PENTOMINOES

Gautham Dayal

Pentomino puzzles were invented (or discovered) in the early 1900s by Henry Dudeney, an English inventor of puzzles (who is unfortunately not as well known as he should be). They then appeared sporadically in recreational mathematical magazines in the 1930s and 1940s. Interest in them was revived when Solomon Golomb wrote about them in the 1950s. They were popularized by Martin Gardener in his column MATHEMATICAL GAMES that appeared in the Scientific American as well as in his books on recreational mathematics. While they are a valuable educational resource in their avatar as puzzles, they can also be used effectively to build spatial intuition.

So what are pentominoes?

Take five identical squares. Now place them one at a time so that (apart from the first square) each square touches at least one of the squares already placed along a complete edge. The various shapes that can result are known as pentominoes ('penta' meaning five and 'mino' to suggest that they are related to dominos).

Keywords: *Pentomino, puzzle, spatial, congruent, transformation, translation, rotation, reflection*

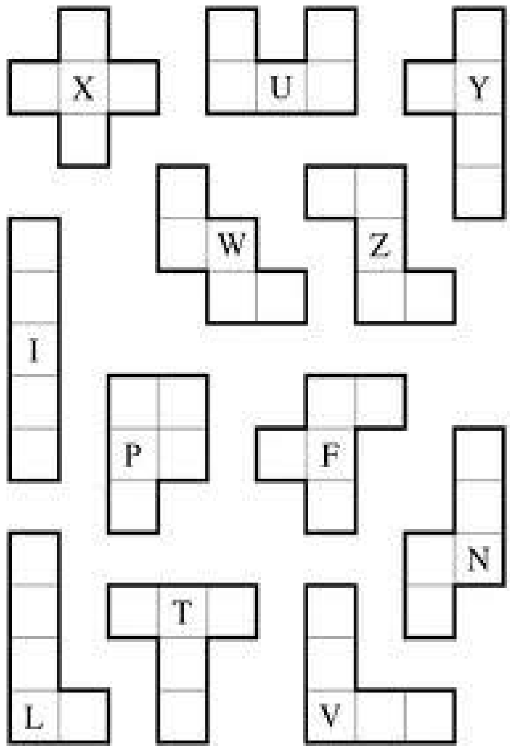


Figure 1: The twelve pentominoes

It turns out that it is possible to make *twelve* such shapes using five squares. These twelve shapes make up a set of pentominoes. In order to talk about the various pieces, it helps to name them. They are usually denoted by alphabets as in Figure 1.

Pentominoes are often used in the same way that *tangrams* are. So, a pentomino puzzle is a shape drawn on a sheet of paper. Solving the puzzle requires one to place the 12 (or sometimes fewer) pentomino pieces on a plane surface to form a shape similar to the one given on the sheet. A Google search gives a large number of sites devoted to such puzzles; the more useful of these sites grades the puzzles in order of difficulty.

In this article, we are not as concerned with *difficult* pentomino constructions (which is a natural and worthwhile place to aim to go once we are familiar with these pieces), as with ways in which pentominoes can be used in a classroom with a group of students to develop geometric intuition. However we give some examples of these construction puzzles at the end of the article, including some references.

Finding pentominoes – an activity

A worthwhile question to investigate with a group of students is that of finding all possible pentominoes and we sketch here one way to do this. While this can be done with paper and pencil, it sometimes helps to have a number of squares cut out of card (or other suitable material).

One begins with the definition of pentominoes and what it can mean for two squares to touch correctly. We observe that there is only one monomino (the square). We see different ways to add a square to this monomino and see that all the possibilities end with the same result (a domino).

