

# George Pólya -

In his own words



COMαC

*George Pólya was a highly influential mathematician of the 20th century. His research contributions span vast areas of mathematics —complex analysis, mathematical physics, probability theory, geometry, and combinatorics. He was at the same time a teacher par excellence who maintained a strong interest in matters of pedagogy right through his long and richly productive career. Among his widely read books are *How To Solve It*, *Mathematical Discovery*, and *Mathematics and Plausible Reasoning* (two volumes). He also wrote (with Gábor Szegő) the influential two volume series *Problems and Theorems in Analysis*. We give below a sampler of Pólya's writings on teaching. (In some cases we have taken the editorial liberty to modify the sentences very lightly.)*

*"What the teacher says in the classroom is not unimportant, but what the students think is a thousand times more important".*

**George Pólya, 1888–1985**

## I. Teaching is not a science

Teaching is not a science; it is an art. If teaching were a science there would be a best way of teaching and everyone would have to teach like that. Since teaching is not a science, there is great latitude and much possibility for personal differences. There are as many good ways of teaching as there are good teachers.

In an old British manual there was the following sentence, *Whatever the subject, what the teacher really teaches is himself*. So therefore when I am telling you to teach so or so, please take it in the right spirit. Take as much of my advice as it fits you personally.

## II. The aims of teaching

My opinions are the result of long experience. . . . Personal opinions may be irrelevant and I would not dare to waste your time by telling them if teaching could be fully regulated by scientific facts and theories. This, however, is not the case. Teaching is not just a branch of applied psychology.

We cannot judge the teacher's performance if we do not know the teacher's aim. We cannot meaningfully discuss teaching, if we do not agree to some extent about the aim of teaching. I have an old fashioned idea about [the aim of teaching]: first and foremost, it should teach young people to THINK. This is my firm conviction.

If you do not regard "teaching to think" as a primary aim, you may regard it as a secondary aim — then we have enough common ground for the following discussion.

"Teaching to think" means that the teacher should not merely impart information, but should try also to develop the ability of the students to use the information imparted: he should stress knowledge, useful attitudes, desirable habits of mind.

## III. The art of teaching

Teaching is not a science, but an art. This opinion has been expressed by so many people so many times that I feel a little embarrassed repeating it. If, however, we leave a somewhat hackneyed generality and get down to appropriate particulars,

we may see a few tricks of our trade in an instructive sidelight.

Teaching obviously has much in common with the theatrical art. For instance, you have to present to your class a proof which you know thoroughly having presented it already so many times in former years in the same course. You really cannot be excited about the proof — but, please, do not show that to your class; if you appear bored, the whole class will be bored. Pretend to be excited about the proof when you start it, pretend to have bright ideas when you proceed, pretend to be surprised and elated when the proof ends. You should do a little acting for the sake of your students who may learn, occasionally, more from your attitudes than from the subject matter presented.

Less obviously, teaching has something in common also with music. You know, of course, that the teacher should not say things just once or twice, but three or four times. Yet, repeating the same sentence several times without pause and change may be terribly boring and defeat its own purpose. Well, you can learn from the composers how to do it better. One of the principal art forms of music is "air with variations." Transposing this art form from music into teaching, you begin by saying your sentence in its simplest form; then you repeat it again with a little more colour, and so on; you may wind up by returning to the original simple formulation. Another musical art form is the "rondo." Transposing the rondo from music into teaching, you repeat the same essential sentence several times with little or no change, but you insert between two repetitions some appropriately contrasting illustrative material. I hope that when you listen the next time to a theme with variations by Beethoven or to a rondo by Mozart, you will give a little thought to improving your teaching.

Now and then, teaching may approach poetry, and now and then it may approach profanity . . . . Nothing is too good or too bad, too poetical or too trivial to clarify your abstractions. As Montaigne put it: *The truth is such a great thing that we should not disdain any means that could lead to it*. Therefore, if the spirit moves you to be a little

poetical or a little profane in your class, do not have the wrong kind of inhibition.

#### IV. The nature of the learning process: three principles of learning

Any efficient teaching device must be correlated somehow with the nature of the learning process. We do not know too much about the learning process, but even a rough outline of some of its more obvious features may shed some welcome light upon the tricks of our trade. [Here are] three “principles” of learning.

