

A Challenging Geometric Construction Problem

JALEEL RADHU

In this article, we study the following construction problem.

Problem. Given any $\triangle ABC$, show that there exist points P on side AB and Q on side AC such that $BP = PQ = QC$ and find a way of locating such a pair of points. (See Figure 1.)

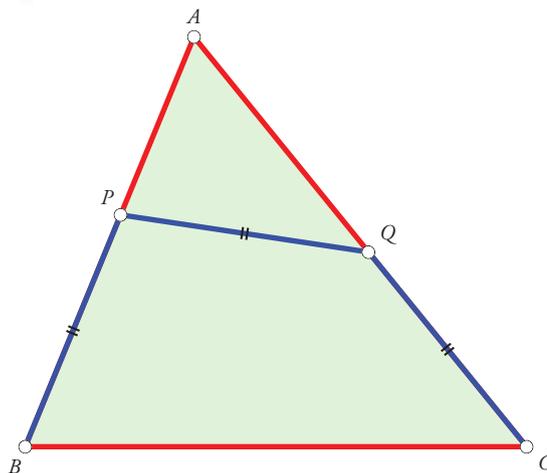


Figure 1.

Remark. The problem claims the existence of a pair of points satisfying the given conditions and also asks for a way of locating the two points. Our solution directly locates the two points and thus also establishes their existence.

Solution. The construction procedure is given below as a series of steps. (See Figure 2.)

- (1) Locate any point D on side AB such that $BD < AC$.
- (2) Through D , draw a line ℓ parallel to side BC .

Keywords: Construction, triangle, parallelogram, similarity

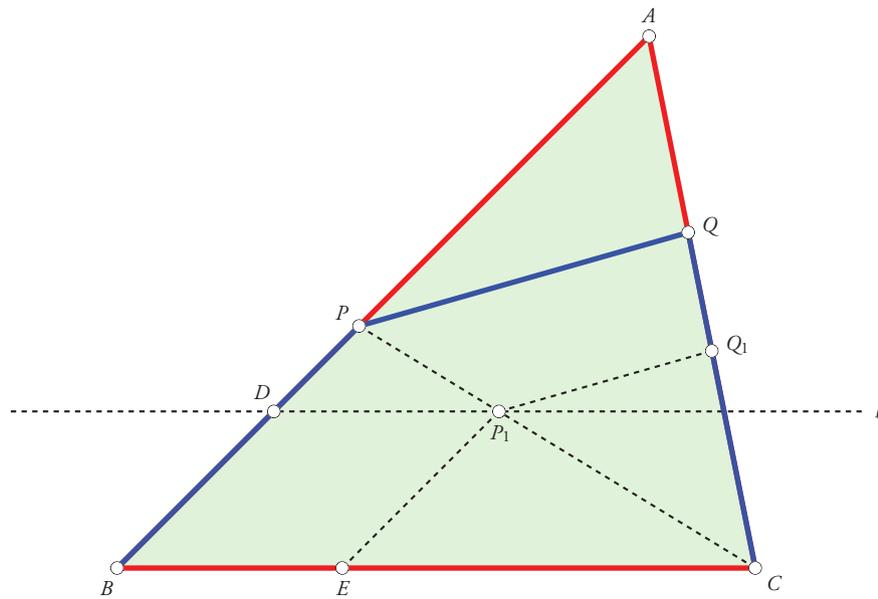


Figure 2.

- (3) Locate a point Q_1 on side AC such that $CQ_1 = BD$. (This can be done by drawing a circle with centre C and radius BD .)
- (4) Locate a point P_1 on ℓ such that $Q_1P_1 = BD$.
- (5) Through P_1 , draw a line parallel to AB ; let it intersect side BC at E .
- (6) Observe that quadrilateral P_1DBE is a parallelogram; so $P_1E = DB$.
- (7) Hence $EP_1 = P_1Q_1 = Q_1C$, by construction.
- (8) Extend CP_1 beyond P_1 ; let it intersect AB at P . Through P , draw a line parallel to P_1Q_1 ; let it intersect AC at Q .

- (9) Then P and Q are the required pair of points.

Proof that the construction works. The claim that $BP = PQ = QC$ is proved as follows.

By similarity of triangles (following from $BP \parallel EP_1$ and $PQ \parallel P_1Q_1$), it follows that

$$BP : EP_1 = PC : P_1C, \quad PC : P_1C = PQ : P_1Q_1.$$

Since $EP_1 = P_1Q_1$, it follows that $BP = PQ$.

In the same way, we have

$PQ : P_1Q_1 = QC : Q_1C$. Since $P_1Q_1 = Q_1C$, it follows that $PQ = QC$.

Hence $BP = PQ = QC$. \square



JALEEL RADHU is from Leh, Ladakh. He is currently studying in class 11 in New Delhi. His academic interests are in Physics, Computer Science, and Mathematics. He hopes to pursue a research career in one of these subjects. Currently, he is preparing for some of the national science exams such as the KVPY. He may be contacted at jaleel3.1428@gmail.com.