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TEACHING SUBTRACTION

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A PRACTICAL APPROACH

At Right Angles
A Resource for School Mathematics

INTRODUCTION

TEACHING SUBTRACTION

Most assessments conducted across the country indicate that the first stumbling block for many children is the subtraction operation, followed by division. On looking more closely we see that the difficulties often arise in subtraction contexts involving double digit or larger numbers. This difficulty is largely caused by three factors: (i) improper understanding of place values (ii) lack of understanding of the rationale behind the formal subtraction procedure, (iii) not seeing the connection between addition facts and subtraction facts.

This article follows the sequence of place values and addition operations and is closely linked to the ideas introduced in the preceding articles of this series. I proceed on the assumption that the reader is acquainted with the earlier ideas and activities talked about.

When do we first introduce children to subtraction? We use the concept of subtraction to introduce 'zero' by removing one object after another: say 10 balls onwards till we reach zero balls. In my childhood it was taught as a nursery rhyme: "Ten green bottles hanging on the wall, if 1 green bottle were to accidentally fall, 9 green bottles hanging on the wall, etc".

ACTIVITY ONE

Counting Backwards From 10 to 1 in Steps of 1

Let children show 10 initially with their fingers and say "10 are open". Close one finger, say "9 open", then close another, say "8 open" and so on, all the way down to "1 open"; then close all and say "zero fingers". Make sure that there is a correspondence between what they say and what they show. Some children have a tendency to count mechanically and not see the correspondence. This naturally leads to inaccuracy in counting and improper development of the number sense.

The ability to recite numbers backwards from 10 to 1 comes a little after mastering forward counting. The challenge can be raised to reciting numbers from 20 to 1, or 50 to 30, or 83 to 65, etc.

If the child has difficulty in reciting backwards one can allow the child to use a number line or number chart or tens and units material as an aid. Providing some visual support is necessary till the child internalises the pattern and observes the transition points. Teachers can draw the number line 1 to 20 on the floor and have children walk back from 10 to 1 in steps of 1 and 2, saying the numbers aloud.

While conducting this activity a teacher will surely notice that the stumbling blocks are the transition points (60, 59; 50, 49; etc). Teachers can use the number chart to make children observe these points.

Reciting numbers backwards is of value at various points in Classes 1, 2, 3, 4; it reinforces the child's understanding of the way numbers are sequenced and their place values. It helps the child to handle numbers in a flexible manner which is a prerequisite for doing mental arithmetic.

One can choose the right challenge from the extensions given for different age groups (classes 2 to 4)

- Extension 1: Counting backwards from 10 to 0 or 20 to 0 in steps of 2.
- Extension 2: Counting backwards from 100 to 0 in steps of 10.
- Extension 3: Counting backwards from 100 to 0 in steps of 5.
- Extension 4: Counting backwards from 100 to 0 in steps of 20.
- Extension 5: Counting backwards from 300 to 0 in steps of 25.
- Extension 6: Counting backwards from 600 to 0 in steps of 75. (Much more challenging.)

ACTIVITY TWO

The Three Subtraction Contexts

In order to familiarise the child with the three subtraction situations, the teacher needs to use the three different contexts (take away, comparison, inverse addition) over a period of time, in a gradual way. Problems posed in one form can be posed in another form so that children begin to see that they lead to the same operation, and are thus able to deal with it in a flexible manner.

Subtraction as take away: Initial introduction to subtraction is through the situation of 'Take away' or 'Remove' from a pile: "Remove 3 seeds from 7 seeds".

How does the child obtain the answer? Usually he counts out 7 seeds, removes 3, again by counting, and then counts the leftover and says "4". When it is given as a problem in the text with an illustration he crosses out the number of seeds to be removed and counts the rest. Counting of the leftover objects begins from 1 again (this changes to forward counting when the child begins to use the number line), so the child counts 1, 2, 3, 4 and gives the answer as "7 take away 3 is 4". Various contexts are used to demonstrate the take away situation. Also the child in his daily life often shares what he has with his siblings or friends, so they have already internalised these subtraction contexts which the teacher can fall back on.

Subtraction as comparison: Comparison situations arise both at home and in school. Teacher needs to make use of these comparison situations consciously to show that they give rise to subtractions.