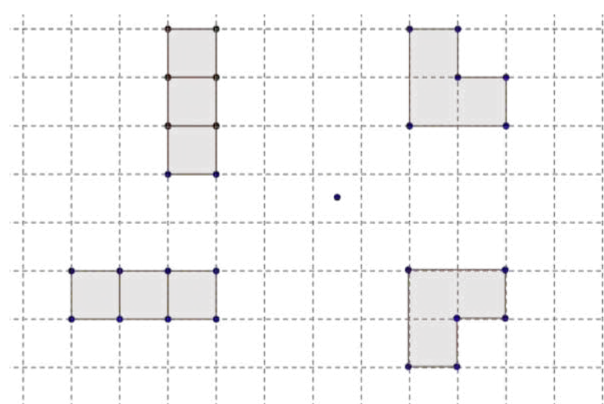


Figure 2: From domino to tromino

To move to trominoes, one sees the various ways to join a square onto a domino. While there are many ways of doing this, there are only two distinct ones ('I' and 'L'; see Figure 2). This is a good time to use words such as 'congruent', 'rotate' and 'reflect'.

Now add a square to the two possible trominoes to get tetrominoes. At this stage, it may be noticed that some pairs of shapes are not identical if only translations and rotations are allowed, but can be superposed if reflection is permitted (that is, flipping over). We find that there are five tetrominoes; see Figure 3.

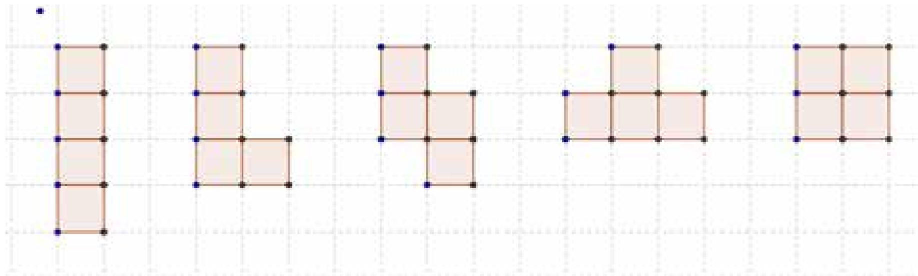


Figure 3: The five tetrominoes

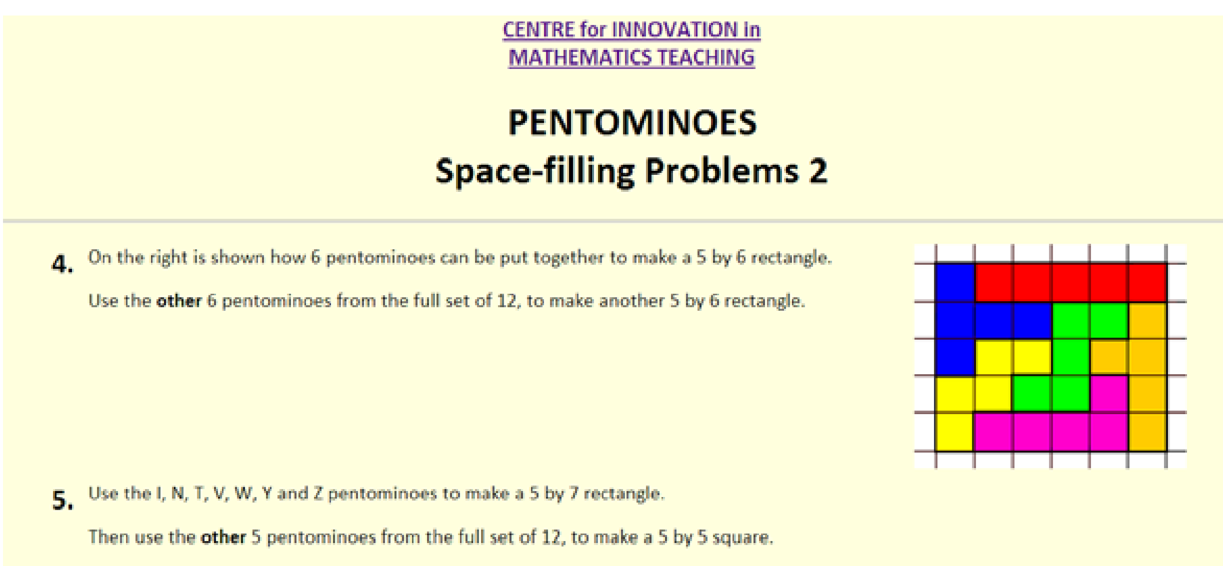
Add another square to obtain pentominoes. Students will need to keep track of the different ways in which a square can be added and the various congruences which occur to find the full list of possible pentominoes. Using the symmetries of each tetromino also makes this enumeration more efficient.