- **Active learning.** It has been said by many people in many ways that learning should be active, not merely passive or receptive; merely by reading books or listening to lectures or looking at moving pictures without adding some action from your own mind you can hardly learn anything and certainly you can not learn much. There is another often expressed opinion: *The best way to learn anything is to discover it by yourself.* Here is another related quote: *What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises.* Less colourful but perhaps more widely applicable is the following statement: *For efficient learning, the learner should discover by himself as large a fraction of the material to be learned as is feasible under the given circumstances.*
- **Principle of best motivation.** Learning should be active, we have said. Yet the learner will not act if he has no motive to act. He must be induced to act by some stimulus, by the hope of some reward, for instance. The interest of the material to be learned should be the best stimulus to learning and the pleasure of intensive mental activity should be the best reward for such activity. Yet, where we cannot obtain the best we should try to get the second best, or the third best, and less intrinsic motives of learning should not be forgotten.
- **Consecutive phases.** Here is an oft quoted piece from Kant: *Thus all human cognition*

*begins with intuitions, proceeds from thence to cognitions, and ends with ideas.*

I am not able (who is?) to tell you in what exact sense Kant intended to use these terms. [So] I beg your permission to present my reading of Kant’s dictum: *Learning begins with action and perception, proceeds from thence to words and concepts, and should end in desirable mental habits.*

So for efficient learning, an exploratory phase should precede the phase of verbalization and concept formation and, eventually, the material learned should be merged in, and contribute to, the integral mental attitude of the learner.

I think that these three principles can penetrate the details of the teacher’s daily work and make him a better teacher. I think too that these principles should also penetrate the planning of the whole curriculum, the planning of each course of the curriculum, and the planning of each chapter of each course.

Yet it is far from me to say that you must accept these principles. These principles proceed from a certain general outlook, from a certain philosophy, and you may have a different philosophy. Now, in teaching as in several other things, it does not matter what your philosophy is or is not. It matters more whether you have a philosophy or not. And it matters very much whether you try to live up to your philosophy or not. The only principles of teaching which I thoroughly dislike are those to which people pay only lip service.

#### V. On problem solving

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime.

Thus, a teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking.

A good teacher should understand and impress on his students the view that no problem whatever is completely exhausted. There remains always something to do; with sufficient study and penetration, we could improve any solution, and, in any case, we can always improve our understanding of the solution.

## VI. Ten commandments for teachers

On what authority are these commandments founded? Dear fellow teacher, do not accept any authority except your own well-digested experience and your own well-considered judgement. Try to see clearly what the advice means in your particular situation, try the advice in your classes, and judge after a fair trial.

1. *Be interested in your subject.* There is just one infallible teaching method: if the teacher is bored by his subject, his whole class will be infallibly bored by it.
2. *Know your subject.* If a subject has no interest for you, do not teach it, because you will not be able to teach it acceptably. Interest is an indispensable necessary condition; but, in itself, it is not a sufficient condition. No amount of interest, or teaching methods, or whatever else will enable you to explain clearly a point

to your students that you do not understand clearly yourself.

Between points #1 and #2, I put interest first because with genuine interest you have a good chance to acquire the necessary knowledge, whereas some knowledge coupled with lack of interest can easily make you an exceptionally bad teacher.

3. *Know about the ways of learning: the best way to learn anything is to discover it by yourself.*
4. *Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their place.*
5. *Give them not only information, but “know-how”, attitudes of mind, the habit of methodical work.*
6. *Let them learn guessing.*
7. *Let them learn proving.*
8. *Look out for such features of the problem at hand as may be useful in solving the problems to come — try to disclose the general pattern that lies behind the present concrete situation.*
9. *Do not give away your whole secret at once, let the students guess before you tell it; let them find out by themselves as much as is feasible.* Voltaire expressed it more wittily: The art of being a bore consists in telling everything.
10. *Suggest it, do not force it down their throats.* In other words: Let your students ask the questions; or ask such questions as they may ask for themselves. Let your students give the answers; or give such answers as they may give by themselves. At any rate avoid asking questions that nobody has asked, not even yourself.

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## Comment from the editors

There is great scope for developing these ideas. For example, take points #5 and #6 in the above list: Give them not only information, but know-how, attitudes of mind, the habit of methodical work; let them learn guessing. What are good mathematical habits? What are good attitudes of mind when it comes to teaching-learning mathematics? Why is ‘guessing’ important? What is mathematical know-how? We invite responses from the readers on these and related issues.

## Recommended books by George Pólya on math teaching and math education

1. How to Solve It: A New Aspect of Mathematical Method
2. Mathematics and Plausible Reasoning: Volume I, Induction and Analogy in Mathematics
3. Mathematics and Plausible Reasoning: Volume II, Patterns of Plausible Inference
4. Mathematical Discovery: On Understanding, Learning and Teaching Problem Solving

## Picture PUZZLE 01

This is a photo taken at the Isha Home School, Coimbatore. Set in tranquil surroundings near Coimbatore, Tamil Nadu, the residential Home School is located at the foothills of the Velliangiri Mountains.



Let's zoom in on one particular curve noticed in the picture:



**If you could transfer the lower curve to graph paper, could you find a quadratic function that modeled it?**

For a soft copy of this photo visit [www.teachersofindia.org](http://www.teachersofindia.org) and search for Snakes at Isha.

Answers may be submitted to [atria.editor@apu.edu.in](mailto:atria.editor@apu.edu.in)  
Do remember to send in your working.