Children should be thoroughly familiar with the usage of language: "How many more?", "How much less?", "What is the difference?" etc.

Subtraction as the inverse of addition: Often we solve subtraction problems by converting them into addition problems.

Ex 1. How many more to be added to 8 to make 12?

Ex 2. A shopkeeper when he has to give change. When given a Rs. 100 note for a purchase of Rs. 67, he would first return Rs. 3 and then Rs. 30, turning it into an addition problem: $67 + 3 + 30 = 100$.

ACTIVITY THREE

Backward and Forward Counting

However at some point the child will need to start visualising the numbers to do subtractions without actual objects and without resorting to counting. An intermediate step to help the child to visualise the number sequence is to use bead chains, number line, 10 frames and number charts to learn backward counting and forward counting. We will discuss here the usage of bead chains and number line to give practice in forward and backward counting.

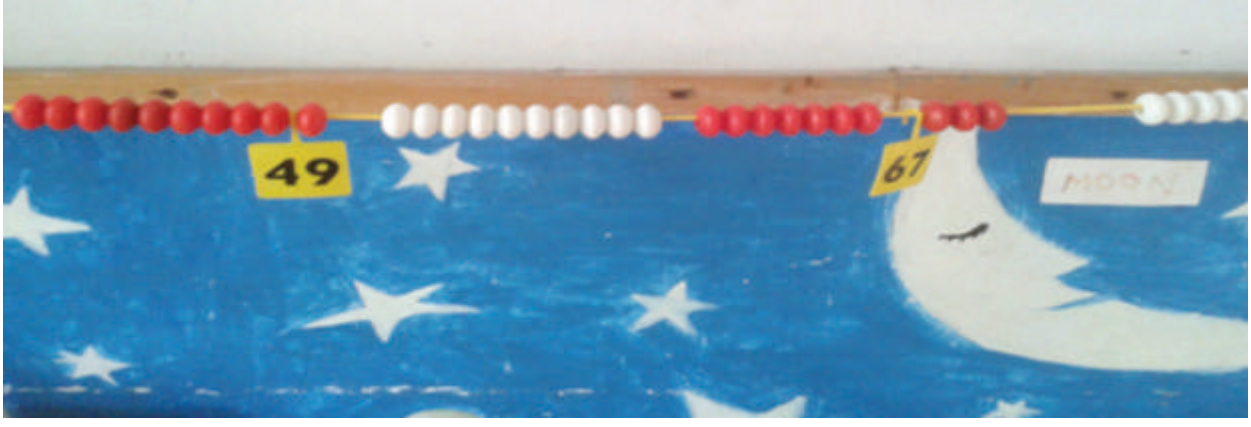
Backward counting: Backward counting is often used when the number to be taken away is small like 2 or 3. If a child has counted 17 beads and wants to remove 2, he may count backwards (perhaps using his fingers to keep track of the number to be subtracted): 17, 16 and arrive at 15 as the answer. But it is of use only in situations where the number to be subtracted is small.

Human number line: If you have a class of 30 students let each child be given a number card in order. Teacher gives a subtraction problem, say 27-3, the children upto the 27th child hold up their cards, starting from 27 three children now put down their cards.

Forward counting: Forward counting has much larger usage and application and needs to be taught in a graded manner using visuals. Bead chains with sets of ten beads in different colours can be used effectively as the change in colour denotes the break at 10, 20 etc.

Ex. 13 – 8 can be worked out as "Separate the first 8 beads from 13". Counting forward from the rest: 9, 10, 11, 12, 13, which becomes 5. (Let the children open one finger at a time while counting forward.)

Depicting 49 to 67 on a bead chain:



Counting to the nearest 10: 'Counting to the nearest ten' and using the understanding of place value is another method used as an extension of forward counting (Classes 3, 4). Example: 17 – 8 may be worked out as 8 +2 (to reach the nearest ten) and seven (as 17 is 10 + 7).

$8 + 2 + 7 = 17$, $2 + 7 = 9$, $17 - 8 = 9$

ACTIVITY FOUR

Relationship of Addition and Subtraction

Appreciation of the relationship between addition and subtraction facts can help children to use their knowledge of addition facts in solving subtraction problems.

