 parts.

Step 5: Fold the fifth strip into 12 equal parts. Colour the first 6 parts.



Step 6: Arrange the strips one under the other.

Step 7: Pose questions to point out the equalities: $1/2 = 2/4 = 3/6 = \ldots$.
Tell them that these are called equivalent fractions.

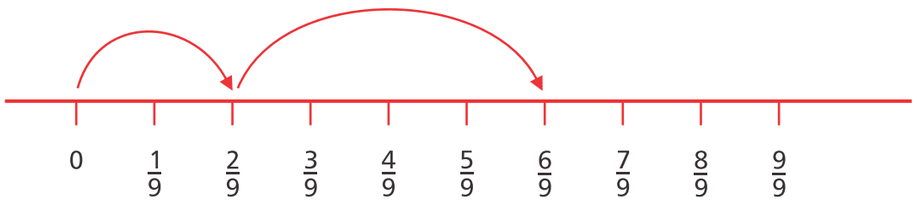
Step 8: Draw out the rule from the students for obtaining equivalent fractions for a given fraction by asking them: "How are the numerators related?" and "How are the denominators related?" Ask them to record the rule in the notebook.

Step 9: Now help them apply the rule for $\frac{1}{3}$ and get the first few equivalent fractions of $\frac{1}{3}$. Discuss: "Into how many parts will you divide the strip?"

Step 10: Extend the activity by asking the children to build equivalent fractions of $\frac{1}{4}$. Children will need guidance to do this till they understand it thoroughly.

ACTIVITY FOUR

Purpose:
To show addition and subtraction of like fractions. Additions and subtractions of like fractions can be easily shown using either a strip folded into the required parts, or on a number line. For example, for $\frac{2}{9}+\frac{4}{9}=\frac{6}{9}$:



ACTIVITY FIVE

Purpose:
To show addition and subtraction of unlike fractions.

Materials required:
8 equal paper strips

Problem:
To show $\frac{1}{2}+\frac{1}{3}$

Step 1: Take the first strip, fold into 2 equal parts and shade $\frac{1}{2}$.

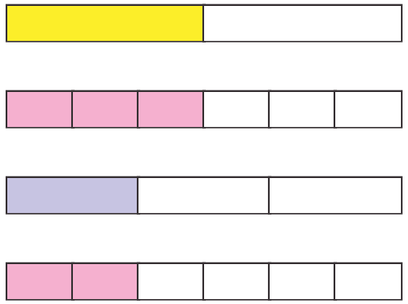
Step 2: Take the second strip, fold into 3 equal parts and shade $\frac{1}{3}$.

Step 3: Point out: The '$\frac{1}{2}$' and '$\frac{1}{3}$' are of two different sizes and cannot be counted together. Discuss their understanding of like and unlike fractions.

Step 4: Ask: "Is there a way of turning them into like fractions without changing their value?" Give a hint about equivalent fractions.

Step 5: Point out: By halving $\frac{1}{2}$ they will get fourths, and halving $\frac{1}{3}$ they will get sixths; but they are still of different sizes.

It is possible that some child will now come up with the solution that $\frac{1}{2}$ can be made into sixths (by folding into three equal parts).



Step 6: Take the first strip with $\frac{1}{2}$ shaded and fold into 3 equal parts to get sixths. Children will now see that the earlier $\frac{1}{2}$ now equals $\frac{3}{6}$.

Step 7: Take the second strip with $\frac{1}{3}$ shaded and fold it into 2 equal parts to get sixths. Children will now see that the earlier $\frac{1}{3}$ now equals $\frac{2}{6}$.

Step 8: Now point out to them that $\frac{1}{2}+\frac{1}{3}$ equals $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

Step 9: Extend this activity to other addition problems which lend themselves to paper folding; for example, $\frac{1}{4}+\frac{1}{8}$.

Step 10: Let children record the results. Then help them find the rule using the LCM of the denominators in finding the equivalent fractions to be added. (We can subtract fractions the same way).

ACTIVITY SIX

Purpose:
To show conversion between a mixed fraction and an improper fraction.

Materials required:
Paper strips or square papers

Problem:
To show $1\frac{1}{2}=\frac{3}{2}$.

Step 1: Take 2 strips or square papers.

Step 2: Fold one strip into half.

