feature

Paper play

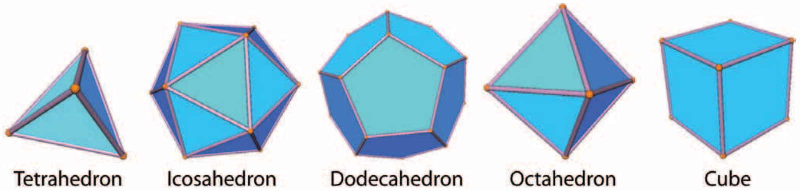
Making a Skeletal Dodecahedron

A solid geometry experience

Vertices, edges, faces, ... so much has been said about them and important connects made between them. But how can a student ever understand these relationships using a 2 dimensional sketch? Constructing your own model personalises the learning in a meaningful and unforgettable way, and nothing can beat that experience.

SHIV GAUR

A dodecahedron is one of the five Platonic or 'regular' polyhedra, and has been known since the times of the ancient Greeks; the other four such polyhedra are the tetrahedron (with 4 triangular faces), the hexahedron (with 6 square faces; better known as a cube!), the octahedron (with 8 triangular faces), and the icosahedron (with 20 triangular faces). The dodecahedron has 12 pentagonal faces. Here are some images of these five polyhedra (source: http://www.ma.utexas.edu/users/rgrizzard/M316L_SP12/platonic.jpg):



What is appealing about all these solids is their high degree of symmetry: their faces are regular polygons, congruent to each other, and at each vertex the same number of edges meet.

So the polyhedron 'looks the same' when seen from above any face. We may associate two numbers with each such solid: m, the number of edges around each face, and n, the number of edges meeting at each vertex (this will equal the number of faces coming together at that vertex). Hence: $(m, n) = (3, 3)$ for a tetrahedron, (4, 3) for a cube, (3, 4) for an octahedron, (3, 5) for an icosahedron, and (5, 3) for a dodecahedron. A dodecahedron has twelve congruent pentagonal faces, with three edges coming together at each vertex.

The numbers indicate certain symmetries that go across the five solids: the cube and octahedron are linked with each other and are said to be duals of each other; so are the icosahedron and dodecahedron. Alone in the family is the tetrahedron, which is self-dual.

In this article we shall show how to make an elegant see-through (skeletal) dodecahedron by first making 30 identical modules which are the building blocks. Later we interlock the modules to form the dodecahedron.

The authorship for the module and design is unknown, and the URL from YouTube where I saw the video for the first time is: http://www.youtube.com/watch?v=JexZ3NIaoEw.

Materials required

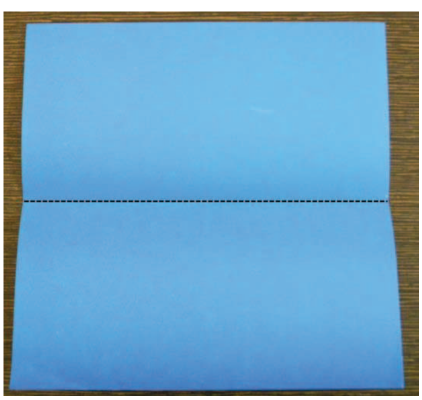
30 coloured square sheets (avoid soft paper which doesn't retain a crease), paper clips, steel ruler

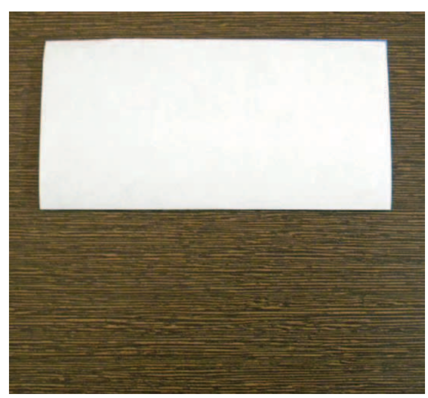
The module

The following are the steps for making a module from one square sheet:

