The gap between 'HOW' and 'WHY' in Mathematics…

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Number tricks are fun to perform and are an excellent way to enhance mathematical skills. When I was young, we used to discuss a lot of number tricks. One amongst those was as follows:

Take a number with a lot of digits. For example, suppose you think of 2134567.
Add its digits: 2 + 1 + 3 + 4 + 5 + 6 + 7 = 28.
Subtract 28 from the original number taken, 2134567. You'll get some answer, say *abcdefg*.

Keeping any one digit (say *e*) as secret, if you say the remaining digits, abcdfg in any order, I can figure out the number '*e*' which you kept hidden from me in less than a fraction of a second…Just one request – that zero not be the hidden digit.

So in this case 2134567-28 = 2134539. If you hide 3 and tell me the remaining digits 1, 2, 3, 4, 5 and 9, I will be able to tell you that the hidden number was 3.

I will illustrate this with one more example. If you thought of 345995, add its digits: 3 + 4 + 5 + 9 + 9 + 5 = 35. 345995-35 = 345960. If you hide 9 and tell me the remaining digits 0, 3, 4, 5, 6 then I will be able to tell you that the hidden digit was 9.

After learning the trick, I amazed many people with it. Try to figure out the trick…

If you're successful …good. Otherwise, let me explain. The trick is add the digits and subtract this sum from the next higher multiple of 9. i.e., 'the multiple of 9 just higher than the digit sum'.

The difference is the hidden digit. When adding the digits, all pairs which add up to 9 and 9 itself can be omitted.

2 + 1 + 4 + 5 + 3 + 9 = 24 and 27 – 24 = 3
(Alternatively you could use 2 + 1 + 3 = 6
(omitting 5 + 4 = 9 and 9 itself) and then 9-6 = 3.)

In the second example: 0 + 3 + 4 + 5 + 6 = 18, its digit sum is 9. 9 – 9 = 0, and as I requested that zero not be the hidden digit, the hidden digit must be 9.

After learning '**How**' to get it, frankly, I never bothered to find out '**Why**' this happens. Our mathematics curriculum encourages us to learn how to solve a problem and we somehow learn the tricks of the trade and ace the examination. Rarely do we question '**Why**' the solution is obtained that way. '**How**' to get the solution is just exercising mathematical thinking. In order to think like a mathematician, we should often ask '**Why**'.

Now let's try to think like mathematicians and find out the reason behind this trick. For that let's start with smaller numbers such as this 2-digit number which we shall call XY.

Adding its digits gives $X + Y$. Subtracting this from the original number which is actually $10X + Y$ we get $10X + Y - (X + Y) = 9X$.

Similarly, if we have a 3-digit number: say XYZ. Adding its digits gives $X + Y + Z$. Subtracting this from the original number which is actually $100X + 10Y + Z$ we get $99X + 9Y$, again a multiple of 9.

Similarly, for a 4-digit number say $WXYZ$ we get the difference as $999W + 99X + 9Y$, a multiple of 9.

It is clear from the pattern that we always get a multiple of 9. Any number is a multiple of 9 if the sum of its digits is a multiple of 9. Thus, when the difference is less than 9, the hidden digit should be the difference between 9 and the difference.

There are plenty of situations wherein we use formulae routinely without actually bothering to know '**why**' the formula is so. For instance, the volume of a cone or the surface area of a sphere, or even the formula for the area of a circle. It will give us a clear understanding of the concepts/formulae if we make the conscious effort to ask '**why**' while learning mathematical concepts.

Admittedly, the gap between '**how**' and '**why**' is not small. Making an effort and constantly thinking about it may sometimes lead us in the right direction. There are many simple sounding but unsolved problems, for instance, the Goldbach Conjecture, conjectures about Twin Primes, Perfect Numbers, …; these conjectures are easy to understand but to date we wonder '**why**' they should be true. Unless we find out '**why**', they remain conjectures that may or may not be true.

http://www.math.utah.edu/~pa/math.html

https://artofproblemsolving.com/wiki/index.php?title = Goldbach_Conjecture