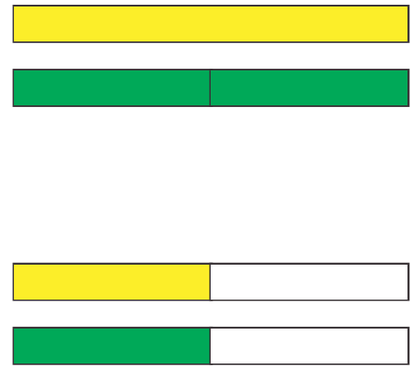
Step 3: Shade 1 whole paper and a half.

Step 4: Pose the question "How much is shaded?" Children respond: 1 and $\frac{1}{2}$. (Or 1 whole and $\frac{1}{2}$.)

Step 5: "Fold the fully shaded whole to make 2 halves. Now how many halves are shaded?" Children respond: 3 halves.

Step 6: Reinforce the fact that 1 whole is the same as 2 halves and therefore $1\frac{1}{2}$ is the same as 3 halves.

Step 7: Extend the activity to conversion of other mixed fractions to improper fractions; e.g., $1\frac{1}{4}=\frac{5}{4}$ and $1\frac{1}{3}=\frac{4}{3}$. Reinforce their understanding that a whole is equal to $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, and so on ($\frac{2}{2}$=2 halves = 1 whole, $\frac{3}{3}$=3 thirds = 1 whole, $\frac{4}{4}$= 4 quarters = 1 whole).



ACTIVITY SEVEN

Purpose:
To demonstrate multiplication of fractions.

Materials required:
Square sheets of paper

Problem:
To find $\frac{1}{4}$ of $\frac{1}{2}$

Step 1: Fold a square sheet of paper vertically in half and shade $\frac{1}{2}$ of it using vertical lines.

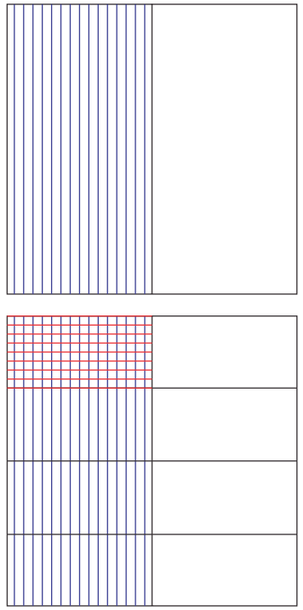
Step 2: Fold the same sheet horizontally in 4 parts. Shade $\frac{1}{4}$ of the $\frac{1}{2}$ using horizontal lines.

Step 3: Open the sheet to show the relationship of the part to the whole. Get the children to notice that one-fourth of one-half equals one-eighth.

Step 4: Extend this activity to demonstrate other multiplications of proper fractions. Ask them to record the results.

Step 5: Now help the children find the rule for multiplication of fractions by asking: How are the first two numerators related to the numerator of the product? How are the first two denominators related to the denominator of the product?

Step 6: Let children record the results in the figure.
For example: $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$, $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.



ACTIVITY EIGHT

Purpose:
To demonstrate division of a whole number by a fraction.

Materials required:
Equal paper strips; use the strips prepared for Activity 1.

Problem: $1 \div \frac{1}{2}$; $1 \div \frac{1}{3}$; $1 \div \frac{1}{4}$.

Step 1: Place the strips one under the other.

Step 2: Revisit the question "How many halves make 1 whole" Children say '2'. Emphasize: 2 halves make 1 whole.

Step 3: Point out: The question "How many halves make 1 whole?" is the same as asking "What is 1 divided by $\frac{1}{2}$?" (You can refer to their understanding of whole number division. "How many 2s in 8?" is the same as: "What is 8 divided by 2?")

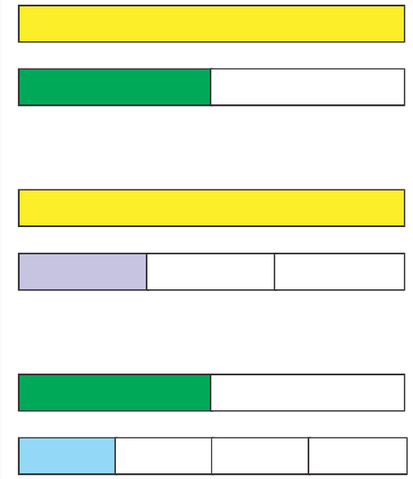
Step 4: Reiterate: "As two halves make 1 whole, therefore 1 divided by $\frac{1}{2}$ equals 2.

