Four Operations for Every Child

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Basic numerical skills are a must-have in today's world. No matter what one does, the ability to add, subtract, multiply and divide is crucial and needed for everyday matters. Naturally, these take up a large portion of the math syllabi at the primary level. However, many reports including the Annual Status of Education Reports (ASER) indicate that our children are not picking up these skills adequately. What can be the reasons?

Reason 1: Numbers, by nature, are quite abstract. They do not exist in nature directly. For example, one can show five fingers or clap five times or count five points from a discussion, but one cannot show five. 5 is the numeral, a symbol, that represents five (and the symbol changes with the script).

Reason 2: Children are expected to learn this abstract concept of numbers at a very early age when their ability to handle abstract ideas are yet to form.

Reason 3: The teacher, who introduces children to the world of numbers, needs to be comfortable with them himself/herself.

Reason 4: These teachers themselves may not have experienced good mathematics pedagogy as children and neither has the teacher education system in our country enabled them to help their students.

As a result, we see two main issues:

Issue 1: Children are unable to solve word problems. Part of the reason can be inadequate reading and comprehension skills. But many children are unable to translate the situation given in a word problem into a math expression. They don't know which operation to use when and often depend too much on keywords.

Issue 2: Children make mistakes in computations involving multi-digit numbers and the standard algorithms. Subtraction involving double-borrowing, like 500 – 283, is more difficult and widely encountered in real life, say, how much change do you get if you give a ₹500 note to buy something worth ₹283. Division seems to be most problematic across the board as per ASER reports and our observations. A lot of this is due to a lack of understanding of place value. So, what can be done?

There are three main points we would like to emphasise in this article. The reader can explore the resources mentioned in the reference for further details.

Meaning-based approach

Step1

First, before introducing any operation, check the *children's understanding of place value*, especially if they are able to connect a given quantity (say, the number of grains in one spoon of rice) with the number name and the numeral. The basic idea of place value (or how we write numbers now) is to make a bundle whenever we get ten. So, for any number ≥ ten, the numeral is a combination of (i) how many bundle(s) and (ii) how many outside bundles i.e. loose one(s). These bundles are called tens and the loose ones are called ones (or units). The moment we reach ten bundles, we have to make a bigger bundle, and we call it a hundred. Similarly, when we get ten hundreds, we make an even bigger bundle called thousand and so on.

To make sense of this bundling and decipher the writing system, known as place value, it is a good idea to have something that children can bundle. Small sticks (from twigs or *tilli*, broomsticks or toothpicks) and rubber bands work very well since these can be bundled and unbundled quickly.

Second, introduce each operation with suitable situations and word problems along with some materials to animate the situation. This should be followed by introducing the relevant symbols i.e. =, +, –, × and ÷ and some practice linking situations or word problems with the corresponding math expressions. The class I NCERT math textbook does a good job of introducing addition and subtraction, while the one for class III does the same for multiplication and division.

Third, get children to create word problems for given expressions like 38 + 14, 72 – 55 etc. This helps children to be creative, fosters their language development and helps them understand what situations can be represented by which operation. It also completes the cycle (Fig. 1) by going from expressions to situations. In addition, the word problems generated by one group of children can be given to another group to solve.

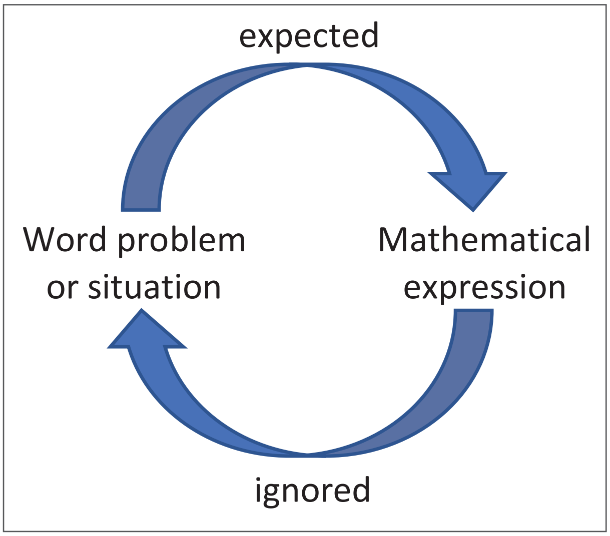

Figure 1

Fourthly, change the situation to emphasise the connection between operations.