Getting to know the pentominoes

There are a couple of ways to develop a sense of familiarity with the pieces.

- One way which works well with younger students is to give them a set of pentominoes and let them construct any figure they like (this strategy also works with other dissection puzzles like the tangram): figures of animals, houses, vehicles, etc. They could be given sheets of squared paper to copy out the silhouette of the figures they have built. They can also be asked to mark out the positions of the pieces on their drawings. Often this gives rise to arrangements that can later be used as puzzles for other children.
- While it is commonly thought that a pentomino puzzle must be made using all the twelve pieces, this is not necessary. One could start off with puzzles using just two pieces and build from there. For examples of such puzzles, please look at the CIMT website from which the Space-filling problems in Figure 4 have been taken.

While these are particularly suited to develop spatial abilities in children, they are also suitable to train children to think methodically and develop reasons for eliminating possible options.



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PENTOMINOES
Space-filling Problems 2

4. On the right is shown how 6 pentominoes can be put together to make a 5 by 6 rectangle.

Use the **other** 6 pentominoes from the full set of 12, to make another 5 by 6 rectangle.

5. Use the I, N, T, V, W, Y and Z pentominoes to make a 5 by 7 rectangle.

Then use the **other** 5 pentominoes from the full set of 12, to make a 5 by 5 square.

Figure 4: Pentomino puzzles from the CIMT website (http://www.cimt.plymouth.ac.uk/)

A board game

While constructing shapes is usually the key feature while using pentominoes, there is at least one board game using the pentomino pieces, which can be played by two or three players.

Mark out an eight by eight grid (or use a chess board whose squares have the same size as the squares making up your set of pentominoes). The first player chooses a piece and places it on the board in such a way that it lies entirely on the board and follows the grid. The next player has to lay a piece in a similar manner, with the additional requirement that it does not lie over a previously placed piece. Players take turns laying pieces until one of them is no longer able to put down a piece. The last player who can lay down a piece is the winner.

The idea behind this game is related to that of pentomino exclusion, an idea that is explored in Chapter 3 of Golomb's book (referred to below).

Making pentominoes

While sets of pentominoes are commercially available, one would recommend making them by hand, especially with students. In this section we suggest a few ways in which a set of pentominoes can be made. Since much of what is involved in this activity can be used to make other puzzles and manipulatives, this opens up many opportunities. You could choose depending of the time, resources and material available.

- Graph paper and card – draw out the twelve pentominoes on graph paper so that each square has a side of 20mm or 25mm. It works better if the pieces are drawn out with no shapes touching. Stick the graph paper on a sheet of heavy KG card. Cut the shapes out using scissors. If this is being done by older students or teachers, use a blade and ruler to cut out the inside corners to get more precise cuts.
- Wood – most plywood shops sell wooden beading which is 3/4th inch or an inch thick. Choose pieces that are neatly cut. Using a tri-square, mark out pieces of the sizes you need and cut them out using a saw with fine teeth (even a hacksaw will work). Don't cut them all into squares to be more efficient (for example, cut out the I piece as a single piece, five units long, the X as one piece three units long and two squares). Using a wood glue (Fevicol SH, for example) stick these together to get the shapes you need.

  You need to keep in mind that wood sticks best 'along the grain' while orienting your pieces. Also, getting precise cuts might take some practice and patience.

