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PLACE VALUE

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A PAPER KIT
APPROACH

**At
Right
Angles**

A Resource for School Mathematics

Teaching of the place value system happens in the context of teaching numbers and is very closely related to counting, grouping objects to aid counting, usage of number decomposition, learning the patterns in number names, learning the written representations of numbers, learning the patterns in the relationships between consecutive places, and developing a proper number sense. Children develop facility with numbers and a sound understanding of the number system only if sufficient care is taken in building all the above mentioned areas.

PRE-REQUISITES BEFORE TEACHING PLACE VALUE SYSTEM

- Recognizing and identifying in terms of objects, the numbers 1 to 9
- Reciting, reading and writing of numerals, number names 1 to 9
- Functional understanding of 0
- Ordering numbers 1 to 9
- Basic addition facts
- Addition facts of 0
- Complementary addition facts of 9 and 10

ACTIVITY ONE

Objective
Introduction of 10 and the relationship between ten and a unit

- Materials required:**
- Loose sticks or straws, rubber bands
 - Loose colour papers, clips
 - Dot sheets
 - Place value card

Importance

Even though this is the first activity in the teaching of place value and is a fairly simple activity for the child it lays the foundation of the place value system. It needs to be done repeatedly in various situations as will be explained later to help children understand the relationship between a ten and a unit.

Initially the teacher should count out the sticks (slowly, saying aloud 1, 2, 3, etc.) till he reaches 10 and show them that he is making a bundle of 10 sticks. He should clearly differentiate between the word **sticks** and **bundle** as the sticks are 10 but the bundle is 1.

Let each child count ten sticks carefully and make a bundle of 10 sticks with a rubber band.

The teacher can pick up 7 sticks and ask: "How many more sticks do I need to make a bundle of 10 sticks?" Since we expect children to know complementary facts of 10 by now, they should be able to answer this.

In a similar way the teacher can pick up 12 sticks and ask: "I need to make a bundle of 10 sticks. What do I do?" The children will suggest that he remove 2 sticks and bundle the rest.

Children can be given some seeds and asked to make a group of ten. It is important however to use and emphasize the right language: "This is a group of 10 seeds."



Tens and units sticks

They can be given coloured square paper sheets which they can count and clip. "This is a bundle of 10 papers."

They can also be given dot paper and asked to line 10 dots or circle 10 dots. "This is a group of 10 dots."

They can now be shown how to write ten using a place value card with headers. The use of place value cards (see photograph) facilitates placing of materials and the corresponding number cards in the right places. From the beginning children must see clearly the relationship between the activity or the manipulative and the procedural rules of recording and writing.

ACTIVITY TWO

Objective

Learning to count in tens: 1 ten, 2 tens, and so on, up to 9 tens; and their number names ten, twenty, etc.

Materials required:

- Loose sticks or straws, rubber bands
- Loose colour papers, clips
- Dot sheets
- Place value card
- Flash cards for number names, numerals, objects
- Beads and string

We now repeat activity 1 by working with more sticks and making several bundles of 10 sticks.

Point out that the bundle that they are making has 10 sticks.

As mentioned earlier one needs to emphasize the language aspect by saying: "Here is 1 bundle of sticks. How many sticks?" Ten. "Here are 2 bundles of sticks. How many sticks?" Twenty.

Now the teacher can ask various children to make different numbers of bundles and teach number names for those. They can record them using the place value cards.

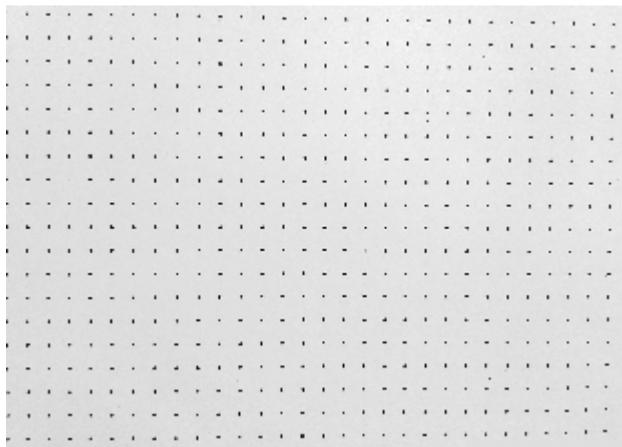
The teacher can pick up some bundles and ask "How many sticks?" They first answer by counting the number of bundles and then verify their answer by opening up the bundle and counting the sticks.

