# An Impossible Construction?

It is well known that there is no general procedure for trisecting an angle using only a compass and unmarked ruler (naturally, we must stick to the rules governing geometric constructions).
In particular, a 60° angle cannot be so trisected. This implies that a 20° angle cannot be constructed using such means. However, here is a construction which appears to do the impossible! Throughout, the notation 'Circle($P$,$Q$)' means: "circle with centre $P$, passing through $Q$" (for a given pair of points $P$, $Q$). We start with any two points $O$ and $A$ (see Figure 1) and follow the steps given below.

1. Draw Circle($O$,$A$) and Circle($A$,$O$). Let $B$ be one of their points of intersection.
2. Draw Circle ($A$,$B$) and Circle($B$,A). Let $C$ be their point of intersection other than $O$.
3. Let $D$ be the point other than $O$ where Circle($A$,$B$) meets ray $OA$.
4. Draw Circle($C$,$D$) and Circle($D$,$C$). Let E be their point of intersection other than $A$. Join $OE$.
5. Measure $\measuredangle EOD$. It appears to be a 20° angle!

So: has the impossible been achieved?

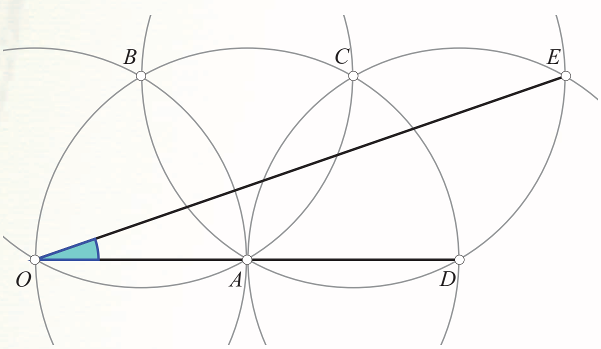


**Figure 1.** Supposed construction of a 20° angle

Construction sent to us by Shri Ashok Revankar of Dharwar. The work is that of his student Subra Jyoti, of KV Dharwar.

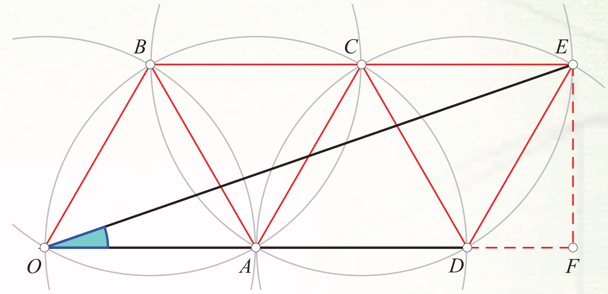


**Figure 2.**

No, the impossible has not been achieved! We shall show that $\measuredangle EOD$ is close to 20° but is not equal to it.

Draw the segments joining the centres of the circles; we get a lattice of equilateral triangles (Figure 2). If we take $OA$ = 2 units, then $B$, $C$, $E$ all lie at a perpendicular distance of $\sqrt{3}$ units from line $OD$. (Use Pythagoras's theorem to see why.) Let $F$ be the foot of the perpendicular from $E$ to line $OD$; then $EF = \sqrt{3}$ and $DF = 1$, hence $OF = 2+2+1 = 5$, and:

$$\tan \measuredangle EOD = \frac{\sqrt{3}}{5}.$$

Using a scientific calculator we get: $\measuredangle EOD = \tan^{-1} \sqrt{3}/5 \approx 19.1066°$. So $\measuredangle EOD$ is quite close to 20°. The difference is small enough that the eye will most likely not notice it.

With some experimentation you will be able to find more such constructions, which come close to 'doing the impossible'. (There are many such impossibilities in plane geometry, and we shall be examining more such examples in the following issues.) In such cases, doing an error analysis of the kind we have done can be most instructive.

The COMMUNITY MATHEMATICS CENTRE (CoMaC) is an outreach sector of the Rishi Valley Education Centre (AP). It holds workshops in the teaching of mathematics and undertakes preparation of teaching materials for State Governments, schools and NGOs. CoMaC may be contacted at comm.math.centre@gmail.com.