TearOut

Fun with Dot Sheets

The 'TearOut' series is back, with perimeter and area. Pages 1 and 2 are a worksheet for students, while pages 3 and 4 give guidelines for the facilitator. This time we explore shapes with given perimeter or given area using the dots or the grid.

You will have to draw various shapes in this worksheet. The corners (or vertices) of each shape must be a dot (or where the lines cross each other on the grid). You can use only sleeping (horizontal) lines and standing (vertical) lines. Do not use any slant line. Check Figure 1.

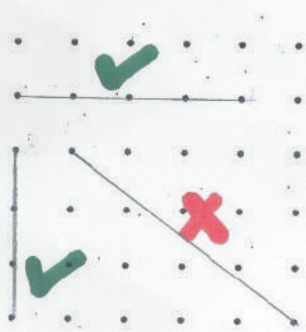

Figure 1

The gap between any two adjacent dots along a horizontal or vertical line should be taken as unit length. Similarly, the area of the smallest possible square with the dots as vertices should be considered one square unit. Likewise, the smallest line segment and the smallest square on a square grid should be taken as unit length and square unit respectively (Figure 2).

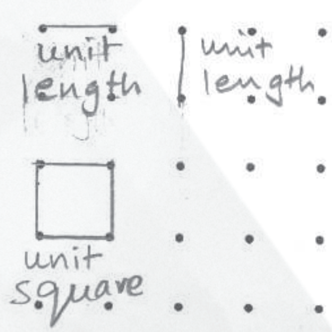

Figure 2

Get some dot sheets (or square grid papers), a pencil, an eraser and, ideally, a scale … you are all set to go… ☺

1. Draw shapes with area 8 sq. units.
   a. Find the perimeter of each shape.
   b. Did any shape have a perimeter of 14 units? If not, draw one such.
   c. Did you get different shapes with the same perimeter?
   d. How long is the shortest perimeter?
   e. How long is the longest one?

2. Draw shapes with perimeter 14 units.
   a. Find the area of each shape.
   b. Could you draw a shape with area 8 sq. units that is different from the one you drew for 1.b.?
   c. Did you get different shapes with the same area?
   d. What is the smallest area?
   e. What is the largest area?

3. Draw the following:
   a. A shape with area less than 8 sq. units and perimeter shorter than 14 units
   b. A shape with area more than 8 sq. units and perimeter longer than 14 units
   c. A shape with area less than 8 sq. units but perimeter longer than 14 units
   d. A shape with area more than 8 sq. units but perimeter shorter than 14 units

	Area	Perimeter
Bigger rectangle		
Smaller rectangle		
L-shaped region		

ClassRoom

4. Draw any rectangle, which is at least 6 units long and 4 units wide. Draw a smaller rectangle inside this bigger one such that they share a corner (Figure 3). Shade the L shaped region in between the two rectangles.

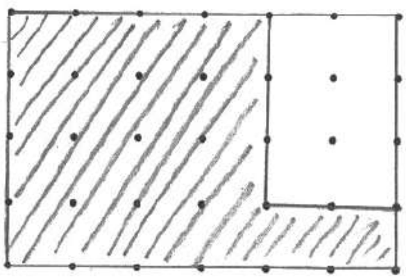

Figure 3

   a. Find the areas and the perimeters of the rectangles and the L shaped region.
   b. How are the three areas related?
   c. How are the three perimeters related?

5. Repeat the above with other pairs of rectangles
   a. Do you see any patterns? What are they?
   b. How can you explain these patterns?

6. Draw another big rectangle with the same length and width as the one you drew for 4. Draw another smaller rectangle with the same length and width as the smaller one you drew for 4. This time draw the smaller rectangle inside the bigger one such that they share one side but do not share any corner (Figure 4). Shade the U-shaped region in between the two rectangles.

