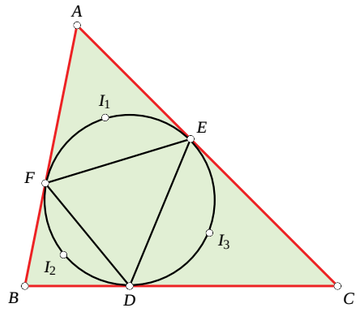

Observe that $\measuredangle AFE = \measuredangle AEF = 90° - \frac{1}{2}A$ and $\measuredangle FDE = \measuredangle AEF = 90° - \frac{1}{2}A$. Again,

$\measuredangle EI_1F = 90° + \frac{1}{2}A$. Hence:

$$\measuredangle EI_1F + \measuredangle FDE = 180°,$$

so I_1 lies on the incircle. Also:

$$\measuredangle I_1FE = \frac{1}{2}\measuredangle AFE = \frac{1}{2}\measuredangle AEF = \measuredangle I_1EF.$$

Thus $I_1E = I_1F$. But then they are equal chords of a circle, so they subtend equal angles at the circumference. Therefore $\measuredangle I_1DF = \measuredangle I_1DE$ and so I_1D is the internal bisector of $\measuredangle FDE$. Similarly we can show that I_2E and I_3F are internal bisectors of $\measuredangle DEF$ and $\measuredangle DFE$, respectively. Thus the three lines I_1D, I_2E, I_3F are concurrent at the incentre of triangle DEF.

ICMI AWARDS 2015

The International Committee for Mathematics Instruction (ICMI) has just announced the recipients of the 2015 Felix Klein and Hans Freudenthal Awards:

- **Felix Klein Medal:** Professor Alan J. Bishop, Emeritus Professor of Education at Monash University, Melbourne.
- **Hans Freudenthal Medal:** Professor Jill Adler, Chair of Mathematics Education, University of the Witwatersrand, South Africa.

The medals will be awarded in the opening ceremony of the International Congress for Mathematics Education (ICME-13) to be held in July 2016 in Hamburg.

Here are brief summaries excerpted from the official citations released by the Awards Committee. See the website http://www.icmihistory.unito.it/ to get a sense of the history of the ICMI and a sense of the work done by Felix Klein and by Hans Freudenthal.

ALAN J. BISHOP

The **Felix Klein medal** is awarded for life-time achievement in mathematics education research. [It] is aimed at acknowledging scholars who have shaped our field over their lifetimes. Past

candidates have been influential and have had an impact both at the national level and the international level. [They have] made substantial research contributions, introduced new issues, ideas and perspectives. Additional considerations have included leadership roles, mentoring, and peer recognition.

Alan J. Bishop, Emeritus Professor of Monash University, Australia is the awardee for 2015. He has been instrumental in bringing the political, social, and cultural dimensions of mathematics education to the attention of the field. His early research was on spatial abilities and visualization, but later he worked on the process of mathematical enculturation and how it is carried out in different countries. Subsequently he developed the notion of mathematics as a cultural product. Over more than 45 years of sustained, consistent work, this led to a great deal of work on the political and social dimensions of mathematics education.

Alan Bishop served as editor of *Educational Studies in Mathematics* from 1979 to 1989. In 1980, he founded and became the series editor of Kluwer's *Mathematics Education Library*. He served as the chief editor for many editions of the *International Handbook of Mathematics Education*. Through his tireless work in the area of publication, he enabled research in mathematics education to become an established field.

His education was at Harvard and later at Hull. After a stint at Cambridge University, he moved to Monash University, Australia. Through the Association of Teachers of Mathematics, he worked as a mentor to numerous teachers and supervised many doctoral students, several of whom became distinguished internationally. Through his work in forging links between research and practice, he helped mathematics educators establish communities of inquiry by teaching courses, speaking at conferences and workshops, directing research and development projects, and serving as a consultant, a project evaluator, and an external examiner.

As noted by one of the nominators, "Alan is an excellent scholar and researcher who shaped our field not only over his lifetime but also over its lifetime, not only in England and Australia ... but also internationally."

review

Review of The Sand Reckoner

by Gillian Bradshaw[1]

'There was a world there, a world without material existence but luminous with pure reason, and they couldn't see it!'