However teachers must take care not to teach it using formal language but help children to assimilate this and use it in problem solving.

a) Show that every subtraction fact gives rise to another subtraction fact (5 – 3 = 2, 5 – 2 = 3).

This can be demonstrated using fingers. Hold up all the fingers. What If I close two fingers? Say aloud, "10 – 2 = 8". What If I close 8? "10 – 8 = 2".

Get the children to try other combinations. Each time, record both the results on the board.

10 – 2 = 8, 10 – 8= 2

10 – 3 = 7, 10 – 7 = 3

10 – 6 = 4, 10 – 4 = 6

10 – 1 = 9, 10 – 9 = 1

Let the children observe the pattern and give the related subtraction fact for a given subtraction fact.

b) Show the relationship between the subtraction fact and the addition fact (5– 3= 2, 2+3 = 5).

Often there are exercises in textbooks which show the related addition fact like this:

7–2 = 5, 5+ 2 = 7.

But this by itself cannot enhance a child's understanding of the relationship between these two facts unless the teacher points out explicitly by using materials or pictures: "Here I have 7 seeds, if I remove 3 seeds there are 4 seeds left. But if I put back the 3 seeds, I again have 7 (i.e., 4+ 3) seeds".

7 – 4 = 3, 4 + 3 = 7

7 – 2 = 5, 5 + 2 = 7

7 – 1 = 6, 6 +1 =7

The fact that the quantity removed and the quantity left sum to what we had started with needs to be internalised by the child by experiencing it.

c) Show that every addition fact gives rise to two subtraction facts.

Teacher can ask, "What do I get when I add 2 and 6?" 8. "What do I get when I take away 6 from 8?" 2. "What do I get when I take away the other number (2) from 8?" 6.

6 + 2 = 8, 8 – 6 = 2, 8 – 2 = 6

4 + 5 = 9, 9 – 5 = 4, 9 – 4 = 5

Several such examples can be done and this can be recorded on the board with pictures.

The child will see that every addition statement can be written as two subtraction statements.

One often finds children trying to subtract by counting on fingers or counting finger segments even in classes 3 and 4. Teachers must help children use number complements and addition facts to arrive at quick answers, and also commit subtraction facts to memory.

In the previous issue we discussed approaches to facilitate the learning of number complements and addition facts.

ACTIVITY FIVE | Subtraction Facts

Subtraction facts: (18 – 9 to 10 – 9, 17 – 8 to 9 – 8, 16 – 7 to 8 – 7, etc, down to 9 – 1 to 2 – 1)

In the previous issue we discussed how to break down the goal of learning addition facts into manageable sub–goals. In a similar manner the learning of subtraction facts can also be broken down into sub–goals as listed below. In order to achieve each goal there is a need to give targeted practice.

- Subtraction of 1: This is fairly simple for the child as he sees it as 'stepping back by 1 step on the number line' or 'counting backwards by 1': 18 – 1, 17 – 1, 16 – 1,.... 2 – 1.
- Subtraction of 2: This can be arrived at by counting backwards or visualising taking 2 steps back on a number line.
- Subtraction of 0: Ex. 7 – 0. Children need to clearly understand that subtracting zero means nothing is being removed. At the outset it may not make sense to them why zero should be written at all and it may baffle them.
- Subtraction ofthe number itself: Ex. 8 – 8. This too is quite clear to the child: that a number subtracted from itself gives 0.
- Subtraction where the first number is twice the second one: 18 – 9, 16 – 8, 14 – 7, 12 – 6, 10 – 5, 8 – 4, 6 – 3.
- Subtraction of 10: Ex. 18 – 10, 12 – 10. Children use their understanding of place value to give the answer. Since 18 is 10 + 8, removing 10 gives 8.
- Subtraction of numbers close to ten: Here, 10 is first subtracted from the number and then 1 is added to compensate for the 1 that was removed. So 17 – 9 is done in two stages: first 17 – 10, which gives 7, and then we add 1 and get 8, which is the answer.
- Subtraction where the first number is 1 more than twice the second one: 17 – 8 can be worked out as: 16 – 8 = 8, 17 is 1 more than 16, so 17 – 8 is 9.
- Subtraction where the first number is 1 less than twice the second one: 9 – 5, 11 – 6, 13 – 7, 15 – 8. Here 11 – 6 can be worked out using knowledge of doubles: 12 – 6 = 6, 11 is 1 less than 12, so the result is 1 less than 6, i.e., 5.

