References

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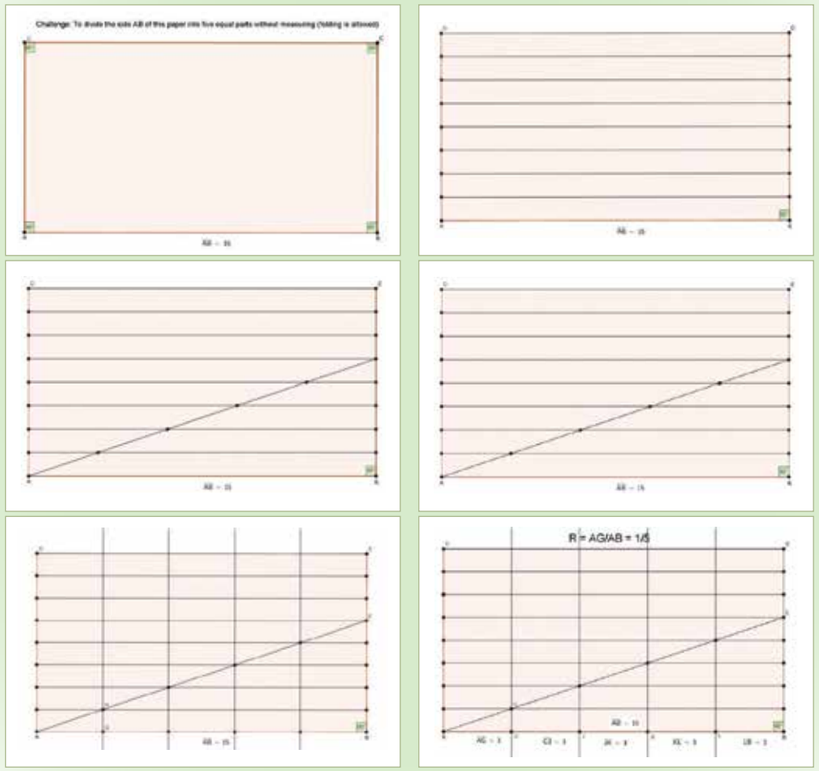
2. http://mathforlove.com/wp-content/uploads/2014/02/ How-Much-Does-Money-Cost-v2-simplification.pdf

The COMMUNITY MATHEMATICS CENTRE (CoMaC) is an outreach arm of Rishi Valley Education Centre (AP) and Sahyadri School (KFI). It holds workshops in the teaching of mathematics and undertakes preparation of teaching materials for State Governments and NGOs. CoMaC may be contacted at shailesh.shirali@gmail.com.

A GRAPHIC MATH STORY



1. What has been done?
2. Why did it work?
3. Will it work for any measure of AB?

Investigations

in the classroom

Tanuj Shah

Investigations are an integral part of the Maths curriculum in the UK, and rubrics have been created to assess children's work in this area. In India, however, investigations have not yet become a part of the maths curriculum, though the activity lends itself in a natural way to interesting mathematical work; in some ways it simulates the way a mathematician works.

An investigation can have a seemingly simple looking problem as its starting point but can lead to lines of inquiry which provide rich insight into a particular area of mathematics. It is important to let children develop their own lines of inquiry, and to have the experience of encountering 'dead ends.' In particular, one must not lead their inquiry but provide broad pointers for developing further lines of inquiry.

In this issue of *At Right Angles* we will look at an investigation which will help children 'discover' an important mathematical relationship which is generally introduced in class 7. Ideally, an hour should be allocated for this investigation, so that even the slowest student in the class is able to complete it, while at the same time there are extensions to this task which can be pursued by the quicker ones. If it is difficult to find a one hour slot, then three quarters of an hour would be sufficient, with the teacher having to provide more hints to the students.

Keywords: *Pattern, arithmetic, algebra, identities, squares, difference, sum, investigation*

Start by posing this question to the students:

Work out

$$9^2 - 8^2.$$

Is there an easier way of computing the above, without having to square the numbers?

If this is the first time the children are doing an investigation, then you could elicit from the students how to move forward with this problem. Some suggestions along the lines of working out a few more problems in the standard way and looking for patterns may come from the students (or the teacher may have to suggest it). You may then let the children get on with the problem, instructing them that when they find something they should write down their answer and then raise their hand, so you can go and check their solution. At no point should they shout out any thoughts they may have regarding the problem (to avoid disrupting other students' lines of inquiry).

