Folding and Mapping Turned-Up Folds (TUFs)

In this issue we take up another 'origamics' exploration by Dr. Kazuo Haga from the chapter INTRASQUARES AND EXTRASQUARES *of his book.*

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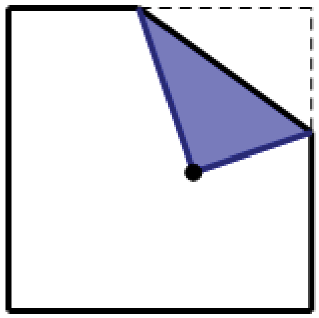
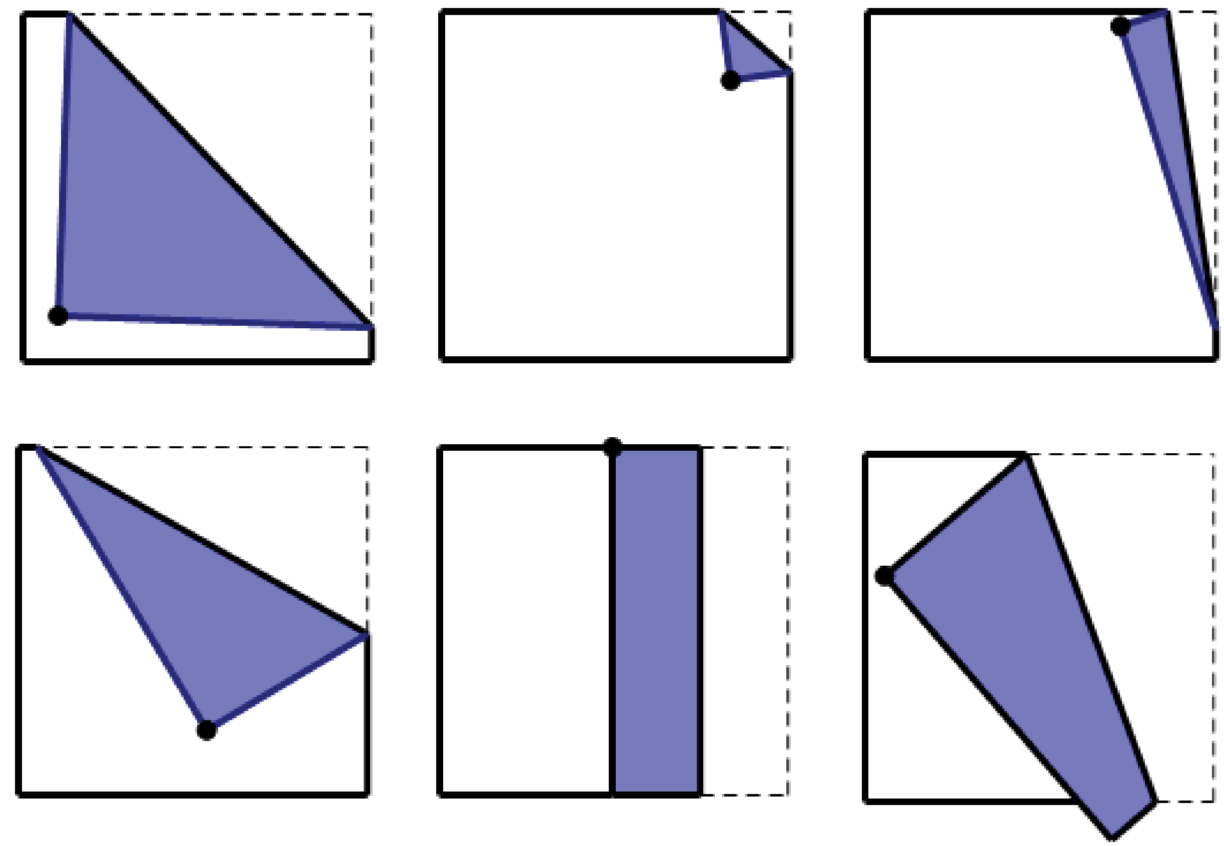
The Task: Take a square piece of paper (colour side down) and choose the upper right corner as your reference point. Pick a point at random anywhere on the paper, and fold the corner to that point. This creates a flap, and we will call it the Turned-Up Fold ('TUF').



Experiment with many TUFs. How many sides does your TUF have? Three? Four? Five? In other words, what types of polygons do you get?

Try to find an answer to the question, "How can we tell how many sides a TUF will have?"

Here are some possible TUFs:



Some possible conjectures relating to the shape of the TUF

Student A: Open the folded paper and examine the line of the fold. If the line connects two adjacent sides of the original origami square, then the flap is a triangle; if the line connects two opposite sides, then the flap is a quadrilateral.

Student B: If during folding only one vertex of the original square moves, then the resulting flap is a triangle; if two vertices move, then the flap is a quadrilateral.

Student C: If the colored portion is contained wholly in the origami square, then it is a triangle; if a part of it is outside the square then the flap is a quadrilateral.

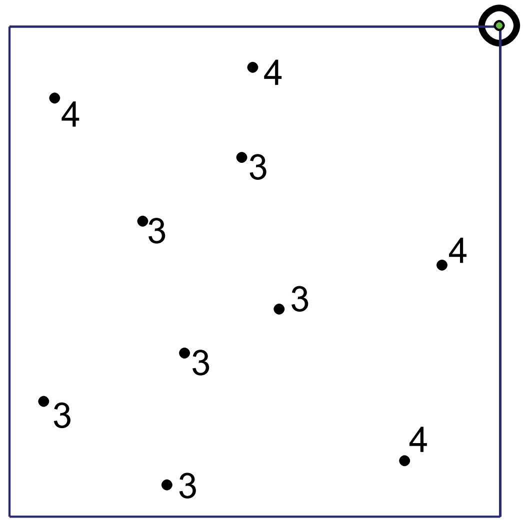
Student D: The shape of the flap depends on the position of the moved vertex on the origami paper.

