

feature

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A Fight With Euclid

BEN ORLIN

*I had a fight with Euclid on the
nature of the primes.
It got a little heated – you know
how the tension climbs.*



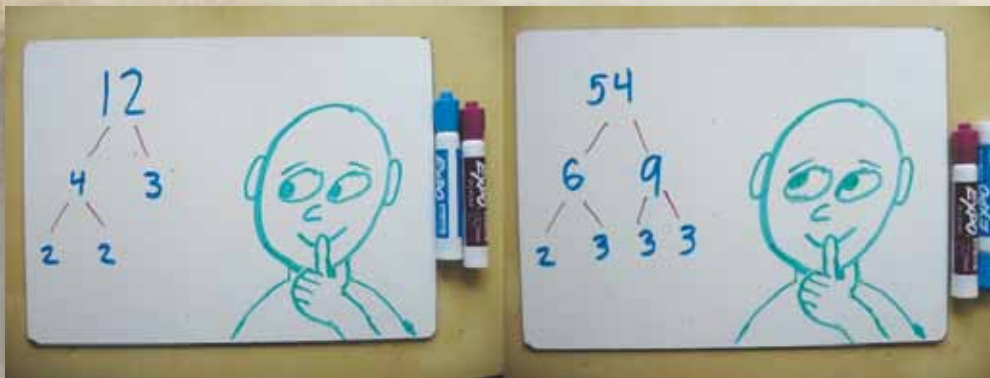
*It started out most civil, with
a honeyed cup of tea;
we traded tales of scholars,
like Descartes and
Ptolemy.*

*But as the tea began to cool,
our chatter did as well. We'd
had our fill of gossip. We sat
silent for a spell.*

*That's when Euclid turned to
me, and said, "Hear this, my
friend:*

*did you know the primes go
on forever, with no end?"*

I took a napkin to my face,
to wipe the tea and shock.
At length I said, "The primes don't end?
My friend, that's crazy talk.



In general, the integers have factors we can find.
Take 12. That's 4 times 3.
Or 54. That's 6 times 9.

But certain numbers can't be broken down in any
way. Take 17. It has no factors.
So it's 'prime,' we say.



At first, the primes are plentiful.
There's 2, 3, 5, and 7.
There's 31 and 43.
There's 19 and 11.



But as our sights climb higher, the primes start thinning out. Long gaps pass without one – throwing Euclid’s claim in doubt. So when he held his ground and said, “It’s true. The primes don’t end.”

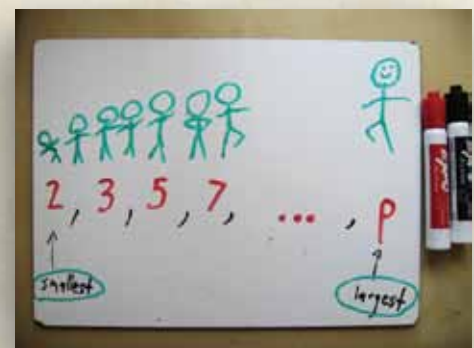
I laughed and told him, “Euclid, boy, you’ve gone around the bend!”

Then Euclid mused, “Suppose you’re right. Suppose they hit a max. In that case, there’s a largest prime. Do you accept these facts?”



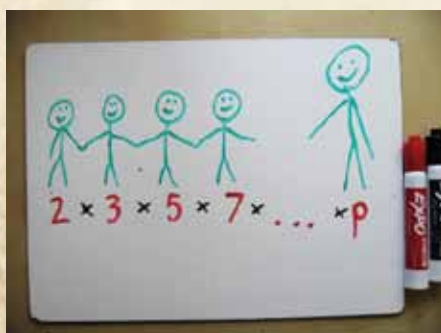
I nodded my acknowledgement, and felt a speck of pride that Euclid, proud and lauded Greek, had come to see my side.

“This largest prime must have a name,” he pressed. “Let’s call it p . The biggest prime of all the primes,” he said. “Do you agree?”



Once again I nodded, with a smile on my face, for I was putting Euclid, mighty Euclid, in his place.

“Now, let’s gather all the primes, from 2 on up to p , and multiply them all together,” Euclid said to me.



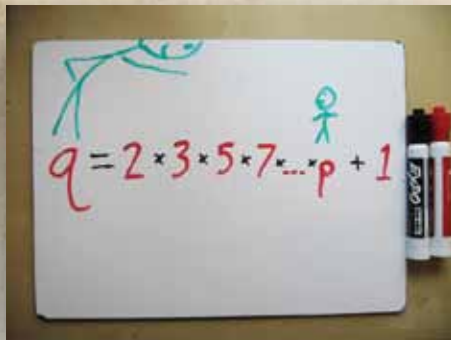
"Fine," I said, a note of worry ringing in my thoughts.
 What was Euclid plotting? Had he given up or not?
 "Multiply out all those primes," he said, "and when you're done,
 take that final product, and simply add on 1."



My confidence was fading. I was swiftly feeling dumber.
 I could not see the reason Euclid conjured up that number.

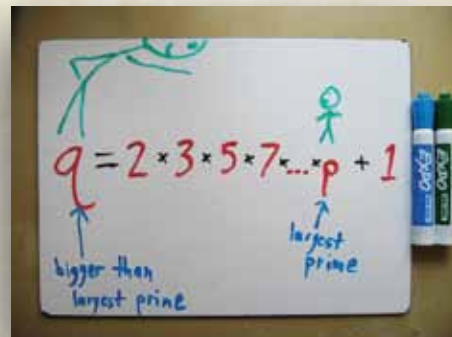
"Let's call this number q," he said,
 "this newly minted figure.
 And notice that, compared with p,
 our q is much, much bigger."

I could not disagree with this. His argument rang true.
 After all, we'd multiplied by p to get to q.



Since q is larger than the largest prime,"
 old Euclid said,
 "our q cannot be prime itself.
 It factors out instead."

I felt a trap was being sprung, but I could not resist.
 A prime that's larger than our p could simply not exist.



"Now, q has factors," Euclid said.
 "What can those factors be?
 We can't divide our q by 2.
 We can't divide by 3.
 We can't divide by 5 or 7.
 Can't divide by p.
 We can't divide by any prime," he said,
 "Do you agree?"



I saw his logic, blinding now. It scorched me like the sun.
 Divide our q by any prime; you'll get remainder 1.
 That means it has no factors.
 That means it must be prime.
 But already, we've said it's not.
 And if that's true, then I'm ...



"You're wrong!" he cried. "You see the flaw?"
I felt like such a dunce.
"You say q 's prime,
then say it's not.
It can't be both at once!"



"So take your claim that 'primes must end,' and
stick it on a shelf.
I've shown you now. That stance is flawed.
It contradicts itself!"

"That only leaves one option.
And now, you see the light.
The primes must never, ever end."
I sighed. The man was right.



We poured another cup of tea,
and smoothed our ruffled shirts.
I said, "Your argument hit hard,
and I confess, it hurts.
I thought you had conceded,
but the whole charade was fake.
You only took my side
so you could show me my mistake."

Euclid sipped his teacup with a twinkle in his eye.
"The proof by contradiction," he agreed, "is rather sly.
You stand upon the sidelines.
Your opponent takes the field.
You let him play against himself,
until his flaw's revealed."

"The truth wins out then, I suppose." I glumly drained my cup.
"The truth will win out even when it seems it's given up."

"So it is," said Euclid, "and so may it always be."
And then he kindly offered up another cup of tea.