Dakshayini Suresh

Archimedes (Syracuse, 287 BC-212 BC) is generally believed to have been the greatest mathematician of antiquity, and certainly one of the three greatest of all time (along with Newton and Gauss). He is probably known best for his articulation of what has come to be known as the Archimedes principle, or rather for the entertaining scene that is said to have ensued upon its discovery. The story goes as follows.

Archimedes was asked by King Hieron of Syracuse to determine whether a gold wreath he had commissioned and subsequently received was, in fact, silver. While turning this problem over in his mind, Archimedes chanced to go for a bath, and it struck him, as he bathed, that the volume of water displaced by his being in the bath was equal to the volume of his own body. When he made this discovery, he is said to have run straight out of the bath and his home naked, shouting 'Eureka, eureka!' ('I have found it!'). He used this rule of displacement to determine whether the crown actually was pure and weighed as much as a pure gold object of the same volume.

[1] *The title of the novel is the name of a treatise by Archimedes in which he sets out to estimate the number of grains of sand it would take to fill the universe. In the opening scene of the novel, Bradshaw's Archimedes is trying to imagine values large enough to express the number of grains of sand in a box. In the novel, Archimedes also designs a catapult that comes to be known as the Sand Reckoner.*

Keywords: *Archimedes, Syracuse, Eureka*

He found that the crown was not made of pure gold and that his king had indeed been cheated.

The *Sand Reckoner* is a historical novel set in Ancient Greece. The young Archimedes is twenty three years old. He has just returned from an exciting and intellectually productive hiatus in Alexandria to find that his father is dying. His family needs an income. They need to sell off one or two of their four slaves. They need the only able male member of their family to find employment.

For about 300 years, during the reign of the Ptolemies, Alexandria was a city largely at peace within itself and with the rest of the world. In this period, it attracted many scholars who came to study and teach in the University of Alexandria. In Archimedes' circles there were scholars who had worked alongside Euclid. Education in Alexandria was certainly a rich opportunity for intellectual growth and exposure to the ideas and methods of the era.

Archimedes himself seems to want nothing more than to draw circles in the sand, to experiment with his abacus. Meals, finances, these things are not important to him. The job of accounting for the responsibilities and monetary concerns of his life falls to his slave Marcus. Finding the household in disarray comes as a rude shock to Archimedes. He must do his duty as a son. From this point onwards, Bradshaw unfurls the many intertwining plot lines that build the book. A diverse cast of characters appears: Archimedes' parents, affectionate, aged; his sister, lovely, supportive, but strangely sidelined; his lady love, the daughter of the reigning Tyrant Hieron, as fascinated with music and its governing technicalities as Archimedes himself is; the competitive and jealous engineers vying for his job. He wins the Regent over with his engineering skills and secures a position as the Tyrant's chief engineer.

The method by which he is said to have won favour with his king was by proving how his use of levers and pulleys could move an object of any weight. He chose to demonstrate this on a large ship, complete with crew and cargo. By operating his pulleys such that the ship did indeed move across the water without any great effort, he secured a position as the king's chief engineer of war machines.

Such is the progression of Archimedes' career, as described by Bradshaw. However, the most complex character in the book is probably Marcus, the slave, and all other plot lines apart, he deserves special mention. He is everything his master is not—decisive, headstrong, emotional and practical—and is caught in the drama of existence, the problems posed by the world, not in the problems of mathematics that Archimedes invents for pleasure. From the outset, it is clear that his attitude towards his master's family and towards Syracuse is not one of servility and fierce loyalty, but rather one of grudging attachment, formed over his years of residence and service. It soon becomes clear that Marcus' being of uncertain descent and his having slipped into Sicily quietly after a war are of more significance than they seem. Syracuse is at war with Rome, and Marcus' identity becomes a point of contention. If he is indeed Roman, as some of the Greek soldiers suspect, then his loyalty, which has till now been taken for granted, will come into question. His having been born free, rather than into a life of slavery, also means in some subtle way that his relationships with those he serves are sometimes ambiguous. Hence, when he returns from Alexandria with Archimedes, he begins a somewhat fraught flirtation with the daughter of the house, Philyra, now a young lady in her own right. It is not long before he realises that the ongoing war is a serious threat to his life, and that without him, the balance of his young master's life will be compromised. Even though *The Sand Reckoner* is technically about Archimedes, the nuanced and tragic character of Marcus is perhaps more engaging. He possesses a sense of spirit and volition which his master seems to lack.

