

# A Challenging Geometric Construction Problem

JALEEL RADHU

In this article, we study the following construction problem.

**Problem.** Given any  $\triangle ABC$ , show that there exist points  $P$  on side  $AB$  and  $Q$  on side  $AC$  such that  $BP = PQ = QC$  and find a way of locating such a pair of points. (See Figure 1.)

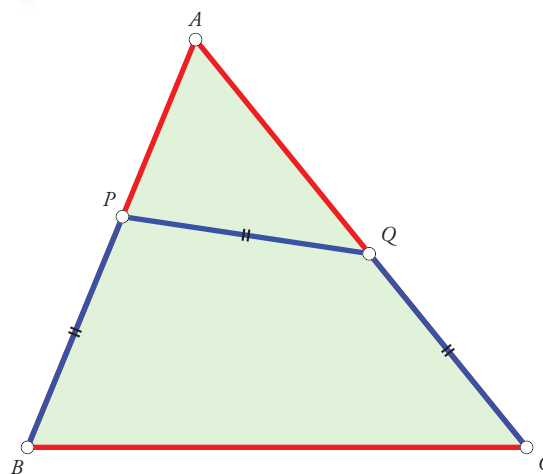


Figure 1.

*Remark.* The problem claims the existence of a pair of points satisfying the given conditions and also asks for a way of locating the two points. Our solution directly locates the two points and thus also establishes their existence.

**Solution.** The construction procedure is given below as a series of steps. (See Figure 2.)

- (1) Locate any point  $D$  on side  $AB$  such that  $BD < AC$ .
- (2) Through  $D$ , draw a line  $\ell$  parallel to side  $BC$ .

*Keywords:* Construction, triangle, parallelogram, similarity

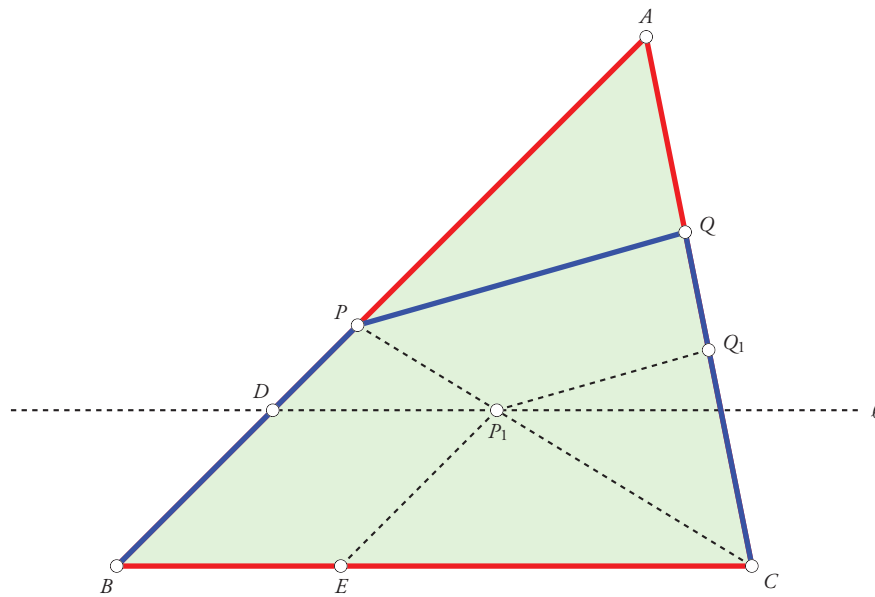


Figure 2.

- (3) Locate a point  $Q_1$  on side  $AC$  such that  $CQ_1 = BD$ . (This can be done by drawing a circle with centre  $C$  and radius  $BD$ .)
- (4) Locate a point  $P_1$  on  $\ell$  such that  $Q_1P_1 = BD$ .
- (5) Through  $P_1$ , draw a line parallel to  $AB$ ; let it intersect side  $BC$  at  $E$ .
- (6) Observe that quadrilateral  $P_1DBE$  is a parallelogram; so  $P_1E = DB$ .
- (7) Hence  $EP_1 = P_1Q_1 = Q_1C$ , by construction.
- (8) Extend  $CP_1$  beyond  $P_1$ ; let it intersect  $AB$  at  $P$ . Through  $P$ , draw a line parallel to  $P_1Q_1$ ; let it intersect  $AC$  at  $Q$ .

- (9) Then  $P$  and  $Q$  are the required pair of points.

**Proof that the construction works.** The claim that  $BP = PQ = QC$  is proved as follows.

By similarity of triangles (following from  $BP \parallel EP_1$  and  $PQ \parallel P_1Q_1$ ), it follows that

$$BP : EP_1 = PC : P_1C, \quad PC : P_1C = PQ : P_1Q_1.$$

Since  $EP_1 = P_1Q_1$ , it follows that  $BP = PQ$ .

In the same way, we have

$PQ : P_1Q_1 = QC : Q_1C$ . Since  $P_1Q_1 = Q_1C$ , it follows that  $PQ = QC$ .

Hence  $BP = PQ = QC$ . □



**JALEEL RADHU** is from Leh, Ladakh. He is currently studying in class 11 in New Delhi. His academic interests are in Physics, Computer Science, and Mathematics. He hopes to pursue a research career in one of these subjects. Currently, he is preparing for some of the national science exams such as the KVPY. He may be contacted at [jaleel3.1428@gmail.com](mailto:jaleel3.1428@gmail.com).