Children can also do some exercises with dot paper. They should also be given worksheets which require them to write the numbers for given pictures and draw pictures for given numbers. They can build bead strings with different tens.

Finally children can be given flash cards consisting of pictures of bundles and corresponding number names for matching.



Bead string for tens



Dot sheet

ACTIVITY THREE

Objective

Counting, recording and writing numbers

- From 11 to 20
- From 20 to 99

Materials required:

- Loose colour papers, clips
- Ten square strips, loose square slips
- Dot sheets
- Place value card
- Flash cards for number names, numerals, objects
- Number line strip (0 to 99); permanent number line can be drawn below the blackboard
- Number cards

We can now repeat Activity II by working with several bundles of 10 sticks and loose sticks.

Let the children count objects not exceeding 100 (objects kept loose). Show how grouping them into tens makes the task easier.

Let them count objects not exceeding 100 (objects kept in tens and ones).

Let them count both discrete objects (seeds, beads) and continuous objects (line of tiles, strings of beads or flowers, paper rolls with regular markings) not exceeding 100.

Show them some tens and some ones.

Ask them to show fewer sticks than what you have put in front of them.

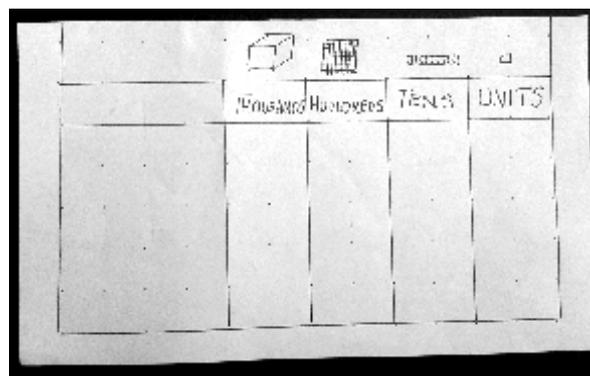
Give them a number and ask them to pick out the required number of tens and ones

Give them a sheet of paper with some dots and let children circle the tens and ones when you call out a number.

Give them various activities which make them record and write different numbers.

Common errors: When asked to write thirty one, a child writes 13. He has not understood that 13 is 1 ten and 3 ones, whereas 31 is 3 tens and 1 one.

One major difficulty with teaching the writing of numbers from 11 to 19 is caused by the mismatch between the way the number is written and the name by which it is called; e.g., 14 is 'fourteen': the word four comes first, which does not happen for numbers



Place value card

from twenty onwards. 61 is 'sixty one'; the number name matches with the way it is written. This problem exists to varying degrees in other languages as well.

Hence while teaching children to record numbers from 11 to 20 it is necessary to emphasize their decomposition: ten and one make eleven, ten and two make twelve, etc., so they associate the tens place digit and units place digit with the correct number.

Practice: The teacher can ask the children to turn to the correct page of a book, given the page number.

Children should also be given worksheets which require them to write the numbers for given pictures and draw pictures for given numbers. The semi-concrete representation is necessary till the children reach the take-off stage.

GAME

Game 1: Double Nine

Objective: Developing number sense

Materials required:

- Ten square strips and loose square slips.
- Dice

Children can be divided into groups of 5. One child becomes a banker and has a stock of loose square slips and strips. Each child throws the dice in turns and collects that many ones (square slips) from the banker. As the children continue to play, they collect more ones. Each time they have a collection of ten ones they exchange it for a strip with the banker. They continue till one of them reaches 99.

ACTIVITY FOUR

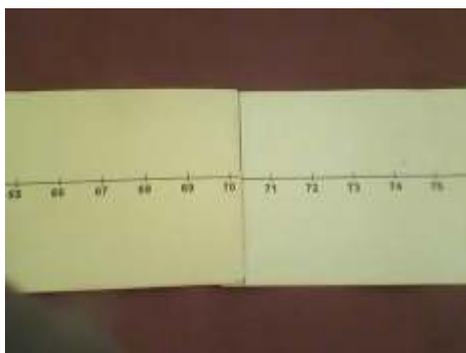
Objective

Developing sequential nature of numbers

- From 11 to 20
- From 20 to 99

Materials required:

- Number line



Number line

Many number line exercises can be created which will help in visualizing the sequential nature of numbers.

Teachers should consciously help children to achieve understanding of the succession of numbers by using different manipulatives.

Both forward counting and backward counting should be practised.