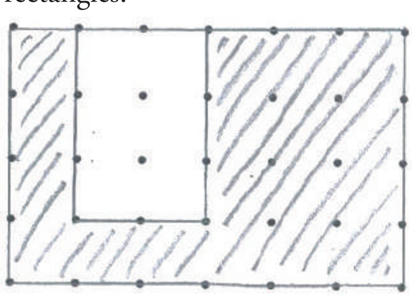

Figure 4

	Area	Perimeter
Bigger rectangle		
Smaller rectangle		
U-shaped region		

   a. Find the areas and the perimeters of the rectangles and the U shaped regions.
   b. How are the three areas related?
   c. How are the three perimeters related?

7. Repeat the above with other pairs of rectangles
   a. Do you see any patterns? What are they?
   b. How can you explain these patterns?

8. Draw any shape and call it Shape 1. Modify it to form a new shape in the following manner. The modified shape and the original one must not differ too much.*
   a. Shape 2 with same perimeter but smaller area
   b. Shape 3 with same area but longer perimeter
   c. Shape 4 with smaller area but longer perimeter
   In each case, explain how you modified the shape.

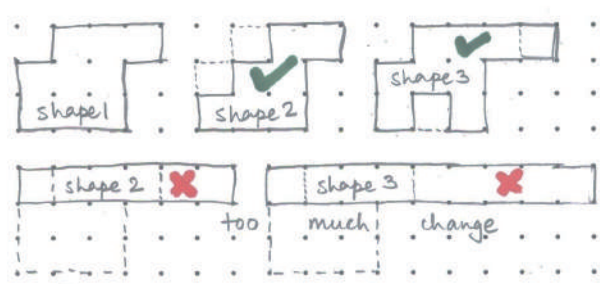

Figure 5

* For example, in Figure 5: top row – shape 1 has 9 squares, shape 2 and shape 1 differ by 2 (out of 9) squares; shape 3 and shape 1 also differ by that much. However, in the bottom row, shape 2 (and shape 3) differ from shape 1 by 6 (out of 9) squares. This ratio should be less than half [i.e., $2/9 < 1/2$ but $6/9 = 2/3 > 1/2$].

This worksheet can be done with Grades 4-5 children once they have an exposure to perimeter and area. The pre-requisite is the understanding of these two measures and not any formula.

The restriction on using only vertical and horizontal lines makes it easier to find the area and the perimeter of the drawn shapes. Perimeter can be found by simply counting the unit lengths along the border of a shape. Similarly, area is given by counting the squares enclosed within the shape. Since dot sheets don't have lines, it is easier to find the perimeter of the shapes drawn compared to a square grid with lines.

The worksheet allows explorations and observations. It encourages one to find patterns and explain them. The last part urges one to use the learnings to create further shapes. This can be used to assess if children can differentiate between perimeter and area. It will help them realize that different shapes can have the same perimeter (Q.2) or same area (Q.1) or both (Q.2b). It breaks the misconception that if perimeter is increased, then area must increase and vice versa (Q.3c, 3d). It also helps them figure out how to reduce area (i) without decreasing perimeter (Q.4, Q.5) and (ii) by increasing perimeter (Q.6, Q.7). They are particularly asked to investigate patterns and articulate the same (Q.5 and Q.7). Finally, they are asked to utilize their learnings to create new shapes with given specifications (Q.8).

1. This question allows children to observe that different shapes can have the same area but possibly different perimeters. All the shapes are essentially octominoes, i.e., polygons made by joining 8 squares. The longest and shortest possible perimeters for octominoes are 18 units and 12 units respectively.

2. The question explores polygons with same perimeter. So it is a relatively harder task on the dot sheet (or grid). In this case, the maximum and minimum possible areas are 12 sq. units and 6 sq. units respectively. In other words, the shapes can be made with 6-12 squares.

3. This question is about testing whether children developed any intuitive idea on how to maximize/minimize perimeter and area. While the first two parts (a and b) are easy to do, the subsequent parts (c and d) pushed them to develop these ideas.

