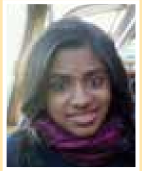
The Sand Reckoner is an enjoyable read for anyone fifteen and up, with basic knowledge of tenth standard mathematics. It may be easier to appreciate this novel if one is already immersed in the world of math and math history, but it will certainly also appeal to readers whose knowledge of the subject does not go beyond the very basic.

But the charm of the book is that it will convey the same sense of awe and excitement to everyone. It will place mathematical discovery and its applications in a historical and social context. It is the ideal way to illustrate the story-like quality of the course of math history to even the most reluctant and intimidated disciples of the subject.



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ICMI AWARDS 2015

JILL ADLER

The **Hans Freudenthal medal** is aimed at acknowledging the outstanding contributions of an individual's theoretically well-conceived and highly coherent research programme. It honours a scholar who has initiated a new research programme and has brought it to maturation over the past 10 years. The research programme is one that has had an impact on our community. It is also intended that a Freudenthal awardee should still have a minimum of a decade of active research work ahead of him or her so as to continue contributing to the field. In brief, the criteria for this award are depth, novelty, sustainability, and impact of the research programme.

Professor Jill Adler, FRF Chair of Mathematics Education, University of the Witwatersrand, South Africa is the awardee for 2015, in recognition of her outstanding research program dedicated to improving the teaching and learning of mathematics in South Africa – from her 1990s ground-breaking research on the dilemmas of teaching mathematics in multilingual classrooms, to her subsequent focus on problems related to mathematical knowledge for teaching and professional development. Her research has served to advance understanding of the relationship between language and mathematics in the classroom.

Over the last two decades, she spearheaded several large-scale teacher development projects aimed at developing mathematics teaching practice at the secondary level, so as to enable more learners from disadvantaged communities qualify for entry to mathematics-related courses at university.

Jill Adler was born in Johannesburg and graduated from the University of the Witwatersrand with a B.Sc. in mathematics and psychology (1972). She taught secondary school mathematics in a so-called 'coloured' school in Cape Town – an experience that she credits for strengthening her concerns about educational inequality and leading her to work in that direction. This was followed by many years spent on developing materials for adults and alienated youth excluded from school mathematics learning in apartheid South Africa. In 1985, she obtained a M.Ed. for her dissertation: *Mathematics by newspaper in South Africa: junior secondary mathematics for adults through the medium of a newspaper. Her doctoral research (1996) was titled: Secondary teachers' knowledge of the dynamics of teaching and learning mathematics in multilingual classrooms.*

In addition to her international research at the cutting edge of the field, she has played an outstanding leadership role in mathematics education research in South Africa, Africa, and beyond, and has helped in adding to the human research capacity in Southern Africa. Her contributions to the development of research and practice have earned her leadership positions in renowned international and national professional associations.

review

A Review of

Math! Encounters with High School Students

Dialogue and Mathematics—Serge Lang Style!

SHASHIDHAR JAGADEESHAN

The notion of dialogue and mathematics may at first seem a strange combination, but if one thinks about it, often in a lively interactive classroom this is exactly what is transpiring. According to the late physicist David Bohm, the root of the word *dialogue* comes from the Greek *dialogus*. The word *logos* in turn can be interpreted as 'meaning of the word' and *dia* means 'through'. So dialogue can then be seen as a process where there is a flow of meaning. All teachers would agree that this is what they would like in their classrooms.

The book under review, *Math! Encounters with High School Students* by Serge Lang, is an old one, published in 1985, but well worth bringing to the notice of students and teachers of mathematics. It is a series of seven dialogues on mathematics with school students and a postscript discussing mathematics teaching.

Apart from the content, which I will discuss later, the book is unique in its style of delivery. Even though we are not in the audience, we can sense the energy and excitement of the exchange. One wonders (without being completely reductionist), what are the ingredients needed for such a flow of energy and meaning to take place between teacher and taught? The obvious

Keywords: *Dialogue, facilitation, pedagogy, creativity*

ones are a mastery of the subject on the part of the teacher, an ability to gauge the level of the students and begin from where they are, a sense of humour, encouraging students to think on their feet, generating a creative tension and finally pulling it all off.

The excerpts below illustrate these points well.

Excerpts from page 20

Serge Lang: . . . $2\pi r = C$. There is your formula. Do you agree that's a proof? [*Serge Lang points to Rachel.*]

Rachel: Yes. [*Her tone is uncertain.*]

Serge Lang: You do?

Rachel: Yes. [*Laughing a little.*]

Serge Lang: What do you mean 'yes'? Is it yes by intimidation or a yes by conviction? Or a little bit of both?

Rachel: A little bit of both. [*Laughter.*]

Serge Lang: Well, where is the intimidation?

Rachel: I don't know.

Serge Lang: You don't know? [*Laughter.*] All right, let's make it all conviction. Look, where do I start from? . . .

Excerpts from pages 34 and 35

Serge Lang: . . .Do you accept all that? [*Students approve . . .*] So we can make a general result:

Under dilation by a factor of *r*, *s*, *t* in the three dimensions, the volume of a solid changes by the factor of the product, *rst*.

