review

An invitation to math

Edward Frenkel's "Love & Math: the Heart of Hidden Reality" - a review

MARK KLEINER

The eminent mathematician Edward Frenkel has written his autobiography in order to convey to the general audience both the human and professional aspects of mathematics.

He explores the human aspect of mathematics by describing his work on several projects. He shows that, in addition to perseverance and hard work, solving a difficult mathematical problem requires imagination and original ideas, and that beauty and elegance usually characterize important mathematical results. The creative process of a mathematician in many ways resembles that of an artist or musician and generates the whole gamut of emotions, the most important of which is love. For it is a genuine love for mathematics that carries a mathematician through the often frustrating and emotionally demanding work. The book describes the erotic film "Rites of Love and Math," which was inspired by the short film "Rite of Love and

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Death" based on a story by the great Japanese writer Yukio Mishima who both directed the film and starred in it. Frenkel invented the plot and played the Mathematician in "Rites of Love and Math." The Mathematician creates a formula for love, but realizes that the formula can be used for both good and evil. To prevent it from falling into wrong hands, he hides the formula by tattooing it on the body of the woman he loves. The idea is that "a mathematical formula can be beautiful like a poem, a painting, or a piece of music" (page 232). The rite of death plays an important role in the Japanese culture. The title of Frenkel's film suggests that mathematics plays an important role in the world culture. He describes his motivation to create the film as follows. "In popular films, mathematicians are usually portrayed as weirdos and social misfits on the verge of mental illness, reinforcing the stereotype of mathematics as a boring and cold subject, far removed from reality. Who could want such a life for themselves, doing work that supposedly had nothing to do with anything?" (page 229). And here is his answer to his own question. "In truth, the process of creating new mathematics is a passionate pursuit, a deeply personal experience, just like creating art and music. It requires love and dedication, a struggle with the unknown and with oneself, which elicits strong emotions. And the formulas you discover do get under your skin, just like the tattooing in the film," (page 233).

Frenkel sheds light on the professional aspect of mathematics by introducing the reader to the Langlands Program, which he perceives as "a Grand Unified Theory of Mathematics because it uncovers and brings into focus mysterious patterns shared by different areas of mathematics and thus points to deep, unexpected connections between them," (page 70). Robert Langlands, currently Professor Emeritus in the School of Mathematics at the Institute for Advanced Study in Princeton, initiated the program in the late 1960s as a conjecture that hard problems of number theory can be solved by using methods of harmonic analysis. Since then, the scope of the program has significantly grown due to contributions of other mathematicians and of physicists, and at this time the Langlands Program is a program of study of analogies and interconnections between four areas of science: number theory, curves over finite fields, geometry of Riemann surfaces, and quantum physics. The first three on the above list are different areas of mathematics. The modern formulation of the part of the Langlands Program concerning these areas was arrived at by the end of the 20th century. The realization that the mathematics of the Langlands Program is intimately connected to quantum physics via mirror symmetry and electromagnetic duality came in the 21st century, around 2006-2007. Frenkel personally participated in achieving the latter breakthrough. His book presents a first-hand account of the work done.

Why is the Langlands Program important? Because it brings together several areas of science that cover a wide range of research, are sufficiently far apart, and use different methods and techniques. Each of the areas has been developing in its own way motivated by results, questions, and conjectures deemed important within it. Once an area is related to another one, the experts in each of these two areas face questions and ideas, as well as methods and techniques, transplanted from the other area, which are often new and unexpected. The situation is similar to a transfusion of fresh blood to a person whose energy level has been stagnant for a while. The interconnections are beneficial to each of the areas involved.

The book discusses in detail precursors of the Langlands Program.

One of them is the Shimura-Taniyama-Weil conjecture, which played an important role in the proof of Fermat's Last Theorem by Andrew Wiles and Richard Taylor in 1994. The theorem, a statement in number theory, had been the most famous open problem since 1637, when the French mathematician Pierre Fermat wrote about it on the margin of an old book he was reading. The reasons for the fame were, on one hand, the simplicity of the statement: there are no positive integers $n > 2$ and x, y, z satisfying

$$x^n + y^n = z^n,$$

and, on the other hand, the number of years the problem had resisted the efforts of people to solve it. Familiarity with middle school mathematics is more than sufficient to understand the problem, and many generations of people, both experts and amateurs, had confused the simplicity of the statement with the ease of proof. The first main step towards the proof was made in 1986 by Ken Ribet who showed that Fermat's Last Theorem follows from the Shimura-Taniyama-Weil conjecture. The second main step was the proof of the latter by Wiles and Taylor in sufficient generality. The Shimura-Taniyama-Weil conjecture, which establishes a connection between elliptic curves over the field of rational numbers and modular forms, relates number theory to harmonic analysis. It is a special case of the Langlands Program.

Another precursor of the Langlands Program is the Rosetta stone proposed by the French mathematician Andre Weil in 1940. The name Rosetta stone is a reference to the famous ancient stele, on which essentially the same text was written in three different languages and which was used to decipher ancient Egyptian hieroglyphs. Weil's Rosetta stone establishes an analogy between number theory, the theory of algebraic curves over finite fields, and Riemann surfaces. He visualized these three areas of mathematics as three columns of Table 1 (below).

