The Game of Craps

JONAKI B GHOSH

The topic of probability is replete with many interesting problems. Some of these, based on games of chance, can lead to interesting explorations in the classroom. Exploring such games actually enlivens the study of probability and also provides opportunities for students to explore many fundamental concepts in probability. In this article we shall explore a very popular game played in casinos called the *Game of Craps* or just *Craps* which is played simply by rolling a pair of dice. It is said that if you want to double your money quickly on a game of pure chance, one of your best opportunities is to bet all your money on one game of craps! Apparently the probability of winning this game is very close to 50%. In this article we will analyse this game by simulating it on the spreadsheet MS Excel. Of course, the simulation can be done on any spreadsheet such as LibreOffice Calc.

The Power of Simulations

Before we proceed, we need to say a little about simulation although a more extended treatment of simulation is beyond the scope of this article. Simulation is essentially a modeling tool, which is used to imitate real-world problems in order to understand system behavior or the behavior of certain phenomena. In particular, *Monte Carlo Simulation* is a problem solving technique used to approximate the probability of certain outcomes of an experiment by running multiple trial runs (called simulations), using random numbers. Simple simulations of real world problems can be explored through spreadsheets such as MS Excel, which have inbuilt functions for generating random numbers. One of the ways of exploring and analyzing games of chance such as Craps is through simulations since such games, by their very nature, are driven by randomness. In this article, we will illustrate how we can play Craps simply by *generating random numbers* instead of actually rolling a pair of dice.

Keywords: Probability, gaming, craps, simulation

The rules of the game

As mentioned earlier, Craps is played by rolling a pair of dice. When a pair of dice is rolled, there are 36 possible outcomes. However, the sum of the numbers on the two faces can range from 2 (with 1 appearing on both dice), to 12 (with 6 appearing on both dice). Sums of 3, 4, 5, 6, 7, 8, 9, 10 and 11 are also possibilities. We also know that the probability of obtaining each of these sums can be easily computed. For instance, a sum of 9 can be obtained if the dice show up the numbers (3,6), (4,5), (5,4) or (6,3). In the language of probability, we say that these four outcomes are favorable to the occurrence of the sum of 9 and the probability of obtaining this sum is 4/36 = 1/9.

Let us now familiarize ourselves with the rules of the game.

A player rolls a pair of dice. This may lead to three possible outcomes:

1. A total of 7 or 11 is obtained on the first roll. In this case the **player wins** and this outcome is referred to as a '**Natural**'.
2. A total of 2, 3 or 12 is obtained on the first roll. In this case the **player loses** the game. Obtaining a 2 is referred to as **Snake eyes**'
3. A total of 4, 5, 6, 8, 9 or 10 is obtained on the first roll. In this case the player neither wins nor loses but can roll again. The number becomes the player's **point** and the game continues, that is, the player rolls the dice again. Two situations can happen:
   a. In subsequent rolls, the player's point appears before a total of 7. In this case the **player wins** the game.
   b. In subsequent rolls, a total of 7 appears before the point. In this case, the **player loses** the game.

   For example, let us suppose that 6 appears on the first roll which is the player's point. He can continue to roll the dice till either 6 (his point) or 7 appears (other numbers are inconsequential). If 7 comes up before 6, the player loses whereas if 6 comes before 7, the player wins.

The probability of a win

Let us now analyse the game using our knowledge of theoretical probability and compute the probability of winning a single game of craps. Clearly, a win takes place if outcomes 1 or 3a, described above, takes place. Let us begin by computing the probability of **winning on the first roll**, or getting a natural, that is, a total of 7 or 11. The possibilities of obtaining 7 are (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) and those for obtaining 11 are (5,6) and (6,5). Thus there are 8 favorable outcomes out of a total of 36 possible outcomes leading to a probability of 8/36 or 22.22%.

Next we calculate the total probability of winning by getting a total of 4, 5, 6, 8, 9 or 10 (which becomes the player's point). In order to find the probability of winning by getting a total of, say 5, we would need to find the probability of getting 5 in the first roll and then getting 5 before getting 7 in subsequent rolls. This is a case of conditional probability and it can be calculated as follows:

The probability of getting a 5 in the first roll is 4/36 (the possible outcomes being (1,4), (2,3), (3,2) and (4,1)). In subsequent rolls, we need to consider only totals of 5 and 7 which leads to only 10 possibilities (4 for a total of 5 and 6 for a total of 7). Among these, a win is possible only when a total of 5 precedes a total of 7, the probability for which is 4/10. Thus the total probability of winning by getting a 5 in the first roll is 4/36 × 4/10 = 0.044. Similarly we can calculate the probability of winning by getting a total of 4, 6, 8, 9 and 10 in the first roll. Table 1 lists the probabilities of winning for all possible outcomes. The total probablity of winning a single game of craps works out to approximately 0.49!