Subtraction by pausing at 10

- Consider the subtraction14 – 9. We do this as follows: 14 – 9 is 1 (i.e., 9 to 10) + 4 (10 to 14), so 14 – 9 = 5.
- Similarly: 17 – 9 is 1 (9 to 10) + 7 (10 to 17), so 17 – 9 = 8.
- Subtraction of 8, pausing at 10 (using the fact that 8 is 2 less than 10): Ex. 11 – 8, 12 – 8.
- Example: 14 – 8 is 2 (i.e., 8 to 10) + 4 (10 to 14), so 14 – 8 = 6
- Subtraction of 7 can also be done by pausing at 10: 12 – 7 = 3 (7 to 10) + 2 (10 to 12) = 5

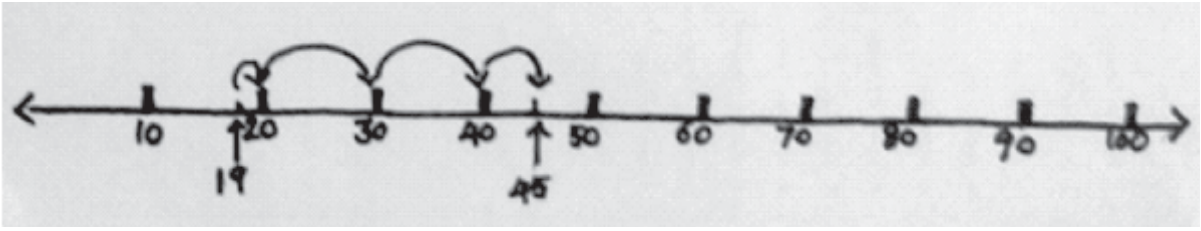
ACTIVITY SIX

Usage of Number Line (Upto 100) for Subtraction

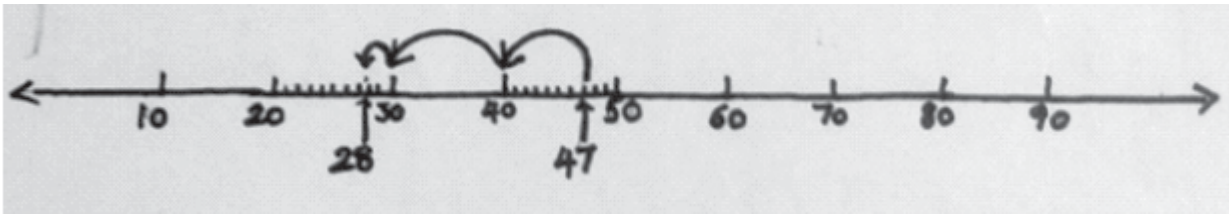
As shown in the picture it is good to get every child to prepare a foldable sturdy number line to use as an aid while solving subtraction problems.

It is good to also draw the number line on the class room wall, running along the space underneath the black board. It is less complex than a 'hundreds' chart, as movement on a number line is "forwards and backwards"; a forward move by 1 step denotes increase by 1, while a backward movement by one step denotes decrease by 1. Number line strengthens both forward and backward counting.

Forward counting: Ex. 45 – 19:



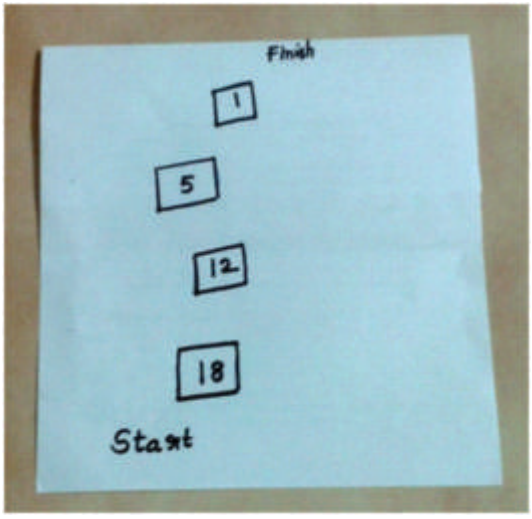
Backward counting: Ex. 47 -28:



You can ask questions which involve related subtraction facts. "From 40 if you take 8 steps backwards where do you reach?" and: "On the number line, if you are on 40 and need to reach 32 how many steps back do you go?"