Archimedes for a long time has very little inkling of the concerns that plague his friends and family. He has a lot of affection for people, but often struggles to express it. The social world puzzles him. Bradshaw successfully conveys that there is something extraordinary about her protagonist aside from his withdrawn manners and daydreaming tendencies. She illustrates what is different about him without embarking on too many complicated, technical math-monologues. She focuses on his wonder and his skill for innovating in a way that makes his brilliance accessible to all readers. Archimedes makes his living building weapons for the king. He offers to engineer catapults and is deeply confident of his ability to manufacture rare machines (one-talenters and two-talenters, catapults that can throw ammunition that weighs as much as one or two men). Rather, he is confident of the mathematical logic behind the construction of these catapults. He explains, and successfully demonstrates that increasing the dimensions of the parts of the catapult while maintaining the ratios of the parts to one another is the key to creating a large weapon. In his mind it is a matter of imagining and calculating. And in the minds of those around him, he has surpassed the limits of what is imaginable.

Archimedes is generally believed to have been happier in the world of theories and abstraction than in the realm of mechanics and concrete application. Plutarch writes that he possessed a 'lofty' spirit and a 'profound' soul. Indeed, most stories about Archimedes portray him as distracted from his physical surroundings (running naked through the streets, for instance) and intent on his mathematics, to the point of putting his own life in danger. (One version of the story of his death says that he provoked a soldier by refusing to obey orders because he was engrossed in solving a mathematical problem.) However, it is the same Plutarch who describes the efficiency and cruelty of the war machines that Archimedes designed to protect Syracuse—machines responsible for the slaughter of thousands of Roman soldiers.

Archimedes finds that for him, the magic of mathematics and mechanics lies in what is new and yet unchartered territory. He invents, and enjoys the process, but producing catapults or pumps (water snails as they come later to be called, because of the spiral tube of reed that enables water to flow uphill and downhill) bores him, and he loses interest once the job becomes repetitive. The author portrays her protagonist as a man whose mind prefers to 'soar' and dream and to attempt to know and make *new* things. He is only grounded by the rules and precision of his subject; no other limits seem to matter.

However, the novel dwells more on his growth as a person than as a mathematician. Through the course of the plot, the young Archimedes discovers that the world is a more practical and brutal place than he imagined, and that limits exist in society, whether or not he wants them to exist in his mind. He faces many confusing challenges: He engages in a wholly impractical romance with the princess Delia, he watches his once brilliant father wither into someone unrecognisable. His friends are endangered. Worst of all, he faces the consequence of his brilliance. However interesting the process of designing a catapult may be, he comes to see that the effect of such a weapon is basically massacre. And no sense of loyalty, or duty, or love for theoretical and concrete wonders of mathematics can change this.

Ultimately, the novel is about much more than Archimedes. It is about the contrast between the inner world of the mind and the more pressing world of real life demands and constraints that exist around us. In this realm duty, loyalty, marriage, war and death take precedence over the power of imagination and logic. Hence, Gillian Bradshaw's Archimedes seems to use his mind as refuge from all that is disorderly in the external world. His mathematics is his safe haven, a space that remains neat, unsullied by all the pain and tragedy of life.

There are several retellings of the story of Archimedes' death. In all of them, he dies at the hands of a Roman solider, after Syracuse has been defeated. In many, he dies merely because he provokes this soldier without intending to, by carrying measuring instruments that are mistaken for treasure, or by refusing to follow the ranks for prisoners when ordered to, or just by not realising that his city had fallen to the Romans, because he was too busy drawing figures in the sand. In nearly every version, Marcellus, the Roman general, sorely regrets the loss of this brilliant mind.