Just like yesterday: area changes by a factor of r^2 if we dilate by r in each direction; a factor of rs if we dilate by a factor of r in one dimension and s in the other dimension; and now volume changes by a factor of rst if you dilate by a factor of r in one dimension, s in the another and t in the third.

And the three dimensions are in perpendicular directions. Now I will deal mostly in three dimensions, but what would be a natural generalization of this? Serge.

Serge: (a student): I don't know.

Serge Lang: What's a generalization of what I have just done there? I started in 2 dimensions, then I went to 3 dimensions . . . .

Serge: [*Interrupts.*] Four dimensions. OK. It's the next product. I see. It's *rst* whatever.

Serge Lang: Ah, *rst* whatever. That's right. So suppose I have a solid in four dimensions. You see the four dimensions? Now I can't draw it.

Serge: Well, you could not draw it either in three dimensions!

Serge Lang: That's a very good remark. You are absolutely right. So the truth of what I am saying does not depend on my ability to draw the picture! . . .

And suppose I have a solid in n dimensions, and I make the dilation by factor of r in all directions, in all n dimensions. How does the volume change?

Serge: r to the power n.

Serge Lang: r^n, and that's how it is in n dimensions. OK? Any problems? Sandra.

Sandra: No. [*The other students nod, and seem perfectly at ease.*]

Serge Lang: . . .But I think it's remarkable how you react to the possibility in n dimensions.

[*Laughter.*] I am slightly taken aback at the way you just went along with it.

Excerpts from page 120

Student: So far you used different methods; you first used one method, then you changed the method; probably a different method would do it for all numbers.

Serge Lang: That is a very weak argument. [*Laughter.*] Because the argument is based on psychology, and I am asking you to deal with mathematical problems. Not psychological ones. [*Laughter.*] So if you start basing your mathematical intuition on my psychology, [*Laughter.*] you're going to have a hard time with it. That's dangerous. Think again.

[*Students talk among each other.*]

Serge Lang was born in 1927 in Paris and died in 2005 in Berkeley, California. Anyone who has studied higher mathematics would be familiar with his name as the author of mathematics books on almost every topic under the sun! On his retirement from Yale University in 2005, where he was a faculty member from 1972, Yale president Richard C. Levin shares a joke about this. "Someone calls the Yale Mathematics Department, and asks for Serge Lang. The assistant who answers says, 'He can't talk now, he is writing a book. I will put you on hold'." He was a prodigious author and wrote more than 61 books (some feel this is an underestimate) and 120 research articles. Most famous amongst his textbooks is *Algebra*, which is a classic in the area. For school teachers, apart from the book under review, I would recommend they refer to [2] and [3].

Lang could not have had a better mathematical lineage. He wrote his PhD thesis under the famous algebraist Emil Artin and did postdoctoral work with André Weil. He won the Cole Prize (1959) and the Steele Prize (1999) of the American Mathematical Society. He was elected to the National Academy of Sciences in 1985.

He was a deeply committed teacher with a great passion for communicating mathematics and devoted a considerable part of his life to teaching. In recognition for his commitment he was awarded the Dylon Hixon Prize for teaching in Yale College. His passions included mathematics, music and politics ('trouble making' in Lang's words). Jorgenson and Krantz pay him the greatest compliment (from my point of view) that a person can receive: "Serge Lang's greatest passion in life was learning" [1]. He demonstrated this by writing books and teaching courses in new areas of mathematics, because he believed that that was the best way to learn. He was famous for cajoling young mathematicians to teach him new mathematics.

Reading the article [1] on Lang by Jorgenson and Krantz, where several famous mathematicians recall their interactions with him, a picture emerges of an extremely colourful and energetic personality, not always the easiest of persons to relate to. Any attempt to categorize him would soon fail, because he seems to be rather short tempered and confrontational, but at the same time kind and generous, especially to young people and his students. He was driven by strong convictions and fought several public battles based on these convictions. It is best to quote Lang on this!

> *I personally prefer to live in a society where people do think independently and clearly. One of my principal goals is therefore to make people think. When faced with persons who fudge the issues, or cover up, or attempt to rewrite history, the process of clarifying the issues does lead to confrontation, it creates tension, and it may be interpreted as carrying out a 'personal vendetta' . . . . I regard such an interpretation as very unfortunate, and I reject it totally.*

Let us turn our attention to the contents of the book. The intention is to make beautiful mathematics accessible to students of roughly class 8 to 10. The first dialogue is called "What is *pi*?" It is extremely important that high school students get a good understanding of this well-known constant of nature. The misconceptions about π that I encounter among teachers and students often alarm me! They remember it as the fraction $\frac{22}{7}$, or 3.14, and very few are aware that it is irrational, let alone transcendental. Lang actually deals with the subtle point as to why the same constant π shows up both in the formula for the circumference and area of a circle.