GAME

Game 2: Flags

Objective: Sequencing numbers

Materials required:

- Long string
- Number cards



Variation 1: Tie the string across the room. Take some number cards at random and let each child pick up one number card. By turn each can clip it on the string ensuring that they are in increasing order.

Variation 2: The teacher can put up a card on the string and ask questions like: "Who has the nearest card to this?", "Who has the card furthest away from this?", "Who has the nearest ten to this card?", "Who has 5 more than this?", "Who has 10 less than this?", "Who has the card where the tens and units are interchanged?" These questions will stimulate discussion amongst students leading to comparison of numbers, adding, subtracting and paying close attention to the place values.

ACTIVITY FIVE

Objective
Developing number sense, approximation and estimation skills

Materials required:

- Newspapers
- A few textbooks
- A collection of seeds

The teaching of numbers should be accompanied by activities which develop a number sense – i.e., a sense of the size of the number, its relationship with other numbers, properties of the number, proximity to multiples of ten, etc.

GAME

Game 3: Guess the number

Objective: Developing number sense

Let each child take a fistful of seeds and pour them out on his table. Let the child guess the number of these seeds and write it down. Now ask the child to count them by making it into groups of ten.

Ask the children to open a particular page in a textbook. Ask them to guess the number of words on the page (ensure that it is less than 100), or in a given paragraph. Let the child record his guess and then count the words to check how close his guess was.

Ask the children to bring a newspaper. Ask them to circle 50 words (by guessing and not counting). Let them later count the words and check how close their guess was to the actual number.

Number patterns: Plenty of number pattern exercises can be done to build number sense leading to an understanding of number behavior.

ACTIVITY SIX

Objective
Fixing place value through headers: tens (t), units (u) and arrow cards

Materials required:

- Ten square strips and loose square slips
- Place value card
- Arrow cards

Common errors: Integrating part and whole:

When asked to write twenty three, a child writes 203.

What has led to this error? If the child were now asked to read it, how does he/she read it?

This is a situation of not being able to integrate parts with the whole – the child is treating the tens separately and the three units separately. What form of teaching will prevent these errors?

Arrow cards help in remedying this kind of a situation and making the hidden values explicit for children.

Let the children show the given number on the place value card with strips and slips.

Let them build the number using arrow cards as shown, one below the other and later by placing one over the other to integrate the parts with the whole.

Let the children write the number for the given picture and build the number.



ACTIVITY SEVEN

Objective
Reinforcing place value through the usage of an abacus

Materials required:

- Abacus, beads
- Strips of ten squares and square slips
- Place value cards

Abacus is a useful device in demonstrating place values. But the teacher must keep in mind that it does involve abstraction as one bead in the tens place represents a ten and a bead in the hundred's place represents a hundred.

Introduction to the abacus needs to be done slowly and carefully by actually showing how numbers from 1 to 9 are represented, and that when we need to represent a ten we move to the tens place as the units place can be used for only nine beads. (It may be best to use a model of an abacus which can only accommodate nine beads.) By placing one bead after another progressively we show how numbers 11 to 99 are represented on an abacus. One has to make sure that children grasp the point that each time we have ten ones an extra bead gets added to the tens place. The transitions from 9 to

10, 10 to 11, 19 to 20, 20 to 21, 29 to 30 and 30 to 31 are important; the teacher needs to make the actions clear by giving a 'running commentary'. It is also important to go backwards from 99 to 1 by removing one bead at a time.

Practice: You can make groups of 3 children and give an abacus to the first child, the strips of ten squares and the square slips to the second, and the place value cards to the third. One child shows a number on the abacus, and the other two show the same with their materials. Another now shows a different number using strips and square slips; the other two have to show that number using their materials. And so on.

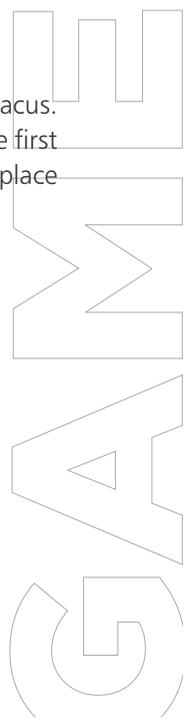
They should record the work in square ruled note books with appropriate drawings and recordings of numbers.

GAME

Game 4: Reach zero

Objective: Exchanging tens and units

Make a group of 4 children. One child can be the banker. Start with any number say 30, represented on the abacus. Children take turns throwing the die. After each throw, they take away that number from the abacus. If in the first round a child gets four the child will have to remove 1 bead from the tens place and exchange for 10 beads and place 6 back on the units rod. They continue to play till they reach zero.