GAME 1 Hopscotch

Objective: To practice subtraction facts (mental arithmetic)



This outdoor game is well known. We can modify it to suit our needs. Children can make a drawing on the ground as shown in the picture, and 3 or 4 children can use one such plan. Numbers can be varied so that children practice different subtraction facts.

'Start' and 'finish' can be fixed. The child has to hop from the starting square to another square by giving the subtraction fact. He first says 18 – 6 =12 and hops to '12'. If he makes a mistake, he can be challenged by others and he loses his turn. If he gets it right he proceeds to the next by giving again the next subtraction fact, 12 –7 = 5. This continues till he reaches the last square,'1'. Then he says 1 –1 = 0, and lands outside on both his feet!

We can also play this as an indoor game by drawing such a plan on stiff paper. Differently coloured counters can be used by each child. As the child moves his counter across the squares, he has to give the correct subtraction fact.

ACTIVITY SEVEN | Using a Hundreds Chart

A hundreds chart also acts as an excellent visual aid in solving subtraction problems. While tracing paths on the chart, children will notice patterns in the subtraction process and in the organisation of numbers, and it caters both to their kinesthetic and visual abilities. Every child should have such a hundreds chart for use as an aid while solving subtraction problems.

- However, a hundreds chart is quite different from a number line. The teacher must take time to point out the way numbers are organised on a hundreds chart. If we move horizontally (left to right) from one square to another, the number gets incremented by 1. When we move vertically downwards, the number gets incremented by 10. When we move diagonally across, it gives rise to another pattern. The teacher can pose questions to help children notice what patterns the different movements yield. Later, the teacher can use these patterns for solving problems, posing them in a graded manner.
- Subtracting 10 from any multiple of 10; ex. 40 – 10, 20 – 10, 30 – 10. The child realizes that on the hundreds chart a vertical move by 1 unit upwards results in subtraction by 10 and reduces the number in the tens place by 1.
- Subtracting 10 from any number; ex. 45 – 10, 28 – 10, 33 – 10. The child notices that subtraction by 10 does not alter the number in the units place.
- Subtracting multiple tens, ex. 50 – 20.
- Subtracting bigger numbers can be broken down into steps and can be done in two ways. Ex: subtract 22 from 45:
  - The child places his finger first on 45, moves vertically upwards 2 steps (to remove 20) and reaches 25 and then 2 squares to the left (to remove 2) to reach 23.
  - Or child locates his finger first on 45, moves 2 squares to the left (to subtract 2) and reaches 43, and then moves vertically up by 2 squares (to remove 20) and reaches 23.
- Subtracting numbers close to multiples of 10, ex. 45 – 19.
- The child moves his finger from 45 to 25 (remove 20) and then moves right by 1 square (add 1 to compensate the extra 1 removed) to reach 26.

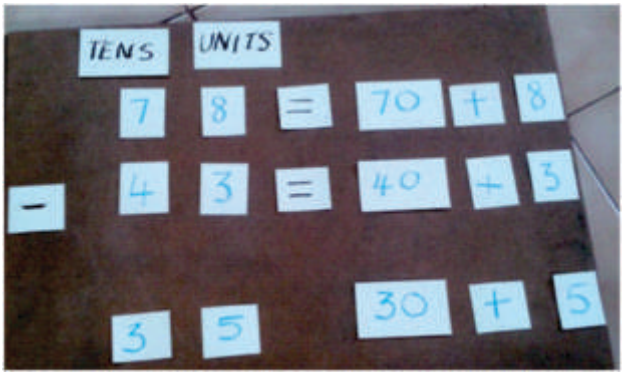
BUILDING CAPACITY FOR EXTENSION

A fundamental principle involved in teaching mathematics is to derive new facts from known ones. In teaching subtraction facts we do this by relating it to addition facts which the child has already learnt. We bring in his understanding of place values. We also use his intuitive understanding of associativity. It is important to give problems which help him to see that a subtraction fact will continue to give the same answer in whatever situation it occurs.