The Sand Reckoner is an enjoyable read for anyone fifteen and up, with basic knowledge of tenth standard mathematics. It may be easier to appreciate this novel if one is already immersed in the world of math and math history, but it will certainly also appeal to readers whose knowledge of the subject does not go beyond the very basic.

But the charm of the book is that it will convey the same sense of awe and excitement to everyone. It will place mathematical discovery and its applications in a historical and social context. It is the ideal way to illustrate the story-like quality of the course of math history to even the most reluctant and intimidated disciples of the subject.

DAKSHAYINI SURESH is an alumnus of Centre for Learning, Bangalore, and has just begun an honours course in English at Lady Shri Ram College. She likes to read, write and cook. She has a fraught history with mathematics but occasionally finds herself looking back fondly on the days when math meant more than grappling with the week's accounts! She can be reached at: dakshasuresh@gmail.com.

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ICMI AWARDS 2015

JILL ADLER

The **Hans Freudenthal medal** is aimed at acknowledging the outstanding contributions of an individual's theoretically well-conceived and highly coherent research programme. It honours a scholar who has initiated a new research programme and has brought it to maturation over the past 10 years. The research programme is one that has had an impact on our community. It is also intended that a Freudenthal awardee should still have a minimum of a decade of active research work ahead of him or her so as to continue contributing to the field. In brief, the criteria for this award are depth, novelty, sustainability, and impact of the research programme.

Professor Jill Adler, FRF Chair of Mathematics Education, University of the Witwatersrand, South Africa is the awardee for 2015, in recognition of her outstanding research program dedicated to improving the teaching and learning of mathematics in South Africa – from her 1990s ground-breaking research on the dilemmas of teaching mathematics in multilingual classrooms, to her subsequent focus on problems related to mathematical knowledge for teaching and professional development. Her research has served to advance understanding of the relationship between language and mathematics in the classroom.

Over the last two decades, she spearheaded several large-scale teacher development projects aimed at developing mathematics teaching practice at the secondary level, so as to enable more learners from disadvantaged communities qualify for entry to mathematics-related courses at university.

Jill Adler was born in Johannesburg and graduated from the University of the Witwatersrand with a B.Sc. in mathematics and psychology (1972). She taught secondary school mathematics in a so-called 'coloured' school in Cape Town – an experience that she credits for strengthening her concerns about educational inequality and leading her to work in that direction. This was followed by many years spent on developing materials for adults and alienated youth excluded from school mathematics learning in apartheid South Africa. In 1985, she obtained a M.Ed. for her dissertation: *Mathematics by newspaper in South Africa: junior secondary mathematics for adults through the medium of a newspaper. Her doctoral research (1996) was titled: Secondary teachers' knowledge of the dynamics of teaching and learning mathematics in multilingual classrooms.*

In addition to her international research at the cutting edge of the field, she has played an outstanding leadership role in mathematics education research in South Africa, Africa, and beyond, and has helped in adding to the human research capacity in Southern Africa. Her contributions to the development of research and practice have earned her leadership positions in renowned international and national professional associations.

review

A Review of Math! Encounters with High School Students

Dialogue and Mathematics—Serge Lang Style!

Shashidhar Jagadeeshan

The notion of dialogue and mathematics may at first seem a strange combination, but if one thinks about it, often in a lively interactive classroom this is exactly what is transpiring. According to the late physicist David Bohm, the root of the word *dialogue* comes from the Greek *dialogus*. The word *logos* in turn can be interpreted as 'meaning of the word' and *dia* means 'through'. So dialogue can then be seen as a process where there is a flow of meaning. All teachers would agree that this is what they would like in their classrooms.

The book under review, *Math! Encounters with High School Students* by Serge Lang, is an old one, published in 1985, but well worth bringing to the notice of students and teachers of mathematics. It is a series of seven dialogues on mathematics with school students and a postscript discussing mathematics teaching.

Apart from the content, which I will discuss later, the book is unique in its style of delivery. Even though we are not in the audience, we can sense the energy and excitement of the exchange. One wonders (without being completely reductionist), what are the ingredients needed for such a flow of energy and meaning to take place between teacher and taught? The obvious

Keywords: *Dialogue, facilitation, pedagogy, creativity*