Factor Fun...

Activities and Questions Around Factorisation

An Exercise in Factorisation

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Here is an interesting exercise in factorisation with boundary conditions, for students of upper primary school – classes 5, 6 and 7.

We generally record dates by writing the day first, followed by the month order and ending with the last two digits of the year ("common era"); i.e., we record the date as *DD-MM-YY*. For instance 17-11-32 would represent the 17^{th} of November of the year '32. In some cases the product of the first two numbers of the triad equals the third number. An example is 15-04-60. The question we pose is: *How many such dates are possible?*

We need to consider the years of only one century as the whole pattern would repeat in other centuries. We can ignore the year ending '00.

This exercise can be approached in three ways.

One is the strictly chronological. We start from the year '01 and proceed till '99. In each year we find the various possibilities (this is basically an exercise in factorisation), keeping in mind the restrictions on the possible dates and month orders. One would need to arrange the dates pertaining to a particular year chronologically again.

The second approach is to go down the month order (1-12), taking one month at a time and writing down all possible combinations of date/month/year satisfying the given condition. Ignore situations that give products exceeding 99.

The third approach is to take the dates in order (1-31), taking one date at a time and listing the possible date/month/year combinations that satisfy the given condition.

Keywords: *Factorisation, HCF, LCM, relatively prime*

The latter two approaches may be described as 'quasi-chronological'.

In practice, different groups of students could work on these approaches and the results compared. The following questions could be posed at the end:

1. How many dates DD-MM-YY satisfy the condition $DD \times MM = YY$?
2. Which year has the largest number of such dates?
3. How many years do not have any such dates?
4. Are there any interesting personal or historical connections with these dates?

Further Questions on Factorisation

Here are some additional questions about factorisation and hints to solve them.

a) **How many zeroes are there at the units' end of the number 100! (when written in its normal decimal expanded form)?** (Here, 100! is the product of the numbers 1, 2, 3, …, 99, 100.)

HINT: Every zero at the end of a number is contributed by a combination of 2 and 5 as factors. So we need to consider the number of 2's and 5's in the fully factorised form of 100!. Now the 2's must outnumber the 5's, i.e., 5 is the 'limiting factor' one needs to consider.

b) **Which 3-digit number has the largest number of factors (prime or otherwise)?**

HINT: Use the following rule here: If $N = p^a \times q^b \times r^c \times \cdots$ where $p,\ q,\ r,\ \ldots$ are primes and $a,\ b,\ c,\ \ldots$ are positive integers, then the number of factors of N is equal to $(a+1)\,(b+1)\,(c+1)\cdots$. Here the list of factors includes 1 and the number itself.

c) **Which 4-digit number has the largest number of factors?**

HINT: Use a similar approach to the earlier question. There seem to be two contenders for this problem. Perhaps the smaller of the two should be given the honours.

d) **Which is the smallest number with exactly 100 factors?**

HINT: This could be considered as a 'converse' to the earlier two questions.

e) **Let a and b be two given positive integers. How many pairs of integers can be found such that their HCF is a and their LCM is b? (The count could include the trivial case of the given numbers themselves forming the pair.)**

HINT: Two numbers can be expressed as the products CX and CY, C being a common factor and $X,\ Y$ mutually prime. Then their HCF is C and LCM is CXY. If we divide the LCM by the HCF we get XY. Now the task is to express XY as the product of two mutually prime numbers. What matters now is the number of *distinct* prime factors of XY, which we denote by d. The required number of pairs is to be expressed in terms of d.

Answers

a) 24
b) 840, with 32 factors
c) 7560 and 9240, each with 64 factors
d) 45360
e) 2^{d-1}

Afterword

The 3-digit number with the greatest number of factors has 32 factors. The corresponding 4-digit number has 64 factors. The 5-digit number with the greatest number of factors has 128 factors. The number in question is 83160. There seems to be a pattern here. Unfortunately it does not carry to the next level. The corresponding 6-digit number (720720) has 240 factors.

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