Weil's main idea was that the middle column is a bridge between the left and right columns, i.e., the results and arguments from the left column can be translated to the right column and vice versa via the middle column. Based on his Rosetta stone, Weil formulated three conjectures, the proof of which "greatly stimulated the development of mathematics in the second half of the twentieth century," (page 104). Weil's Rosetta stone is another special case of the Langlands Program.

Frenkel formulates "four qualities of mathematical theories . . . : universality, objectivity, endurance, and relevance to the physical world," (page 228), of which objectivity seems to be the only controversial one in that it claims that mathematical concepts and ideas inhabit the Platonic world of mathematics that exists independently of the physical reality or the mental world. He quotes several famous mathematicians and physicists who support his point of view.

The book is aimed at the reader who may have no background or interest in mathematics or even hates math. Using plain language and examples from everyday life and minimizing the use of formulas, Frenkel presents highly nontrivial mathematical facts. At the same time, if the reader gets interested and decides to delve into the described theories more deeply, the Notes contain many formal definitions, rigorous proofs, and heuristic arguments. The author also describes numerous applications of mathematics to support his point that an advanced society should be mathematically literate.

The book discusses mathematics in the context of the author's mathematical career and, therefore, places the reader in the Soviet Union between years 1983, when Frenkel was in the ninth grade of high school, and 1989, when at the age of 21 he became one of the first four recipients of the Harvard Prize Fellowship after his mathematical papers had been smuggled abroad and *perestroika*, which was initiated by General Secretary of the Communist Party Mikhail Gorbachev, had lifted the iron curtain. Frenkel has lived in the United States since 1989.

Although the focus of Frenkel's narrative is his learning and doing mathematics, as well as his interaction with other mathematicians, by default he paints a picture of life in the Soviet Union.

Number Theory	**Curves over Finite Fields**	**Riemann Surfaces**

Table 1

The reader learns about the widespread anti-Semitism when Frenkel describes how he, the son of a Russian mother, was denied admission to *Mekh-Mat* of *MGU (Moskovskiy Gosudarstvennyy Universitet)*, that is, the Department of Mechanics and Mathematics at the Moscow State University, because he had a Jewish father. He discusses careers of other Jewish mathematicians, some of them truly exceptional, who were mistreated by the Soviet system. The reader also learns about prominent Soviet mathematicians, both Gentile and Jewish, who recognized Frenkel's talent, mentored him, and helped him find his place in mathematics. They did this of their own volition and without any remuneration.

I cannot resist adding a personal touch here. I graduated from *Mekh-Mat* of Kiev State University in 1969, being one of the less than ten graduates with straight A's. Out of the graduating class of 175, more than half were admitted to the graduate school at Kiev State, but the door was shut for me because of my Jewish descent. I got a job as a computer programmer, continued doing mathematics in my spare time, and defended my Ph D thesis in 1972. An academic position was beyond my reach, so I immigrated to the United States in 1979. That was possible due to the Jackson-Vanik amendment to the US federal law, which forced the Soviet government to lessen the restrictions on the emigration of Jews.

The reader gets a glimpse of the great mathematician Israel Gelfand and his weekly seminar at MGU, "an important mathematical and social event, which had been running for more than fifty years and was renowned all over the world," (page 61). Many outstanding mathematicians considered it an honor to meet Gelfand and give a talk in the seminar. It was open to both Gentiles and Jews. Since most of the latter were not formally affiliated with MGU, the seminar provided "a safe haven" where they could develop as mathematicians. The core of Gelfand's seminar comprised the "Gelfand mathematical school," of which Frenkel was a member.

The book also mentions correspondence schools, organized by famous Soviet mathematicians, Gelfand among them, that reached out to talented high school students who lived outside of major cities and did not have access to special mathematical schools. In contrast, very few prominent American mathematicians get directly involved with teaching high school students.

To summarize, the book is a lively account of the life and work of a prominent mathematician.

The issues raised are diverse enough to make the book interesting to a wide range of readers, from a person having little or no knowledge of mathematics to an experienced professional mathematician.

MARK KLEINER was born in the Soviet Union in 1946. He received his MS and PhD from Kiev State University. He worked later as a computer programmer in Kiev, but in his spare time he worked on representations of algebras. He was a postdoctoral student at Brandeis University (USA), and has been a professor of mathematics at Syracuse University (USA) since 1981. He may be contacted at mkleiner@syr.edu.

Error Regretted

Dear Readers,
In the previous issue of At Right Angles, November 2014, we had missed acknowledging the contributor for the article "Folding a 45º, 60º, 75º Triangle from a square sheet of paper in 6 easy steps" on page 24.

The article was contributed by ***Roopika Jayaram****, (Sahyadri School (KFI), Chas Khaman Dam, Tiwai Hill, Rajgurunagar, Pune – 410513. E-mail: roopikajayaram@gmail.com)*

The error is regretted.