DIY Problems for the MIDDLE SCHOOL

A. RAMACHANDRAN

The tasks of this set require you to get down to some actual cut and paste work. Arm yourself with a chart paper, a roll of cello tape and a pair of scissors.

Task 1. Cut out a rectangular piece of chart paper with dimensions 16 x 9 units. Now this has the same area as a square piece of side 12 units. Your task is to cut the 16 x 9 piece into just two parts and put them together again to form the square.

Task 2. Take a piece of chart paper the size of a post card (roughly 15 cm. x 10 cm.). Post cards are not much in use now but that is how the author first encountered this problem. Your task is to cut a hole in this, through which you can pass yourself. (You could hang it around your neck and then let it slip down; you then step out of it.)

Task 3. A tetrahedron is a Platonic solid – its 4 faces are congruent equilateral triangles. You could also think of it as a triangular pyramid – a 3-D shape with a triangular base and 3 triangular sides meeting at an apex. Cut out two copies of the net (Figure 1), fold and stick each of these to get two wedge-like shapes, and then assemble them into a tetrahedron which can be sliced to give two identical 3D pieces, revealing a square cross section.

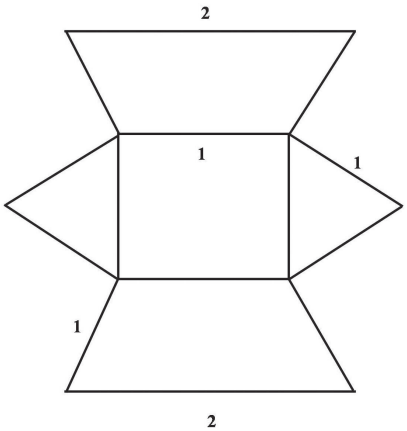

Figure 1

Keywords: Tetrahedron, cube, skew pyramid, foldable map.

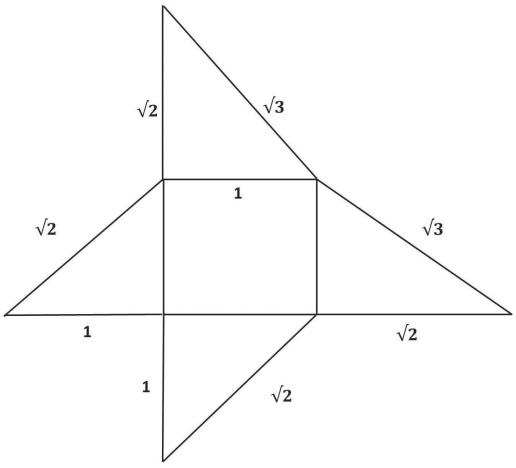

Figure 2

Problem 4. A cube is another Platonic solid – its 6 faces are congruent squares. You could also think of it as a square prism – a 3-D shape with congruent squares at top and bottom and rectangular (here, square) sides. Cut out three copies of the net (Figure 2), fold and stick each of these to get three skew pyramids, and then assemble them into a cube which can be sliced into three identical pyramids.

Problem 5. You may have used foldable printed maps in geography lessons or while traveling. They are usually rectangular in shape, divided into convenient sized smaller rectangles or squares by vertical and horizontal lines. Such maps are generally kept folded along these lines. After opening and using one, you may be at a loss as to how to fold it back. There are many alternative ways to fold a map and this task is an attempt to track them down.

Cut out a square piece of chart paper. Divide this into four squares. Mark them A, B, C, D as in Figure 3. This is your 2 x 2 'map.' Fold this along the dividing lines in as many ways as possible. You could do the vertical fold first and then the horizontal, or vice versa. Every fold can be forward or backward. Once you have folded the map into a square, note down the order in which the letters A, B, C, D appear, reading from top to bottom. How many different ways of folding can you find (by different ways, we mean different orders of the four letters after the folding)?

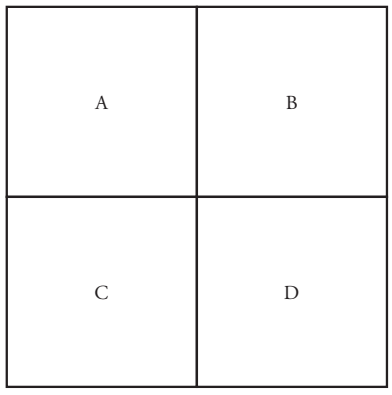

Figure 3

Discussion

1. Cut the 16 x 9 piece as shown in Figure 4. Move the part on the right towards the bottom left and join it up again to get the square.

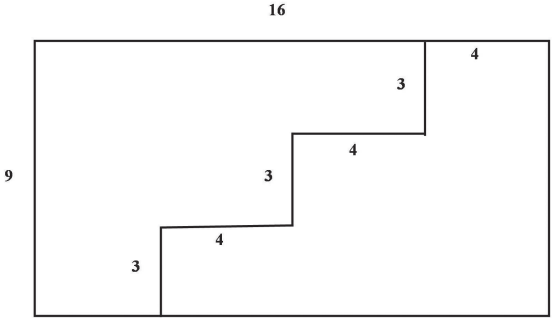

Figure 4

Exploration: Can you think of other rectangles for which such a dissection can be done? The task is the following. We are given a certain integer sided rectangle, and we are required to cut it into just two pieces and to then assemble the two pieces to form a square. Try it out for different integer sided rectangles and see when it works out and when it does not. What pattern is needed in the sides of the original rectangle for such a procedure to work?

2. Figure 5 should give you an idea as to how to proceed. You need to redraw the figure with more twists and turns. If you cut along the lines shown you get a loop which should be sufficiently long. Theoretically if one settles for a strip width of 1 cm one should get a

loop 150 cm long. For a strip width of 1.5 cm, the loop length would be 100 cm.

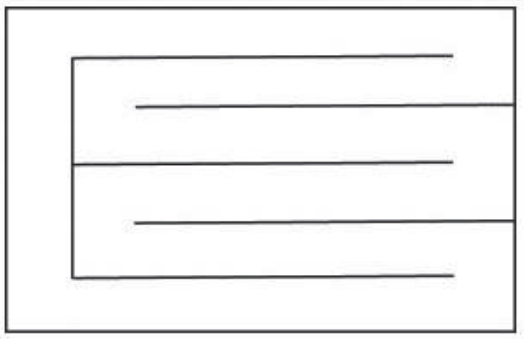

Figure 5

3. The phrase 'square cross-section' should be a giveaway and you should not have any difficulty carrying out this task.
4. The side lengths $\sqrt{2}$ and $\sqrt{3}$ should give you a lead. These are the lengths of the surface diagonals and interior diagonals of a cube of unit side.
5. You should have found eight different ways to fold your 2 x 2 map. The order of the letters after the folding would be as below:

BACD CDBA DCAB ABDC BDCA ACDB CABD DBAC

There are 4! = 24 ways of ordering four distinct objects/letters. But in actually folding the above map, squares A and D cannot come next to each other; nor can squares B and C. You could write down all the permutations of the letters A, B, C, D and then cross out the ones where the pairs A, D or B, C or both occur in adjacent positions. You should be left with the above eight orderings.

Hope that was not too tiresome. However, the problem becomes much more complex if we take up larger 'maps'. A 3 x 2 map can be folded in 60 different ways. Some ways of folding involve inserting one part of the map into another folded part. This is only a small part of the 720 permutations of 6 objects/letters.

A STAR FILLED SUM

Is it possible to replace each star (☆) in the following equality with a different nonzero digit (1-9) so as to make the following equality true?

$$\frac{☆}{☆☆} + \frac{☆}{☆☆} + \frac{☆}{☆☆} = 1.$$