Step 5: Now ask: "What is 1 divided by $\frac{1}{3}$?" "What is 1 divided by $\frac{1}{4}$?" "What is 1 divided by $\frac{1}{5}$?" Establish the division facts.

Step 6: Get children to observe the pattern ($1 \div \frac{1}{2} = 2$, $1 \div \frac{1}{3} = 3$, $1 \div \frac{1}{4} = 4$, etc), and deduce the rule for getting the answer. There are 2 halves in one ($2 \times \frac{1}{2} = 1$), there are 3 thirds in one ($3 \times \frac{1}{3} = 1$), there are 4 fourths in one ($4 \times \frac{1}{4} = 1$), etc. Show them the relationship between division and multiplication.

Step 7: Now repeat these questions using 2 wholes, 3 wholes, and so on ($2 \div \frac{1}{2} = 4$, $3 \div \frac{1}{2} = 6$, $4 \div \frac{1}{2} = 8$ and so on). Explain that 1 whole = 2 halves, 2 wholes = 4 halves, and 3 wholes = 6 halves.

Step 8: Now explain the idea of reciprocal. Reciprocal of $\frac{1}{2}$ is 2, reciprocal of $\frac{1}{3}$ is 3, reciprocal of $\frac{1}{4}$ is 4, etc.



ACTIVITY NINE

Purpose:
To demonstrate division of a fraction by another fraction.

Materials required:
Equal paper strips; use the strips prepared for Activity 1.

Problem: $\frac{1}{2} \div \frac{1}{4}$.

Step 1: Fold a sheet of paper in half, i.e., to get $\frac{1}{2}$.

Step 2: Fold another sheet in half and then again in half to get $\frac{1}{4}$.

Step 3: Place the strips one under the other. Ask: "How many one-fourths make $\frac{1}{2}$?" Children say: 2.

Step 4: Point out: 2 one-fourths make $\frac{1}{2}$. Emphasize the 'one-fourth' part as they should not think of the 2 as '2 wholes'.
So: $\frac{1}{2} \div \frac{1}{4} = 2$. Let them record it as 'there are 2 one-fourths in half.'

Step 5: Extend the activity to demonstrate various other fraction divisions: $\frac{1}{2} \div \frac{1}{8} = 4$, $\frac{1}{4} \div \frac{1}{8} = 2$, $\frac{1}{3} \div \frac{1}{6} = 2$ and so on. Point out: there are 4 one-eighths in half; 2 one-eighths in one-fourth; 2 one-sixths in one-third.

Step 6: Get children to observe the pattern and deduce the rule for division of fractions using reciprocal. Let them write the above in the following form: $\frac{1}{2} \div \frac{1}{8} = \frac{1}{2} \times 8 = 4$.

GAME

Game 1 : Memory
Purpose : Practice in equivalent fractions
Number of players : 2 to 4

Materials: Make a set of cards like the ones shown. Shuffle them and lay them face down. Each player picks two cards in turns. If they are equivalent fractions he retains them, else he puts them back in the same place. Players must remember where the equivalent pairs are and 'capture' as many as possible. The game finishes when all the cards have been captured. The aim is to capture as many pairs as possible.

$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	$\frac{5}{15}$	$\frac{6}{18}$
$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{4}{16}$	$\frac{5}{20}$	$\frac{6}{24}$
$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$	$\frac{12}{18}$
$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{15}{20}$	$\frac{18}{24}$

GAME

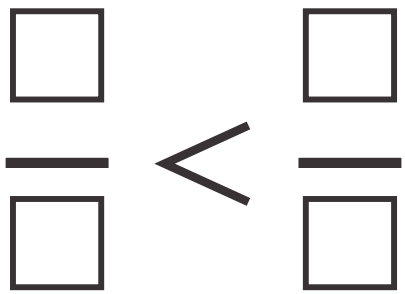
Game 2

Purpose: Practice in comparison of fractions

Number of players: 4

Materials: Number cards from 1 to 10 (2 sets), plain sheets of paper.

Each player makes a drawing like this on his sheet:



Shuffle the number cards and place them upside down. Each player takes one number card at a time and places it in one of the boxes. Once it is put in the box it cannot be changed. If a player manages to get the relation right he gets a point.

Padmapriya Shirali

Padmapriya Shirali is part of the Community Math Centre, in Rishi Valley School (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box'. Padmapriya may be contacted at **padmapriya.shirali@gmail.com**