Are these conjectures valid? Please do some explorations and make your own conjectures!

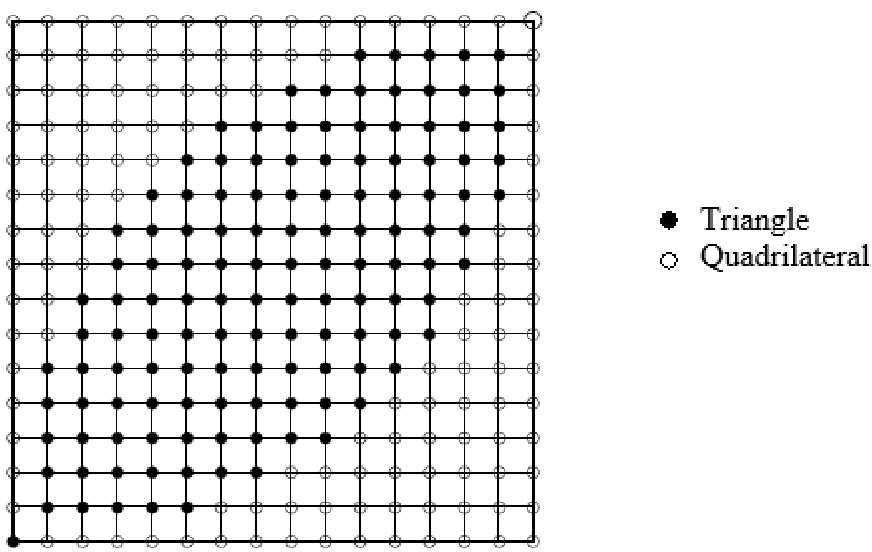
Making a Map

We continue looking for conditions that lead to the formation of a triangular or a quadrilateral flap. Use a new sheet of square paper.

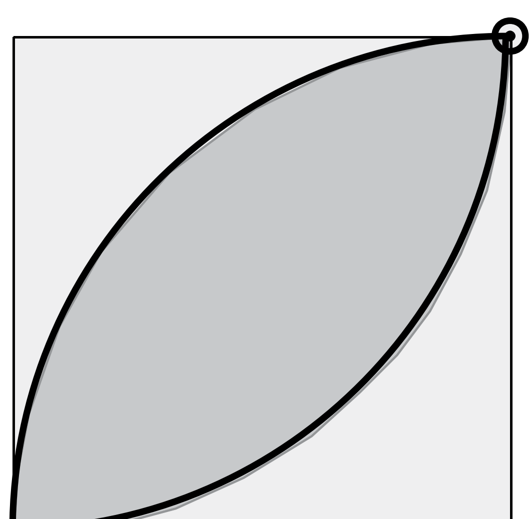
Mark at random ten points on the sheet. Select the upper right corner of the sheet to be the reference vertex. Carefully move it to a marked point and determine the shape of the flap formed. Write “3” or “4” beside the point or color it, to indicate the number of the vertices of the polygonal flap.



Next we get more organized and make a grid and the distribution area gradually appears.



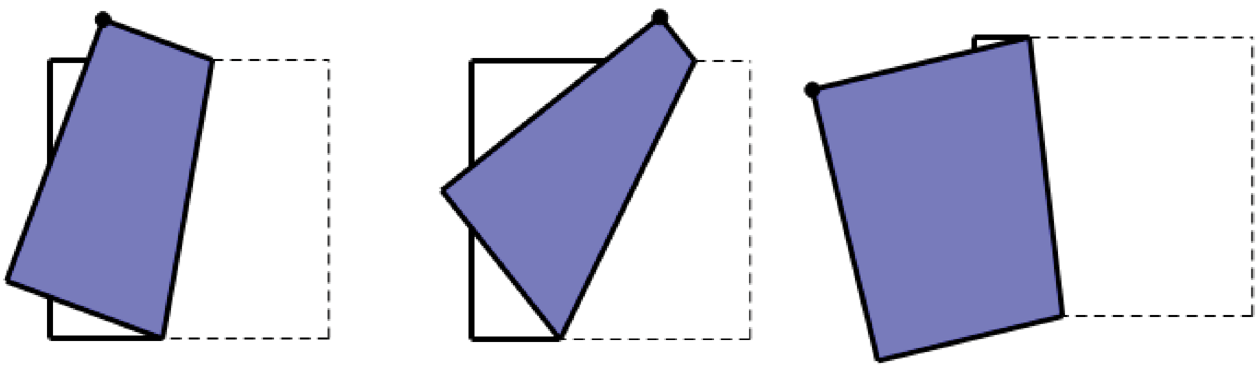
The eye-shaped region inside the square is the "triangle region" whereas the outsides form the "quadrilaterals region".



The boundary between the Triangle and Quadrilateral regions is the path traced by the reference vertex as it moves, with one end of the line of the fold fixed at one of the vertices adjacent to the reference vertex. It would be a nice challenge to explain this curious shape.

Extended Task: What if we allow the point to be outside the square? Then what are the possibilities? As an exercise try extending the above map to include all possible shapes which emerge when the reference vertex moves outside the square.

Here a few possible TUFs when the vertex moves out of the square:



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