Step 01

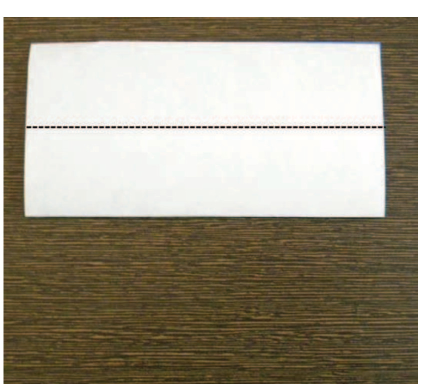
Colour side up, fold the paper in half inwards as shown on the right

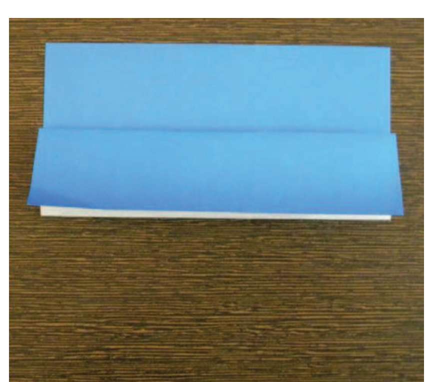




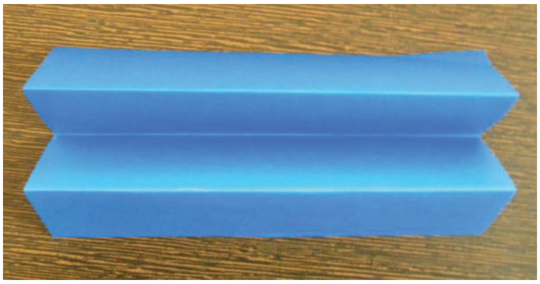
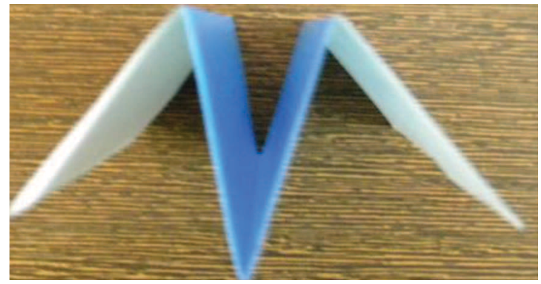
Step 02

Fold both sides in half outwards as shown on the right



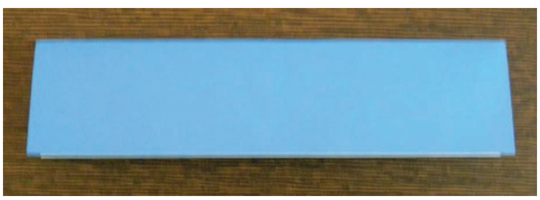
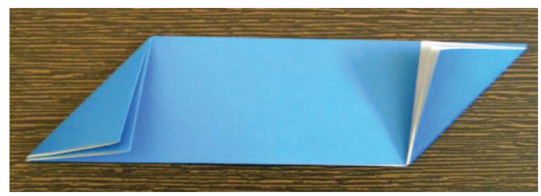


A top and a side view of what you should get at the end of Step 02
(it looks like the letter M from the side):

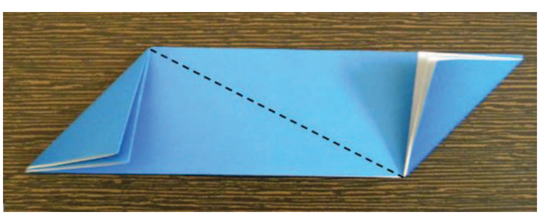
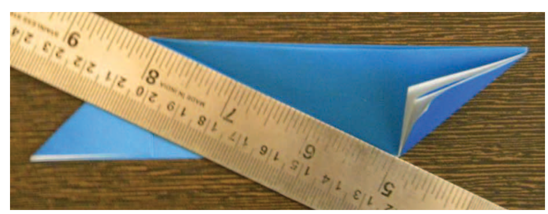
Step 03

