in the classroom

An 'Origamics' Activity: X-lines

ORIGAMICS: Activities based on exploration, conjecture and proof by Kazuo Haga

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Dr. Kazuo Haga is a retired professor of biology at the University of Tsukuba, Japan. During his career as a biology professor, while waiting for his experiments to progress, he used to while away the time doing paper-folding and noting his mathematical findings through these paper-folding sessions.

He devised a set of activities and classified them under the name 'Origamics' (coined by him) as the end product was different from Origami. Unlike Origami, his exercises don't produce paper models but rather they lead to the study of the effects of the folding and seek patterns.

Haga's Origamic activities require students to explore simple, geometric properties found when we fold paper in prescribed ways.

The aim of these activities is to give students easy-to-explore paper-folding puzzles so that they can experience a micro-version of the three stages of mathematical research: exploration, conjecture and proof.

Here we look at one such activity from the chapter "X-Lines with lots of Surprises".

Keywords: *Kazuo Haga, origamics, paper folding, exploration, conjecture, proof, dynamic geometry, Geogebra*

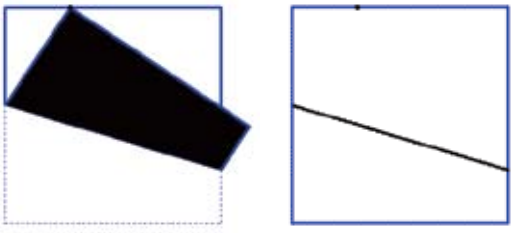
Observe the following procedure:

Step 01:

Take an arbitrary point on the upper edge of a square sheet of paper.

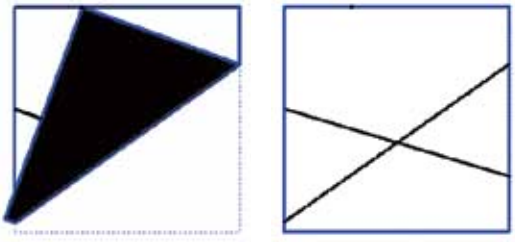
Step 02:

Place the lower left vertex onto the arbitrary point and unfold. We obtain one creased line.



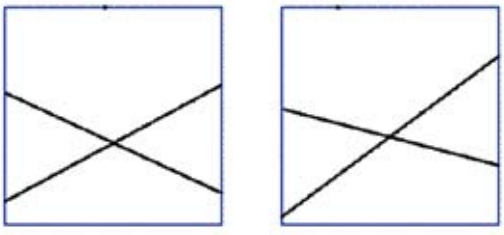
Step 03:

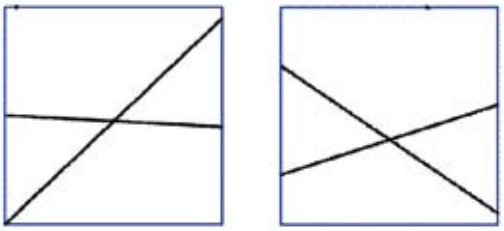
Place the lower right vertex onto the same point and unfold. We obtain two X-shaped creases. We shall call the pair of creases obtained as X-creases.



Repeat this procedure on different pieces of paper with different starting points.

Different X-creases will be obtained by different starting points, and therefore the position of the point of intersection may vary.

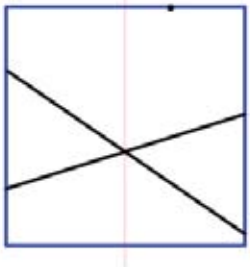




Now take one piece of paper. Make a vertical book fold to obtain the vertical midline of the square. Do likewise with your other X-creases.

What do you observe from your various X-creases?

It seems that regardless of the starting point, the intersection falls on the midline!



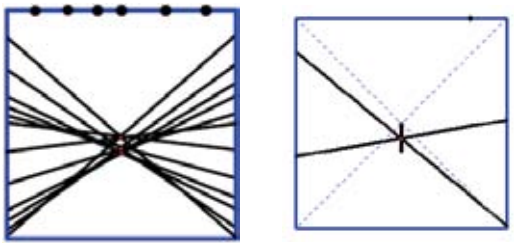
We now state our first observation:

The points of intersection of the X-creases fall on the vertical midline.

Pile up the pieces of paper which you used to make X-creases, and hold the pile up to the light.

What's your observation?

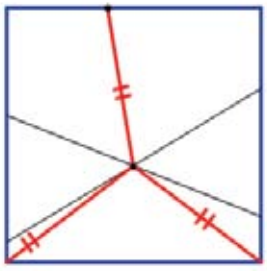
You see that the points of intersection seem to vary up and down along the midline, although within a small range.



We state our second observation:

The points of intersection of the X-creases lie along the midline and lie below the centre of the square within a certain range.

Next select a starting point and the corresponding X-creases. Draw a line from the intersection point to the starting point. Also draw lines from the intersection point to the lower vertices of the square. Then fold along an X-crease and hold the paper to the light. It comes out that two of the spokes coincide. Repeat with the other X-crease. It appears that the third spoke also has the same length!



We state our third observation:

The distances from the point of intersection to the starting point and to each of the lower vertices are equal.

We'll leave it as an exercise for the reader to prove the first and the third observation. Proofs will be provided in the next issue of At Right Angles.

Please note you can use any Dynamic Geometry Software such as Geogebra to simulate the above mentioned paper folding exercise.

Reference

[1] ORIGAMICS: Mathematical Explorations through Paper Folding, Kazuo Haga (World Scientific Publishing Co. Pvt. Ltd)

A B.Ed. and MBA degree holder, SHIV GAUR worked in the corporate sector for 5 years and then took up teaching at the Sahyadri School (KFI). He has been teaching Math for 13 years, and is currently teaching the IGCSE and IB Math curriculum at The Gandhi Memorial International School, Jakarta. He is deeply interested in the use of technology (Dynamic Geometry Software, Computer Algebra System) for teaching Math. His article "Origami and Mathematics" was published in the book "Ideas for the Classroom" in 2007 by East West Books (Madras) Pvt. Ltd. He was an invited guest speaker at IIT Bombay for TIME 2009 and TIME Primary 2012. Shiv is an amateur magician and a modular origami enthusiast. He may be contacted at shivgaur@gmail.com.