Table 1: Probabilities of winning a Game of Craps

Initial total	Probability
4	0.027
5	0.044
6	0.063
7	0.167
8	0.063
9	0.044
10	0.027
11	0.056
Total probability of winning	**0.491**

Similarly we can calculate the probability of losing on the first roll, that is, getting a total of 2, 3 or 12 (snake eyes). The possibilities are (1,1), (1,2), (2,1) and (6,6) leading to a probability of 4/36 or 11.11%

Simulating the Game of Craps in Excel

Having worked out the various possibilities of winning the game we can now try to simulate it in Excel. In order to perform the simulation we need to be familiar with the following basic functions of Excel.

IF – a logical function, which specifies a logical test to be performed or a condition to be checked.

OR – a logical function which returns TRUE if *any* of the arguments of a statement is true.

AND – a logical function which returns TRUE if *all* of the arguments of a statement are true.

SUM – a function that adds up all the values of an argument

COUNT – a statistical function which counts how many numbers are in the list of arguments.

RANDBETWEEN – returns a random number between two specified numbers.

We shall now try to simulate the game and find out the average number of throws required to win the game. The steps to be followed are:

Step 1: Enter the numbers 1 to 20 in column A. To do this, we enter 1 in cell A2 and = **A2 + 1** in cell A3. Dragging cell A3 till A21 will create a column of numbers 1 to 20. These indicate the number of rolls of the pair of dice.

Step 2: To simulate the numbers appearing on a pair of dice we enter **=RANDBETWEEN(1,6)** in B2 and C2 as shown in Figure 1.

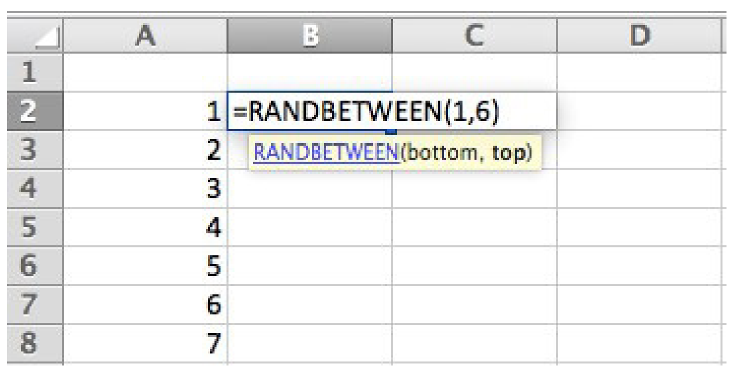

Figure 1: The RANDBETWEEN command in Excel randomly generates an integer between 1 and 6 (inclusive of 1 and 6).

Step 3: After this, we select the cells B2 and C2 and double click in the corner of cell C3. This will fill both columns with numbers from 1 to 6, thus simulating 20 rolls of a pair of dice as shown in Figure 2. To obtain the sum of the numbers appearing on the pair of dice we enter **=SUM(B2,C2)** in cell D2. Column D will show the sums of the dice rolls. The entry in cell D2 (highlighted in red) will determine if the player has won, lost or if the game can be continued.

	A	B	C	D
1		Die1	Die2	Sum
2	1	4	6	10
3	2	6	2	8
4	3	6	2	8
5	4	1	6	7
6	5	6	6	12
7	6	1	5	6
8	7	5	6	11
9	8	3	1	4
10	9	1	5	6
11	10	6	5	11
12	11	2	1	3
13	12	6	1	7
14	13	2	3	5
15	14	1	6	7
16	15	6	3	9
17	16	2	3	5
18	17	5	3	8
19	18	3	4	7
20	19	6	1	7
21	20	4	6	10

Figure 2: Excel simulates 20 rolls of a pair of dice. The entry in cell D2 (shown in red) determines if the player wins, loses or if the game will continue.

Step 4: To check the outcome of the first roll we enter the conditional statement **=IF(OR(D2=7,D2=11), 1,0)** in cell E2 as shown in Figure 3. This will ensure that the value of cell E3 will be 1 if the player obtains a sum of 7 or 11, that is, if the player wins, and will be 0 otherwise.