ACTIVITY EIGHT

Objective
Introducing hundred

Materials required:

- Abacus, beads
- Hundred square sheets, Ten square strips and Square slips
- Place value cards
- 100 square board

Initially count out the ten square strips (slowly, saying aloud: 91, 92, 93, etc.) till you reach 99 and show them that when one more square slip is added there will be 10 ten square strips (the 10 loose units get exchanged for a ten strip). Now tell them that 10 such strips together make a hundred (they can be exchanged for a hundred square) and show them how it is written. (Let the children verify for themselves that the hundred square is made up of ten strips.) They must see that a hundred equals 10 tens and also 100 units.

You can demonstrate this on an abacus, counting from 99 onwards. It is important to approach the teaching of a new place value in a progressive way, so that children see its relationship to numbers they have met earlier, and the place values.

Now use the place value cards by progressively changing cards from 91 to 99 and point out how the units and tens places both have a 9, and how as you add one more unit to it, a new place (hundred gets created), and the units and tens places both become zero.

Many a time, teachers conduct activities with children without adequate commentary, explanations and questions; without pausing and drawing children's attention to the crucial aspects. The activity will not produce the desired benefits (in terms of improving understanding) if this is not done. Also, the activity needs to be repeated by the teacher and the students a sufficient number of times for the concept to be internalized. It is important to correlate the activity with the materials and the writing by asking questions such as: "How many units do I have here now?" Nine. "What happens when I bring in 1 more unit?" There are 10 units. "Now I exchange the ten units for a strip.



Hundreds, tens and units

So how many ones do I have now?" Zero. "So I write a zero in the ones place." ... "How many tens did I have in the beginning?" Nine. "How many tens are there now?" 10 tens. "I can exchange 10 Tens for a hundred square. How many tens are there now?" Zero. "So I write a zero in the tens place. How many hundreds do I have now?" One. "So I write a 1 in the hundreds place." And so on.

Explore: Let children write all the numbers from 1 to 100 in a 10 by 10 square. There are many patterns in a number square which the children can notice and share. For example, if they look at the numbers vertically (along the columns) they see 23, 33, 43, 53, etc., leading to understanding of addition by tens. If they look at the numbers along the diagonals they see how the units and tens places are changing. They notice what happens when they increase any number by nine. A modified version of snakes and ladders can be played using a 100 square board.

ACTIVITY NINE

Objective
Teaching 101 to 999

Materials required:

- Hundred square sheets, Ten square Strips and Square slips
- Place value cards
- Abacus, beads
- Arrow cards

The initial focus is on numbers 100 to 200.

Each child must have a place value kit (hundreds, tens and units material, arrow cards, place value card, abacus) which can be used for depicting any number between 100 and 200.

Handling the concrete material should be followed by a representation (semi-concrete) in the square ruled note book, accompanied by the written form of the number.

Common errors: When asked to write the number which comes after 129, a child writes 1210.

What could be the causes for this?

The child has not understood that when the units increase to ten, it alters the tens place and the units place.

The child has also not grasped that any place can hold only one digit.

The child may not be reading the number as a whole –not as ‘one hundred and twenty-nine’ but as ‘one two nine’.

A child who has handled concrete materials for a sufficient length of time would have internalized the relevant concept, and this would have prevented and corrected these types of problems.

It is important to focus on these transition points in numbers: 119-120-121, 129-130-131, 139-140-141, etc. Children need to perceive the patterns present here. Many text books do have exercises

which expect children to fill a 10 by 10 square grid with numbers from 101 to 200. This exercise is meaningful if teachers pose questions based on this, requiring them to observe and record different kinds of patterns and helping them to generalize from the observations.

Once children are thorough with numbers from 100 to 200, one can proceed to 200 to 999.

Common error: When asked, “How many tens are there in 342?” a child responds by saying ‘4 tens’.

This error comes from the child not having understood that each higher place is composed of the lower places.

The teacher will need to show that the hundreds are composed of tens and 3 hundreds are composed of 30 tens. So the number 342 contains 34 tens and 2 units. While discussing place value it is important to help the child to realize that tens are composed of units, hundreds are composed of tens and units and so on.

As practice one needs to pose exercises like: $254 = \underline{\quad}$ tens + $\underline{\quad}$ units, with the blanks to be filled.

Common error: While comparing numbers, a child writes ‘ $97 > 102$ ’.