For example, consider the situation: you have 8 flowers and I have 5, then how many flowers do we have together? This maps to 8 + 5 = __, an addition expression. Modify this to: you have 8 flowers and together, we have 13 flowers, then how many do I have? This results in 8 + __ = 13 which is an addition equation. Using role-play, the other child can ask a similar question resulting in 13 – 5 = __? So, while together does imply addition, it may be an expression (8 + 5) or an equation (8 + __ = 13) resulting in subtraction (13 – 8). This helps children understand the situation as a whole and be not overly dependent on keywords.

More importantly, it emphasises how every addition can be represented as two subtractions. A similar approach can be taken to link multiplication and division.

Step 2: Algorithms – when, how and why

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Instead of jumping into the standard algorithm immediately after introducing each operation, let the children figure out different ways of solving problems. Solving sums and differences with numbers (< 100) can be aided by the 10 × 10 board of numbers as well as the ganitmala modelling the number line. For example: 37 + 25 can be solved in any of the following ways:

- 37 + 10 + 10 + 3 + 2 i.e. 37 → 47 → 57 → 60 → 62
- 37 + 3 + 20 + 2 i.e. 37 → 40 → 60 → 62
- 37 + 30 – 5 i.e. 37 → 67 → 62

Note that these are very different from the standard algorithm which separates the ones and the tens and adds the ones before adding the tens, that is,

$37 + 25 = (30 + 7) + (20 + 5) = (7 + 5) + (30 + 20) = 12 + (30 + 20) = 2 + (10 + 30 + 20) = 2 + 60 = 62$

The need for standard algorithm emerges as we add more than two numbers, for example while finding the total of a bill, or when we add bigger numbers (≥ 100).

Next, help the children construct standard algorithms with the help of suitable manipulatives. Bundle and sticks work very well for addition-subtraction with number < 100. Children should be asked to write down each step as they work with the bundles and the sticks to make it effective.

2D base-10 blocks, known as *flats* (hundreds), *longs* (tens) and *units* (ones) or FLU work very well for numbers < 1000 as well as for multiplication and division. The reader can find more details in the reference section.

It is also important that children get answers to their questions regarding the standard algorithms. For example, why does division start from the left-hand side while the remaining three start from the right-hand side? (Check the second reference on division). Children should be allowed to figure out which method they prefer and why. For example, 376 + 285 can be done on the number line in two ways, left to right i.e. starting with the hundreds or right to left i.e. starting with the units (Fig. 2). While right to left involves no re-write (in the standard algorithm), left to right provides as with better estimate of the sum after the first step.