	A	B	C	D	E	F
1		Die1	Die2	Sum		
2	1	2	4	6	=IF(OR(D2=7,D2=11),1,0)	
3	2	2	6	8		
4	3	3	4	7		

Figure 3: The IF conditional statement is used to check if the player wins in the first roll.

Similarly we will enter the conditional statement **=IF(OR(D2=2,D2=3,D2=12), 1,0)** in cell F2 as shown in Figure 4. This will produce the output 1 in cell F2 if the sum on the first roll is 2,3 or 12, that is, if the player loses and will be 0 otherwise.

	A	B	C	D	E	F	G	H
1		Die1	Die2	Sum				
2	1	2	5	7	1	=IF(OR(D2=2,D2=3,D2==11),1,0)		
3	2	5	2	7	OR(logical1, [logical2], [logical3], [logical4], ...)			
4	3	3	2	5				

Figure 4: The IF conditional statement is used to check if the player loses in the first roll.

Step 5: Finally we enter the conditional statement **=IF(AND(E2=0,F2=0), "YES","NO")** in cell G2. The output of this statement (YES or NO) indicates if the game will continue. The output will be YES if the game continues beyond the first roll and NO otherwise. Note that if the game does not continue beyond the first roll, we will have to ignore the remaining rows of the ouput.

	A	B	C	D	E	F	G	H	I
1		Die 1	Die 2	Sum					
2	1	1	6	7	1	0	=IF(AND(E2=0,F2=0),"YES","NO")		
3	2	5	2	7					
4	3	1	1	2					

Figure 5: The IF conditional statement is used to check if the game continues beyond the first roll.

Step 6: If the game continues beyond the first roll, we need to insert conditional statements in cells E3, F3 and G3 as follows

In cell E3 enter **=IF(OR(G2="NO",G2=""),"",IF(D3=$D$2,1,0))**

In cell F3 enter **=IF(OR(G2="NO",G2=""),"",IF(D3=7,1,0))**

In cell G3 enter **=IF(OR(G2="NO",G2=""),"",IF(AND(E3=0,F3=0),"YES","NO"))**

After inserting these statements, we will need to select the cells E3, F3 and G3 and double click in the corner of cell G3 so that the above formulae get evaluated in all the cells till row 21.

	A	B	C	D	E	F	G	H
1		Die1	Die2	Sum				
2	1	6	3	9	0	0	YES	
3	2	3	2	5	=IF(OR(G2="NO",G2=""),"",IF(D3=$D$2,1,0)			
4	3	5	4	9	IF(logical_test, [value_if_true], [value_if_false])			
5	4	5	1	6				
6	5	6	3	9				

	A	B	C	D	E	F	G	H
1		Die1	Die2	Sum				
2	1	4	3	7	1	0	NO	
3	2	2	4	6		=IF(OR(G2="NO",G2=""),"",IF(D3=7,1,0))		
4	3	5	6	11		IF(logical_test, [value_if_true], [value_if_false])		

	A	B	C	D	E	F	G	H	I
1		Die1	Die2	Sum					
2	1	4	1	5	0	0	YES		
3	2	6	6	12	=IF(OR(G2="NO",G2=""),"",IF(AND(E2=0,F2=0),1,0))				
4	3	1	2	3					
5	4	1	4	5					

Figure 6: The IF condition is used to compute the cells E3, F3 and G3 to continue the game further.

Step 7: After entering the IF conditions in the cells E3, F3 and G3, the excel sheet is ready for simulation. Click on any cell of column B or C which represent the die rolls. This will generate a new simulation and the outcome of the game can be easily interpreted as we shall see in the next section.

	A	B	C	D	E	F	G
1							
2		Die 1	Die 2	Sum			Game continues?
3	1	2	5	7	1	0	NO
4	2	5	5	10			
5	3	6	6	12			
6	4	3	1	4			
7	5	3	6	9			
8	6	6	4	10			
9	7	3	3	6			
10	8	2	5	7			
11	9	3	4	7			
12	10	2	1	3			
13	11	4	4	8			
14	12	4	6	10			
15	13	3	2	5			
16	14	2	5	7			
17	15	6	4	10			
18	16	1	4	5			
19	17	5	2	7			
20	18	1	1	2			
21	19	2	5	7			
22	20	3	3	6			

Figure 7: The player wins by rolling a 7 in the first roll.

