Squaring the Circle

Editor's Note

Elsewhere in this issue, there are two articles dealing with geometric constructions, i.e., constructions using compass and ruler. One of the articles deals with the general theory behind such constructions, while the other deals with a particular construction problem from an earlier issue for which a reader had offered a solution. We would therefore like to give some background to these articles.

The ancient Greeks had invented a kind of game for themselves, in which they explored questions of the following sort: using only a compass and an unmarked straightedge (which we shall call a 'ruler' for convenience; note that it is *unmarked*), what geometric constructions are possible? Starting with two given points, with the distance between them forming the unit for measurement, what lengths can we construct? What angles can we construct? For example, can we construct a segment whose length is the square root of 2? Of course, we can; we only need to make use of Pythagoras's theorem. Can we construct an angle whose measure is 30°? Of course, we can. Can we construct an angle whose measure is 72°? We can, but this is far less obvious. Can we construct a segment whose length is the cube root of 2? It turns out that this is not possible (remember that we are constrained to use only a ruler and compass) – but this is very far from obvious! Can we construct an angle whose measure is 20°? Yet again, it turns out that this is not possible, and once again, it is very far from obvious why this should be the case. (You may wonder how we can ever know 'for sure' that certain constructions are just not possible. We will go into some of these questions in later articles.)

The ancient Greeks asked many such questions. In particular, they asked the following: Given a circle, is it possible to draw a square (using only a ruler and compass) whose area is equal to that of the circle? This question, along with a few others of its kind, troubled geometers for many centuries, until it was finally resolved in the 19th century. To the disappointment of many, it turned out that the stated task was not possible! This being so, we can at most ask for *approximate* constructions. That is, we can ask for a way to construct a square whose area is *close* to that of the circle (with not too large an error). The construction offered in the article by Gaurav is of this type.