4. This question helps children figure out how an L-ing of a rectangle reduces its area – since that of the smaller rectangle is subtracted – while keeping the perimeter same.

5. This takes it up a notch. The goal is to get children to observe that an L-ing would always reduce the area but would always keep the perimeter intact. They should be encouraged to explain why the perimeter remains unchanged. This will help them see that the perimeters of the bigger rectangle ABCD and the L-shape ABCGFE are the same because ED and DG get exchanged with FG and EF respectively. Since ED = FG and EF = DG being opposite sides of the smaller rectangle EFGD, this exchange causes no change in the perimeter (Figure 6).

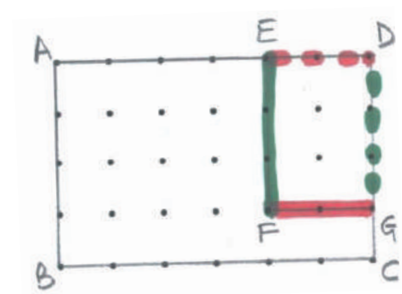

Figure 6

6. As a continuation of the above, this question considers the often un-intuitive situation where area reduces while perimeter increases. This introduces children to the U-ing of a rectangle with the same effective change in the area as in 4.

7. This question is similar to 5. It gets children to conjecture that a U-ing of a rectangle should always reduce the area while increasing the perimeter. Children should be encouraged to observe and argue how the U-ing results in two extra sides FG and EH of the smaller rectangle getting added to the resultant U shape ABCDEHGF, thereby causing a longer

perimeter (Figure 7). Note that FE in the rectangle ABCD gets replaced by GH in the U-shape.

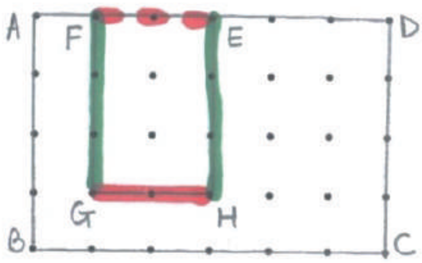

Figure 7

8. Takes this learning further and asks children to apply them to start with a shape and modify it according to given specifications. It may be natural for children to start with a rectangle. This will work fine for 8a and 8c if the minimum dimension is at least 2 units. But it may not work for 8b. So, the starting point has to be a more interesting shape. The first part (a) is a direct application of L-ing, the second one (b) can be achieved by changing an L to a U and the last one can be achieved by applying U-ing.

If this worksheet is used for higher grades, then slant lines can be allowed. The basic aspects of L-ing and U-ing can be generalized by cutting out non-rectangular shapes as well. The starting shape does not need to be rectangular either. Below are some examples of L-ing and U-ing with polygons made with slant lines. Will L-ing always keep the perimeter same? If not, then when will it? Figure 8 includes three cases:

a. U-ing
b. L-ing with no change in perimeter – Why?
c. L-ing with changed perimeter – Increased or decreased? Justify.

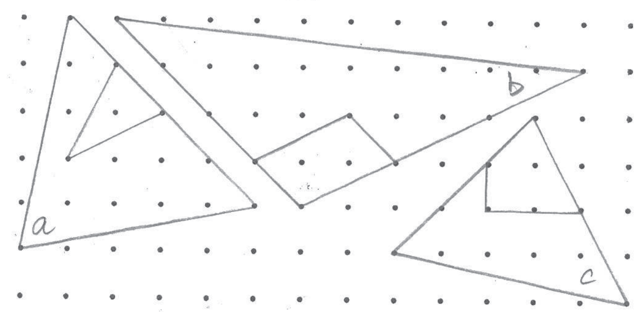

Figure 8

CAPTURED MATHEMATICS

The ground area of this camp has been beautifully designed by removing grass of a fixed square unit on alternate positions

Design of a camp area near Sela Pass, Tawang, Arunachal Pradesh

Photo & Ideation: ***Kumar Gandharv Mishra***

Mathematical Relevance: Tessellation