Ex. $7 - 4 = 3$. What is $17 - 4$? What is $47 - 4$? What is $70 - 40$?

ACTIVITY EIGHT | Using a Place Value Kit

78 is shown by using tens and units materials. This is written down in the place value table along with expanded notation and the number to be removed is recorded underneath. As mentioned, the teacher needs to constantly bring it to the child's attention that units are being subtracted from units and tens are being subtracted from tens.



It should be read as: '8 units minus 3 units gives 5 units'; '7 tens minus 4 tens gives 3 tens'.

To reinforce this a few problems can be written initially in expanded form.

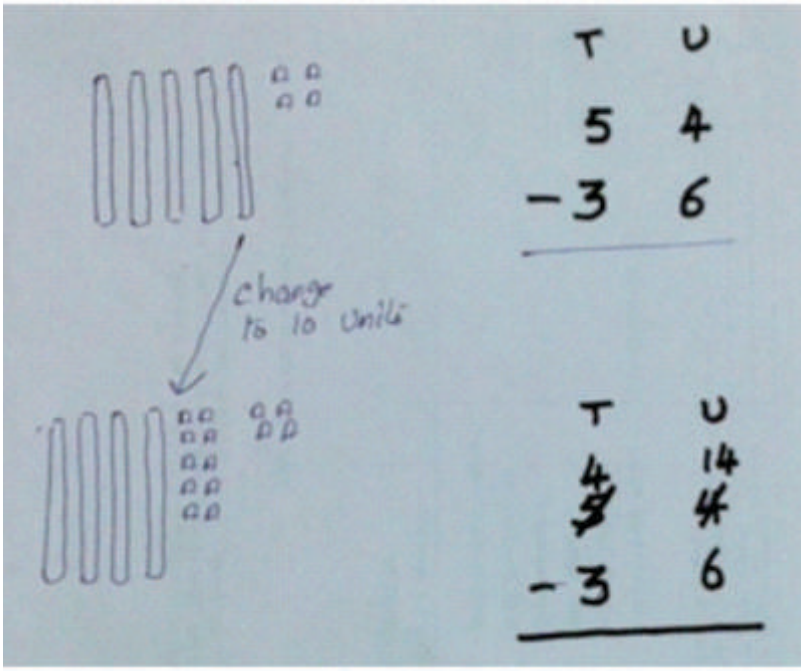
Problem exercises should initially contain visuals. Also let children use materials (tens and ones) till they gain confidence and drop usage of aids on their own.

ACTIVITY
NINE

Subtraction of Tens and Units with Exchange

Materials required: Place value kit.

Subtractions like 54 – 36 require exchanging a ten into units and recording the result appropriately. Let the child pick up tens and units corresponding to 54 and place them on the place value card. The number should also be recorded using tens and units as headers. The child now has to remove 36 from this collection. He writes the number to be removed in the second row of the place value card. Then, realising that there are not an adequate number of units for removal, he exchanges a ten for 10 units. When this exchange is done the teacher needs to help the child do corresponding recording by striking out the 5 tens and writing 4 on top in the tens place, and striking out the 4 units and writing 14 on top in the units place (some teachers may follow the convention of writing 10 on top in the units place and not strike out the number in the units place).



Extension: Subtraction of tens and units (zero in the units place) with exchange

Ex. 50 – 36

This can be performed in the same way as the earlier one, stressing on the importance of the need for exchange of a ten for units.

Many children often make errors in subtraction from zero. A child who has had plenty of opportunity to handle materials while learning subtraction is unlikely to make such mistakes.

GAME 2

Race to Zero !

Objective: Practice of subtraction involving exchanging, aids in conceptual understanding of subtraction procedure.

Materials required: Kit containing flats, longs and units (Hundreds, tens, units material) and two dice.

Number of players: 4 players, 1 Banker and 1 shopkeeper

All the materials are initially kept with the banker. He then issues to each player a flat (hundred square). One child throws 2 dice and totals the number he has got (say 9). He now has to pay the shopkeeper that amount. Since he has only a hundred he goes to the banker to exchange 10 tens, and if needed he exchanges 1 of the tens for 10 units. He records the transaction in the note book as $100 - 9 = 91$. Now he pays that amount (9) to the shopkeeper.