Turn the corners inwards as shown in the picture on the right

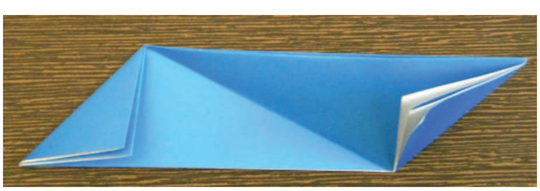
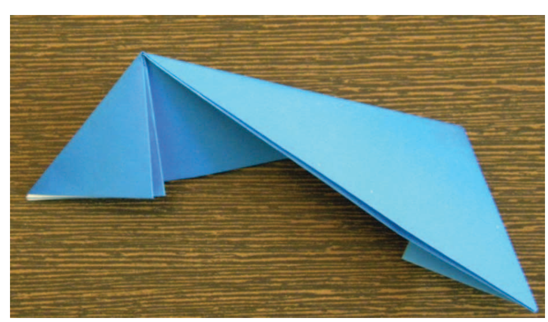



Step 04

Along the dotted diagonal, fold a valley crease (a steel ruler is helpful for accuracy and also speeds up the process).

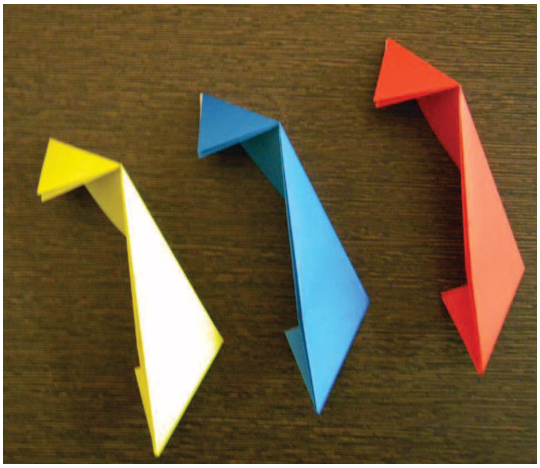



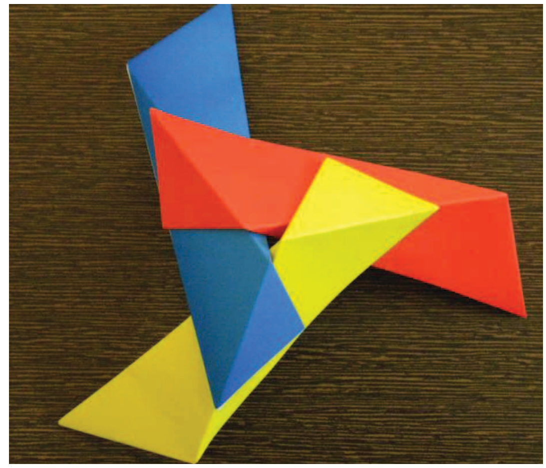
End of step 04 gives us the required module (two views of the same).

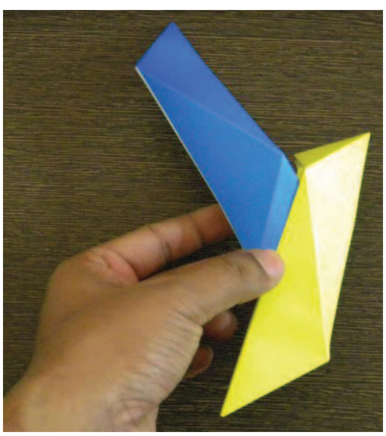
Make 30 modules of different colours as shown above.

