Generating PPTs

Letter Received from a Reader

We have received an interesting letter from **Mr Aravind Badiger**, in which he describes another way of generating Primitive Pythagorean Triples (PPTs). Let x be a positive rational number, and let $a = x$, $b = (x^2 - 1)/2$, $c = (x^2 + 1)/2$; then a, b, c are rational numbers such that $a^2 + b^2 = c^2$ and $c - b = 1$. To ensure that a, b, c are in ascending order we must have $x > \sqrt{2} + 1 \approx 2.414$. By choosing x appropriately and after multiplying by an appropriate constant to clear fractions we get a family of PPTs in which $c - b$ has a constant difference; i.e., the difference between the largest two numbers in the triple is a constant. Here are some examples.

Let x take the odd integral values 3, 5, 7, . . .; we get the PPTs (3, 4, 5), (5, 12, 13), (7, 24, 25), . . . in which $c - b$ has constant value 1.

Let x take the even integral values 4, 6, 8, . . .; after doubling to clear fractions we get the PPTs (8, 15, 17), (12, 35, 37), (16, 63, 65), (20, 99, 101), . . . in which $c - b$ has constant value 2.

Let x take the fractional values 5/2, 7/2, 9/2, 11/2, 13/2, . . .; after clearing fractions we get the PPTs (20, 21, 29), (28, 45, 53), (36, 77, 85), (44,117,25), (52,165,173), . . . in which $c - b$ has constant value 8.

Let x take the fractional values 11/3, 13/3, 17/3, 19/3, . . . (fractions of the type odd number/3); after clearing fractions we get the PPTs (33, 56, 65), (39, 80, 89), (51, 140, 149), (57, 176, 185), . . . in which $c - b$ has constant value 9.

Let x take the fractional values 8/3, 10/3, 14/3, 16/3, . . . (even number/3); we get the PPTs (48, 55, 73), (60, 91, 109), (84, 187, 205), (96, 247, 265), . . . in which $c - b$ has constant value 18.

If we put $x = m/n$ we get $(a, b, c) = (m/n, (m^2 - n^2)/2n^2, (m^2 + n^2)/2n^2)$, and after scaling by $2n^2$ we recover the familiar formula for generating PPTs.

The interesting feature of this method is that it permits us to group PPTs into families in which the two largest numbers of the triple have a constant difference. And as a bonus, it makes it clear that *this difference is always of the form* k^2 or $2k^2$ (where k is some integer). This is an interesting finding in itself.

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