Chika's Test for Divisibility by 7

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At the primary and upper primary levels, students encounter and easily master the tests for divisibility by 2, 5, 3, 9, 4, 6, 8 and 11. They do not always study why the tests work, but they are easy to execute and students do not forget them easily.

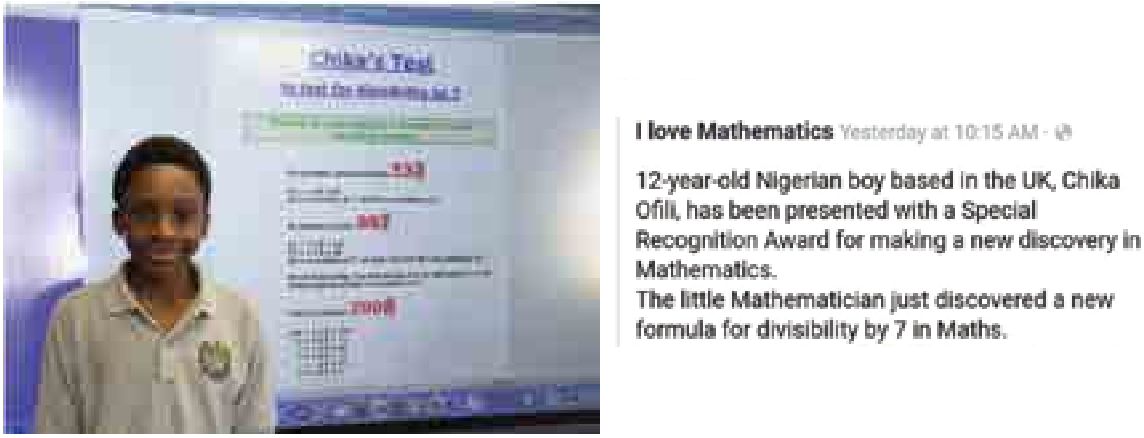
A few observant students wonder at the absence of the number 7 from this list. Occasionally, a keen student may discover such a test for himself or herself. Of course, it is a thrilling experience when this happens, for the students as well as the teachers. Such a test was discovered by Chika Ofili, a Nigerian student in Westminster Under School, London, UK. This article documents the instance.

Chika's test for divisibility by 7. The test is easy enough to perform. We describe it here in the words of his math teacher, Miss Mary Ellis [1]:

> *In a bored moment, Chika had turned his mind to the problem and this is what he came up with. He realised that if you take the last digit of any whole number, multiply it by 5 and then add this to the remaining part of the number, you will get a new number. And it turns out that if this new number is divisible by 7, then the original number is divisible by 7. What an easy test!*

She adds: "The opposite is also true in that if you don't end up with a multiple of 7, then the original number is not divisible by 7." The picture below shows Chika in his classroom demonstrating the test.

Keywords: Divisibility, number, test for divisibility



Illustrative examples. We consider a few examples to illustrate how this works.

- Take the number 532. Its last digit is 2, so the operation prescribed is:

  $$532 \longmapsto 53 + (5 \times 2) = 63.$$

  Note that both 63 and 532 are multiples of 7.

- Take the number 973. Its last digit is 3, so the operation prescribed is:

  $$973 \longmapsto 97 + (5 \times 3) = 112.$$

  We can repeat the same operation with the number 112. Its last digit is 2, so the operation prescribed is:

  $$112 \longmapsto 11 + (5 \times 2) = 21.$$

  Note that both 21 and 973 are multiples of 7.

- Take the number 873. Its last digit is 3, so the operation prescribed is:

  $$873 \longmapsto 87 + (5 \times 3) = 102.$$

  We can repeat the same operation with the number 102. Its last digit is 2, so the operation prescribed is:

  $$102 \longmapsto 10 + (5 \times 2) = 20.$$

  Note that both 20 and 873 are *non-multiples* of 7.

Justification. We now justify why this procedure works. Let N be any number (i.e., a positive integer). Let b be its units digit, and let a denote the remaining part of the number (i.e., the number obtained by deleting the units digit). Then, clearly:

$$N = 10a + b.$$

The procedure replaces N by the number $n = a + 5b$. So we have the following statement to prove:

$10a + b$ is divisible by 7 if and only if $a + 5b$ is divisible by 7.

It may not be immediately obvious how this is to be shown. But the trick is to find a combination of the two expressions, $10a + b$ and $a + 5b$, which is visibly a multiple of 7. Noting that $10 - 3 = 7$, which is a multiple of 7, we may try to subtract 3 times the second expression from the first one, and this immediately works:

$$\begin{aligned}(10a + b) - 3(a + 5b) &= 7a - 14b \\ &= 7(a - 2b).\end{aligned}$$

This means that $(10a + b) - 3(a + 5b)$ is a multiple of 7. That is, $N - 3n$ is a multiple of 7, say $N - 3n = 7r$. Our task is now almost over.

- Suppose that n is a multiple of 7, say $n = 7s$. Then we have:

  $$N = 3n + 7r = 21s + 7r = 7(3s + r),$$

  showing that N is a multiple of 7.

- Suppose that N is a multiple of 7, say $N = 7k$. Then we have:

  $$3n = N - 7r = 7k - 7r = 7(k - r),$$

  showing that $3n$ is a multiple of 7 and therefore that n itself is a multiple of 7 (this works because 3 and 7 are coprime and therefore do not interfere with each other's divisibility).

It only remains to note that the procedure can be iteratively carried forward. At each step, the number of digits decreases by 1, so we soon obtain a number for which divisibility by 7 can be checked mentally. So the procedure is very efficient in its operation.

A final note. There are tests of this kind for many different divisors. Indeed, one can find such a test for any odd divisor which is not a multiple of 5. We leave the proof of this to the reader.

References

1. Westminster Under School, "Chika's Test", https://www.westminsterunder.org.uk/chikas-test/

The **COMMUNITY MATHEMATICS CENTRE** (CoMaC) is an outreach arm of Rishi Valley Education Centre (AP) and Sahyadri School (KFI). It holds workshops in the teaching of mathematics and undertakes preparation of teaching materials for State Governments and NGOs. CoMaC may be contacted at shailesh.shirali@gmail.com.

CAPTURED MATHEMATICS

A Natural Protractor

Mathematical relevance: The plant in the picture is a species of palm tree. Its fan-shaped leaves are symmetrically placed about the central axis. Interestingly, the arrangement of leaves from one end to the other reminds us of a *Protractor* useful for measuring angles.

Photo & ideation:
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