You may want to orient the children in terms of the attitudes and qualities required when doing an investigation. They would need patience, and they must not be in a hurry to find the answer. In fact, there may be more than one answer; sometimes they may discover patterns which even the teacher may not have seen. Sometimes, they will encounter 'dead ends,' so the need to persevere should be emphasised. They should also get the sense that there is sufficient time to tackle the investigation and, therefore, that they are not working under any time pressure.

You will find that the children adopt a wide variety of approaches in tackling an investigation. Some children are well organized and systematic in their approach, and will have looked only at square numbers which are consecutive; they will soon be able to come up with the observation that adding the two numbers is an easier computation. They could be then asked to extend this investigation to square numbers which are not consecutive. Meanwhile, you will also see that some children are not systematic in their approach and therefore unable to see any patterns. They could be asked to look at the example again and see the relationship between the two numbers. Eventually, for those not able to get it even after the hints that you have given, you could tell them that the numbers in the example are consecutive numbers.

Student 1	**Student 2**
$4^2 - 3^2 = 7$	$5^2 - 3^2 = 16$
$7^2 - 6^2 = 13$	$8^2 - 7^2 = 15$
$10^2 - 9^2 = 19$	$10^2 - 8^2 = 36$

Children who would have seen the need to be systematic would soon be trying examples where the gap between the two numbers is 1, then 2, then 3, etc. It will not be long before they will begin to see that if the gap is 1, then you need to add the two numbers; if the gap is 2, then you need to add the two numbers and double your answer, and if the gap is 3, you need to add the two numbers and multiply by 3. They should then be encouraged to write down such statements in algebraic form. You may begin to get generalisations like this:

$$x^2 - y^2 = (x + y)\, g,$$

where g is the gap between the two numbers. A little bit of prodding to make them think about how the gap can be represented algebraically will lead them to the identity:

$$x^2 - y^2 = (x + y)(x - y).$$

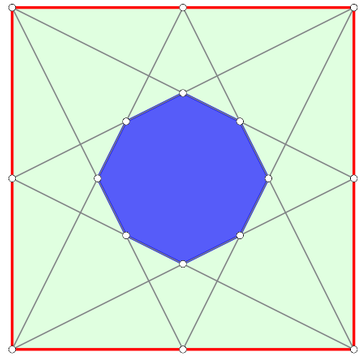
Those who are quick at coming upon the identity could be encouraged to extend the investigation by studying $5^3 - 4^3$, just to see if they could come up with a way of getting the answer in which they do not need to cube the numbers. You could give them a hint that squares of the numbers may help here.

Sometimes, children come up with interesting discoveries which we ourselves may not have seen earlier. For example, in studying the differences between consecutive cube numbers, children may come up with the following relation: $x^3 - y^3 = 3xy + 1$, where x and y are consecutive numbers. Others may come up with the more standard form $x^3 - y^3 = x^2 + xy + y^2$. You could ask the students to show how these two expressions which look very different are actually the same. Once a certain buzz has been created, you will find children attempting to look at differences of higher powers. Looking at fourth powers would be well within their reach.

One of the things about an investigative activity is that it is accessible for ALL the students in a class and the teacher can use his or her judgment on the level of support to be given to different individuals. For example, the teacher would tell the slower students to choose smaller numbers when generating their own examples. When children 'discover' the identity for the difference between two squares, they have an ownership of it and thus a greater chance of retention and an ability to use it in different situations.

TANUJ SHAH teaches Mathematics in Rishi Valley School. He has a deep passion for making mathematics accessible and interesting for all and has developed hands-on self learning modules for the Junior School. Tanuj Shah did his teacher training at Nottingham University and taught in various Schools in England before joining Rishi Valley School. He may be contacted at tanuj@rishivalley.org.

OCTAGON IN A SQUARE



The picture above shows a square in which line segments have been drawn from each vertex of the square to the midpoints of the two sides remote from that vertex (i.e., the sides which do not touch that vertex). Eight line segments have thus been drawn within the square, creating an octagon, shown in blue. Here are two questions for you relating to this octagon.

1. Is the octagon regular? (Recall that a polygon is said to be regular if its sides haveequal length and its internal angles have equal measure.) Prove your answer!
2. What is the ratio of the area of the octagon to that of the square?

E-mail your answers to AtRiA.editor@apu.edu.in.