Dialogues 2 to 5 deal with derivations of the formulae for the volume of a pyramid, cone and sphere and the formulae for the circumference of the circle and the surface area of the sphere. Lang uses essentially Archimedes' method of 'exhaustion' for these derivations. As far as I am aware, standard school mathematics textbooks rarely derive these formulas. Perhaps there is a feeling that these derivations are too difficult, or that they are best done using integral calculus. But, as Lang so aptly demonstrates, they are very accessible to younger students, and in fact if done before the students see integral calculus, it serves to show not only the power of calculus, but also the limitations of the method of exhaustion.

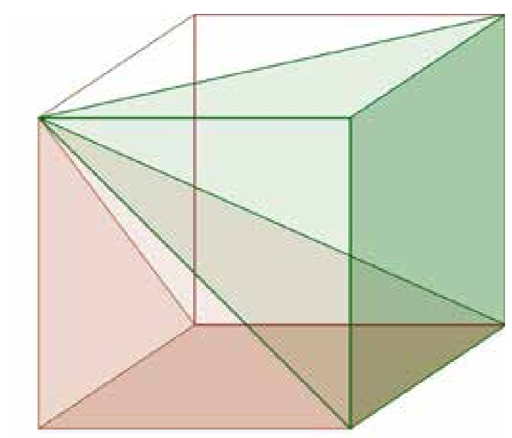

Figure 1.

One of my favourite parts is the derivation of the volume of a pyramid. Lang and his students stumble upon the special case of the cube, which can actually be divided into three congruent pyramids (see Figure 1). This helps us to understand where the $\frac{1}{3}$ factor comes in. It is also a nice activity to get students to make nets of these pyramids, as many surprises await the student in doing so!

Dialogue 6 deals with Pythagorean triplets. Here students are introduced to the problem and the complete solution is demonstrated with the help of the parametric representation of the unit circle

$$x(t) = \frac{1-t^2}{1+t^2} \text{ and } y(t) = \frac{2t}{1+t^2},$$

explaining the geometric significance of the parameter t. Here t is the slope of a special line, and one gets a very elegant connection to double angle formulae from trigonometry.

The last mathematical dialogue deals with infinities, taking students from the very basics all the way to the result that real numbers are not denumerable.

I have used this little book in many ways as a teacher: as a reference book, as a model to conduct mathematical dialogues and also a source for students to read on their own and make presentations. In short, I would highly recommend it to students and teachers of secondary and high school.

I would like to end the review with comments and excerpts illustrating Lang's views on mathematics education from the preface and from the Postscript, which is also a dialogue among Lang, educators and a student. Lang has strong views on the curriculum: to quote him from the preface,

A lot of the curriculum of elementary and high schools is very dry. You may never have had a chance to see what beautiful mathematics is like. . . . I have many objections to the high school curriculum. Perhaps the main one is the incoherence of what is done there, the lack of sweep, the little exercises that don't mean anything.

In reaction to the feeling that school students are not mature enough to see proofs,

There is the scandal! Those proofs are very beautiful, it's real mathematics. They allow you to appreciate mathematics, to see why something is true by using arguments which are quite understandable.

But in the course of the same dialogue, almost contradicting himself, he insists that memorization of formula is essential!

There is no way to avoid this, so you must ask kids to repeat the formula ten times. . . . It must be driven into their ears like music. You shouldn't ask every time why the formula is true.

One may not agree with Lang's philosophy or ideas all the time, but he does force you to think about what we are doing as teachers. He ends the dialogue on a more humane note and we will leave the readers with that.

Each teacher must do according to his own way, his own taste. Each one must use their own means to excite the students. One needs everything without exclusivity.

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REVERSIBLE PRIMES

A **reversible prime** is one which when one reverses the order of its digits remains a prime number. Examples: the primes 13 and 17. Obviously, the definition is base dependent; here it is assumed that we are working in base 10 (the decimal system). A question of interest: *How common are such primes?* The table below gives the relevant data. We use the following notation: $f(n)$ is the number of n-digit primes, and $g(n)$ is the number of reversible n-digit primes. Note that $f(1) = g(1)$, since any single-digit prime trivially satisfies the definition of reversibility.

n	$f(n)$	$g(n)$	Examples of reversible n-digit primes
1	4	4	2, 3, 5, 7
2	21	9	11, 13, 17, 31, 37, 71, 73, 79, 97
3	143	43	101, 107, 113, 131, 149, 151, 157, 167, ...
4	1061	204	1009, 1021, 1031, 1033, 1061, 1069, ...
5	8363	1499	10007, 10009, 10039, 10061, 10067, ...
6	68906	9538	100049, 100129, 100183, 100267, 100271, ...
7	586081	71142	1000033, 1000037, 1000039, 1000117, 1000159, ...

We see that reversible primes occur rather more frequently than one may expect! For example, among the four-digit primes, close to 20 percent of them are reversible.

We close with an interesting question: how does the fraction of primes that are reversible change with the number of digits? From the data it is evident that the fraction slowly decreases with n. However, the exact nature of this decrease is difficult to predict. Clearly, a more detailed study will be required to ascertain this.