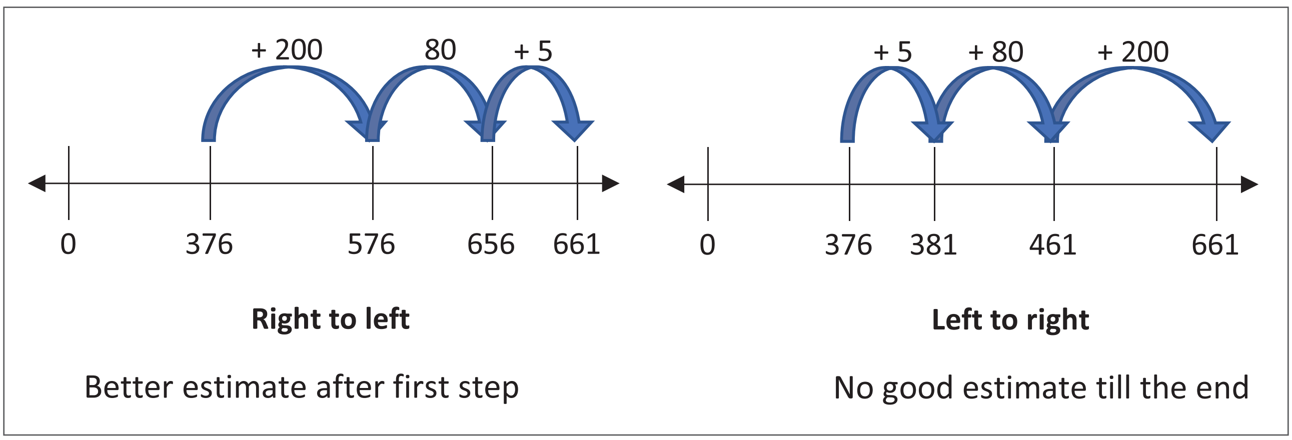

Figure 2

Step 3: Practice

While meaning-making is crucial, there is no substitute for practice. It is needed to gain mastery of any skill. Automatisation of addition (and subtraction) with single-digit numbers as well as quick recall of products of single-digit numbers smoothen the path of gaining proficiency in computation. Ten-frames help in automatisation of addition facts with single digit numbers, especially for numbers ≥ 5. Likewise, multiplication tables should be constructed by children. Also, they should be taught how to recall products involving 6, 7 and 8 using tables of 10 or 5, for example

$6 \times 8 = 5 \times 8 + 8 = 40 + 8 = 48$ or $8 \times 7 = 10 \times 7 - 7 - 7 = 70 - 7 - 7 = 63 - 7 = 56$

This enables children to find the products without revisiting the table from the beginning. 9 times tables can be constructed this way and children discover various patterns in it. So, products involving 9 are easier to recall.

Practice can be made interesting in a number of ways. The Wall activity and the Random Digits game are few such options. The Thinking Skills pull-out is rich with several explorations which automatically provide a lot of practice along the way.

We would like to end this article with two often ignored areas:

(i) operations with zero; and,

(ii) some properties of these operations

It is important that children consider zero not just as a place holder but a number. The best way to achieve this is to consider how this number takes part in the four operations. NCERT and other textbooks have included addition and subtraction with zero. However, multiplication with zero is often ignored. This omission generates the false notion that the product is always larger than the whole numbers that were multiplied. Division with zero is understood even less. $0 \div 4$ can still be explained, but division by zero needs to be examined more closely. Consider $6 \div 0 =$ __. This can be written as a multiplication equation i.e. __ $\times 0 = 6$. Clearly, there is no number that can fill this blank. On the other hand, consider $0 \div 0 =$ __ as the equation __ $\times 0 = 0$. Now, every number we know works! How can we choose one number from so many? Therefore, the situation is exactly the opposite of $0 \div 4$!! Therefore, division of any number, zero or non-zero, by zero is undefined.

Commutative, associative and distributive properties of addition and multiplication are usually glossed over at this stage. However, they are crucial for standard algorithms and they can be explored in a child-friendly way. The splitting of 2-digit numbers in tens and ones and combining for the standard addition algorithm (see 37 + 25 mentioned above), involves several applications of the commutative and the associative properties of this operation. Multi-digit multiplication on that other hand uses associative and distributive properties directly. $4 \times 30 = \mathbf{4 \times (3 \times 10) = (4 \times 3) \times 10} = 12 \times 10 = 120$: We thus use the associative property of multiplication while describing how to find such products. Also, it is a good idea to let children explore if these properties hold for subtraction and division. Distributivity does hold in cases like $(40 - 12) \div 4$ (Why?).

The issues regarding children mastering the four basic operations have been known and are unfortunately continuing. But it need not be so because the solutions have also been known for a long time. While changing a large system like teacher education – pre-service and in-service – take a lot of time and effort, we hope this article can provide some guidelines for willing teachers to try something different. It would require exploring the resources indicated below and modifying pedagogical practices. But we can assure that the effort is completely worth the trouble as experienced by many teachers across the country.

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