This is an error of incorrect application of procedures. The child is comparing the starting digits in both the numbers without reading the whole numbers with their place values.

ACTIVITY TEN

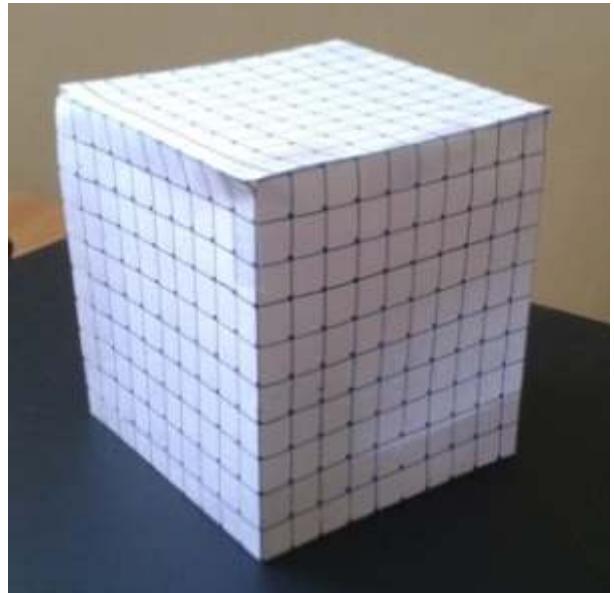
Objective
Introducing thousand

Materials required:

- Wooden cube or a card board cube (as shown in the picture), hundred square sheets, ten square Strips and Square slips
- Place value cards
- Abacus, beads

Use materials or drawings to show 991, 992, 993, etc., till you reach 999, and ask the children what would happen when one more unit is added. Lead them to discover that one more unit will increase the tens by one more ten, one more ten will lead to an increase in hundreds by one more hundred and that we will then have 10 hundreds. Now you can tell them that 10 hundreds is called a thousand and is written as 1000. You can show the model for 1000. If you show them a cube (as shown in the picture) you will need to discuss with the children the layers of hundreds in them, get them to count these layers so that they are able to visualize the 1000. They need to see that a thousand equals each of the following: 10 hundreds, 100 tens, 1000 units. As the numbers grow larger and counting is not an option any longer, children need to notice relationships and patterns, generalize them and develop the capacity for abstraction.

You can now demonstrate this on an abacus, counting from 990 onwards.



Paper cube

ACTIVITY ELEVEN

Objective
Thousands and larger numbers

Materials required:

- Abacus, beads

Beyond thousand it is not necessary to use concrete materials as the child would have internalized the relationships between different places, and would be in a position to extend his understanding to other places.

However usage of abacus does help many children, particularly children who have difficulties with visualization and abstraction. It aids in gaining a functional understanding of zero. Many children make

errors while writing large numbers with several zeroes in them. Abacus as a visual aid helps in strengthening the memory of the place value order.

Some activities and games given above can be used for teaching thousands and larger numbers.

It is important that the teacher helps the children to develop both factual understanding as well as procedural understanding. In the context of place

GAME

value, children must understand that a thousand is equal to 10 hundreds, 100 tens, 1000 units. They must also understand that each place is created by taking 10 times the lower place. Once they are clear on the procedures, they will then be able to generalize them and apply them to higher place values (up to lakhs or millions at the appropriate age).

Right from the beginning we must help children to learn the place values in order from the right most number. For example: 32,504. We need to point to 4 (while saying units) and move step by step mentioning each place value so that the child notes the order.

This will address the problem of wrong reading of numbers. The point that one has to stress is that the value of a place is determined by reference to the right most place.

"What number precedes 2,01,010?" A child writes 2,00,009.

We find that the child has not completely understood the way numbers increase.

Teachers should consciously help children to achieve understanding of succession of numbers by discussing many such problems.

GAME

Game 5: Twenty questions

Tell the children that your number lies between 100 and 200. The children have to find the number by asking twenty questions. They can only ask questions of the kind which require an answer "yes" or "no". They may ask a question like "Is the number more than 130?" Teacher can draw a number line on the board, and after each yes/no answer, cross out the irrelevant part to help children in visualizing the range within which the number lies. It also helps children learn how to ask good questions, how to eliminate the unnecessary parts, and how to use diagrams in problem solving.



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Padmapriya Shirali is part of the Community Math Centre based in Sahyadri School (Pune) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box'. Padmapriya may be contacted at padmapriya.shirali@gmail.com