  If one is able to find a workshop that cuts beading, it is worthwhile trying to get some lengths of wood that are square in cross-section. Cutting these out into cubes allows one to make pentominoes suitable for three-dimensional puzzles as well as experiment with making other puzzles like the 'soma cube'.
- Acrylic or other plastics – this is the most "high-tech" method. One can draw out the pentomino pieces using sketching software (such as Inkscape or SketchUp, both available on the net without a fee). The files can then be given to a laser cutter to be cut out on a variety of materials. Tracking down a laser cutter involves enquiring at shops that make rubber stamps or trophies. Often operators of these machines will also be able to sketch out the pieces on appropriate software for those reluctant to use Inkscape / SketchUp (though learning to use these software tools is a worthwhile skill to learn). The advantage of this method is that it is possible to make a number of sets fairly quickly, and access to a laser-cutting workshop could lead you to explore and make other *dissection* puzzles.

Resources

The book "Polyominoes: Puzzles, Patterns, Problems, and Packings" (revised and expanded second edition) by Solomon Golomb (pub: Princeton University Press, 1994) has a number of excursions into the geometry of pentominoes. While not directly accessible to most school students, it is certainly possible for teachers to adapt some of the material in it for use with students.

The internet has a large number of sites devoted to pentominoes for students of all ages, and a particularly good one is the CIMT website http://www.cimt.plymouth.ac.uk/resources/puzzles/pentoes/pentoint.htm which has pentomino problems accessible to school children, many of which are inspired by Golomb's book mentioned above.

Appendix: Some pentomino puzzle sites

There are many good sites for such puzzles, for example:

- http://puzzler.sourceforge.net/docs/pentominoes.html
- http://isomerdesign.com/Pentomino/
- http://gp.home.xs4all.nl/PolyominoSolver/Polyomino.html

Here are some typical pentomino puzzles which we have taken from the site http://isomerdesign.com/Pentomino/. (In Figure 5, the regions shown shaded represent 'holes'.)

EIGHT BY EIGHT SQUARES WITH 'HOLES' IN FOUR DESIGNATED SQUARES

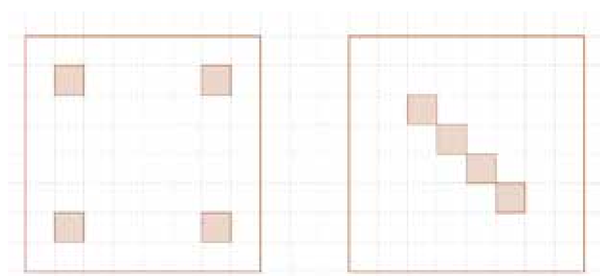

Figure 5: Two pentomino puzzles requiring making a eight by eight shape

TRIPLICATION: USING NINE PENTOMINOES TO CREATE THE FOLLOWING SHAPES

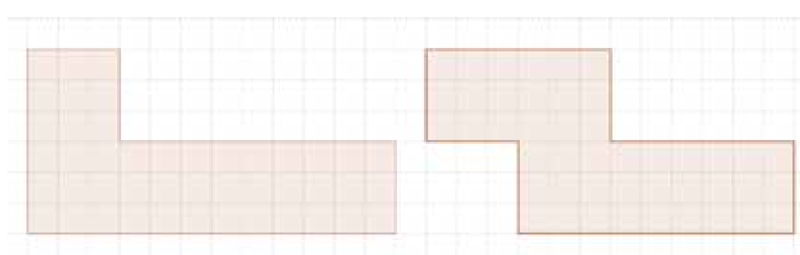

Figure 6: Using nine pentominoes to create a triplicate 'L' and a triplicate 'N'

After completing his Ph.D at the Chennai Mathematical Institute, GAUTHAM DAYAL taught Mathematics (and some Physics) at the middle and high school levels both in the context of an international school as well as in learning centres for the urban poor. He is presently working at the Centre for Education Research Training and Development (CERTAD), Srishti School of Art, Design and Technology, Bangalore.
Gautham's interests are in the ways that Making and Play can be used as ways to learn math and physics. He has more recently been exploring the opportunities that technology provides in teaching/learning. In Gautham's words: ' I enjoy working with materials,particularly wood. While I do use this interest to make puzzles and toys, I find that I am not very good at solving these puzzles.' Gautham may be contacted at gautham.vanya@gmail.com