Analysing the game through simulation

Using the above code we will explore four scenarios. Note that each time we click on the Excel sheet a new data set of 20 dice rolls is generated and the sheet gets updated.

Case 1: The sum of the numbers of the two die in the first roll is 7 (Figure 7) and the player wins. The game doesn't continue beyond the first roll. Here we will ignore the output from row 2 onwards as these rows are no longer relevant.

Case 2: The player loses the game in the first roll as the sum is 3 (Figure 8). Once again the game doesn't continue beyond the first roll. Here we will ignore the output from row 2 onwards as these rows are no longer relevant.

Case 3: The player neither wins nor loses in the first roll. In Figure 9, the sum in the first roll is 9, which becomes the player's point. The game continues till either 9 or 7 appears. Since 9 appears in the 5th roll, the player wins the game in this roll. The remaining rows of the output may therefore be ignored.

In Figure 10 the players' point in the first roll is 5. However 7 appears before 5 in the subsequent throws and the player loses the game.

To compute the average number of rolls required to win the game we can conduct several simulations and record the number of rolls each time. Let us suppose that the number of rolls for 10 simulations are recorded. The total number divided by 10 will give us the mean number of rolls. The reader is encouraged to try the experiment for 20 simulations, then 50 and then 100. What do you observe about the mean number of rolls as the number of simulations of the game is increased?

	A	B	C	D	E	F	G
1							
2		Die 1	Die 2	Sum			Game continues?
3	1	1	2	3	0	1	NO
4	2	6	3	9			
5	3	3	5	8			
6	4	6	2	8			
7	5	6	5	11			
8	6	5	4	9			
9	7	1	2	3			
10	8	1	1	2			
11	9	5	3	8			
12	10	1	6	7			
13	11	4	3	7			
14	12	6	4	10			
15	13	1	2	3			
16	14	2	3	5			
17	15	4	1	5			
18	16	6	5	11			
19	17	2	5	7			
20	18	5	3	8			
21	19	3	1	4			
22	20	5	6	11			

Figure 8: The player loses as the sum of the first roll is 3.

	A	B	C	D	E	F	G
1							
2		Die 1	Die 2	Sum			Game continues?
3	1	6	3	9	0	0	YES
4	2	5	3	8	0	0	YES
5	3	4	1	5	0	0	YES
6	4	2	4	6	0	0	YES
7	5	5	4	9	1	0	NO
8	6	6	3	9			
9	7	6	4	10			
10	8	3	6	9			
11	9	5	5	10			
12	10	6	4	10			
13	11	6	2	8			
14	12	6	3	9			
15	13	3	5	8			
16	14	2	5	7			
17	15	3	4	7			
18	16	6	4	10			
19	17	1	3	4			
20	18	3	3	6			
21	19	6	3	9			
22	20	1	5	6			

Figure 9: The player wins in the fifth roll, as 9, the point is obtained before obtaining a 7.

	A	B	C	D	E	F	G
1							
2		Die 1	Die 2	Sum			Game continues?
3	1	1	4	5	0	0	YES
4	2	3	4	7	0	1	NO
5	3	2	5	7			
6	4	2	6	8			
7	5	5	1	6			
8	6	5	3	8			
9	7	3	3	6			
10	8	1	3	4			
11	9	3	6	9			
12	10	2	2	4			
13	11	4	1	5			
14	12	3	4	7			
15	13	3	3	6			
16	14	1	1	2			
17	15	1	6	7			
18	16	6	1	7			
19	17	2	4	6			
20	18	2	3	5			
21	19	3	5	8			
22	20	1	3	4			

Figure 10: The player loses in the second roll, as 7 is obtained before obtaining the point, that is, 5.

In general the probability of a player winning on a point is about 27.07%, that is, about once in every 3.7 rolls. However the probability of winning the game (on the first roll or on a point) is once in every two rolls! That is why it is said that the probability of winning a single game of craps is about 50%.

We hope this article will motivate the reader to further explore this very interesting game and analyse it using the rules of probability.

Bibliography

[1] Basic Excel Formulas retrieved from https://corporatefinanceinstitute.com/resources/excel/study/basic-excel-formulas-beginners/

[2] Sun, Yaqin (2014). A Mathematical Analysis of a Game of Craps. Undergraduate Review, 10, 149-154, retrieved from

[3] https://pdfs.semanticscholar.org/b788/f306ae3f76ea814b8a83592f43842e6ff2f1.pdf

[4] Craps retrieved from http://mathworld.wolfram.com/Craps.html