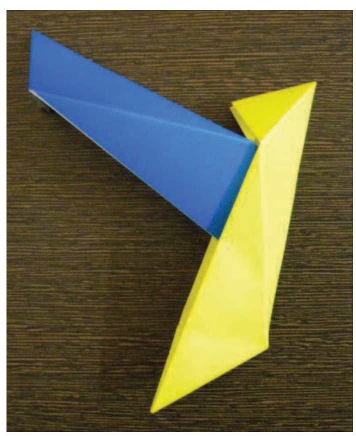
The connecting process



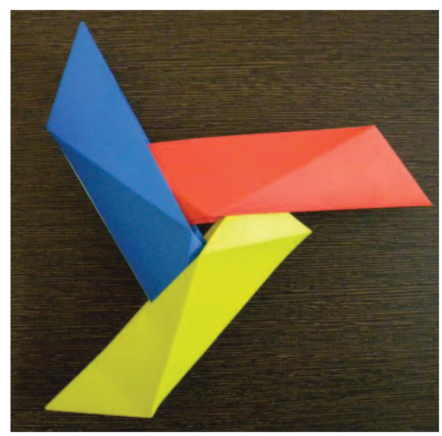


Take 3 modules and keep the corners together. This is the key idea. We need a small mountain and the 3 corners need to go into the side pockets of the neighbouring colours. So the blue corner will go into the yellow side pocket, the yellow corner into the red side pocket, and the red corner into the blue side pocket.



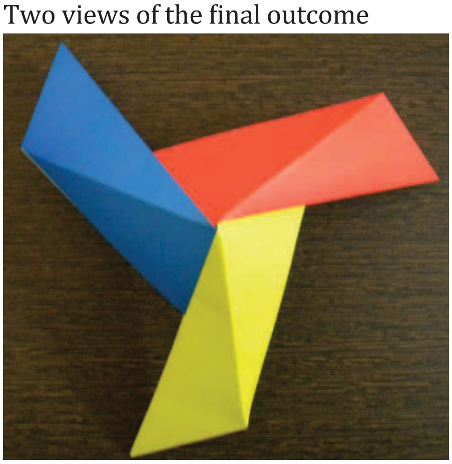


The blue corner being slid into the yellow side pocket.
Do likewise for the red module.

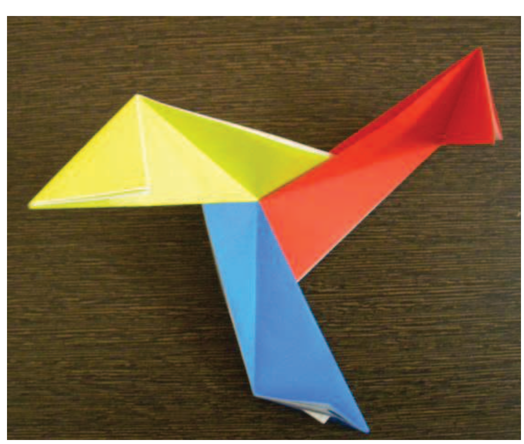


The mountain about to be completed.