This same process is followed by the other three children in succession. Each records his transaction in his notebook.

After the first round it is now again the turn of the first child. As the game continues situations which require exchange and do not require exchange both will occur deepening the child's understanding of place value and the relationship of hundred to tens and ten to units.

Towards the end it is possible that the number left with the child is small, say 6, but when he throws the dice he may get a larger number. He will then skip his turn till he gets the required number.

Whoever reaches zero first is the winner.

(If the game seems to take too long, one can have a different goal post, say 50.)

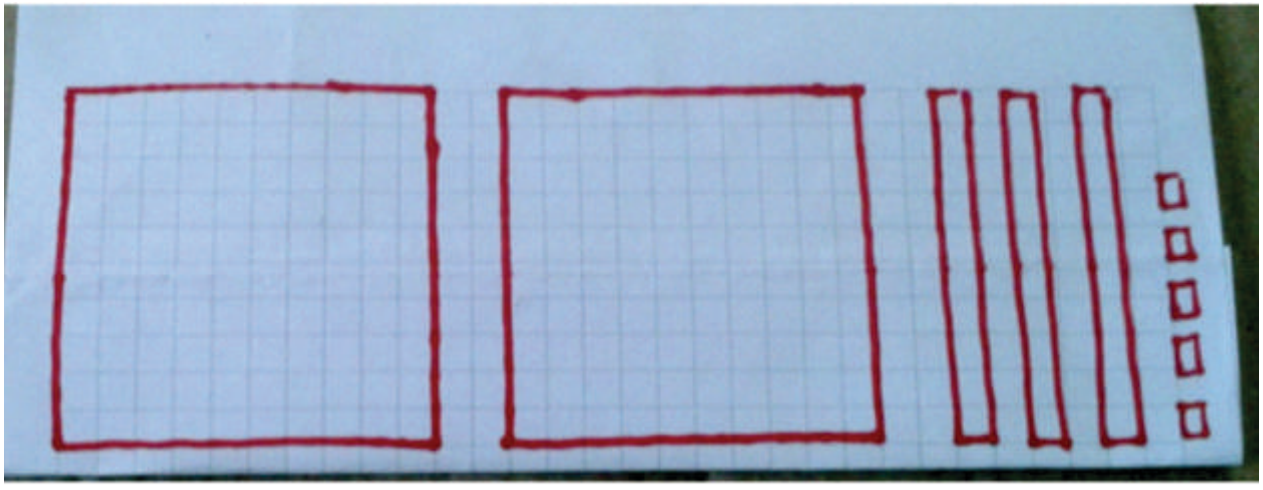
ACTIVITY
TEN

Subtraction of Hundreds, Tens and Units with Exchange

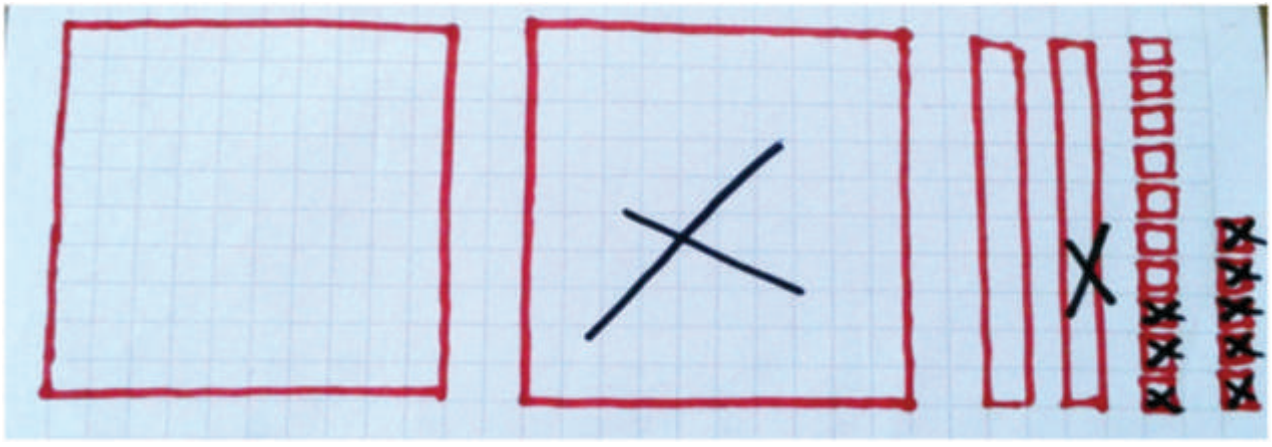
Materials required: Place value Kit.