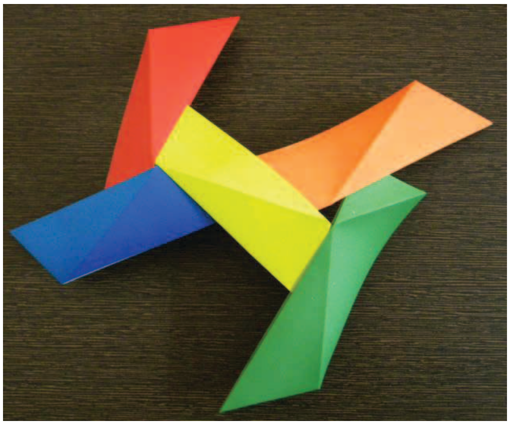
Two views of the final outcome

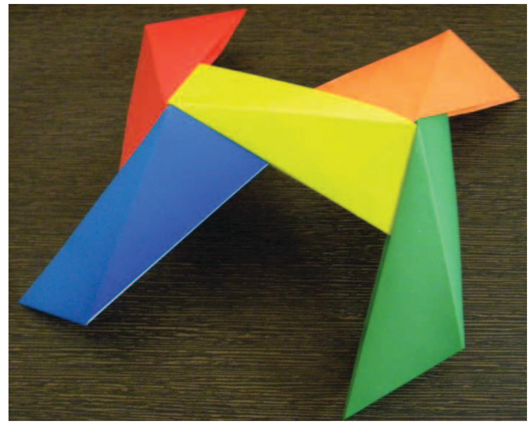


A top view

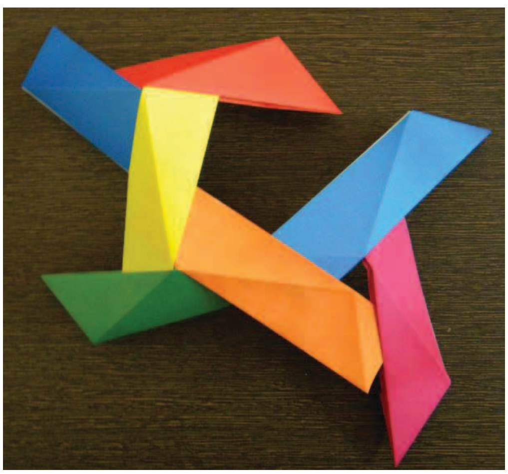


A bottom view

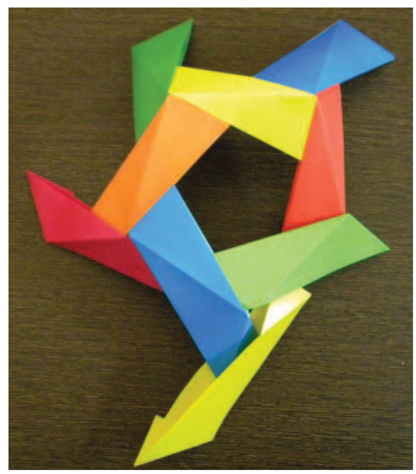




From here onwards we apply the same process to **each loose corner** we see till we get a **pentagon**.



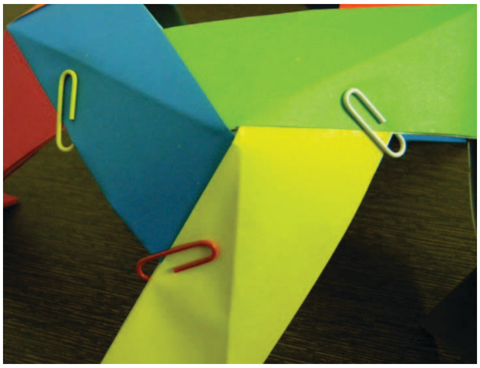
The third mountain being formed



The fifth and the final mountain being formed

The pentagon face finally(A top view)!

A view from the bottom




At this stage it is a good idea to clip the corners with paper clips. Continue **working pentagons along every length** and a curvature will emerge.

The halfway mark

Close to completion



The last mountain and finally, the completed dodecahedron on the right!

Reference

http://www.youtube.com/watch?v=JexZ3NIaoEw

A B.Ed. and MBA degree holder, SHIV GAUR worked in the corporate sector for 5 years and then took up teaching at the Sahyadri School (KFI). He has been teaching Math for 12 years, and is currently teaching the IGCSE and IB Math curriculum at Pathways World School, Aravali (Gurgaon). He is deeply interested in the use of technology (Dynamic Geometry, Computer Algebra) for teaching Math. His article "Origami and Mathematics" was published in the book "Ideas for the Classroom" in 2007 by East West Books (Madras) Pvt. Ltd. He was an invited guest speaker at IIT Bombay for TIME 2009. Shiv is an amateur magician and a modular origami enthusiast. He may be contacted at shivgaur@gmail.com.