Subtraction problems need to be taught in a graded manner: initially teach problems which require exchange of ten to units, eg., 235 – 118, and then teach problems which require exchange of hundred to tens, Ex., 342 – 161 followed by problems which require exchange of hundred to tens as well as ten to units, Ex. 245 -168.

Depiction of 235



Backward counting: Ex. 47 - 28:

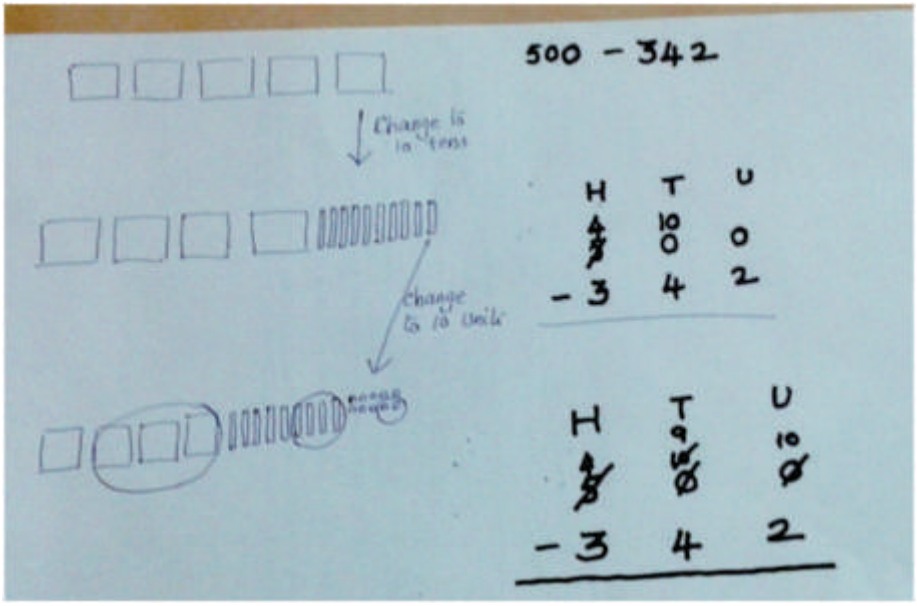


ACTIVITY ELEVEN | Subtraction Problems with Zero in Tens and Units Place

Problems involving zero could initially be with zero only in the units place, followed by problems with zero in the tens place and problems with zero both in tens and units place.

Problems with zero in both tens and units are often not understood clearly by children because they require two sequential exchanges. Unless it is demonstrated repeatedly and performed as well by the child with the help of materials, it will not be understood and the child may just pick up the procedure mechanically or may continue to make mistakes.

Ex. 500 – 342:



The teacher first takes 5 hundred square flats and poses the question, "How do I remove 342 from this?" The problem is written on the board in the usual vertical column way with place values written on top. Simultaneously there can be a visual drawn to depict the 5 flats. "I need to remove 2 units which I do not have. So I first exchange 1 hundred for 10 tens. This is how I record it here." Teacher alters the drawing by striking out 1 flat and drawing 10 tens, as well as the written form by striking out the 5 in the hundreds place and writing 4 instead and writing 10 in the tens place.

Caution: Many books and teachers tend to skip the crucial step here and write directly 9 in tens place and 10 in the units place. This step should not be skipped as it is not obvious to the child how the digit in the tens place becomes a nine.

Now the teacher shows the need for further exchange of 1 ten for 10 units and simultaneously shows it in the drawing by striking out 1 ten and drawing 10 units, and in the written form by striking out the 10 tens, rewriting 9 in the tens place and 10 in the units place.

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