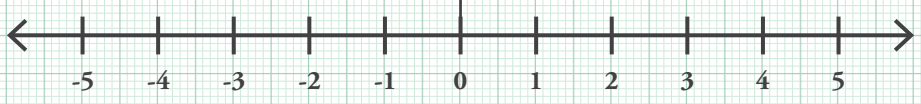
At Right Angles
A Resource for School Mathematics

PARALLEL LINES

PADMAPRIYA SHIRALI



Azim Premji University

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PARALLEL LINES

Many elements in buildings – beams, pillars, windows, doors, window bars, flooring tiles – incorporate parallel lines. Line dividers on roads, railway lines, power lines are all examples of parallel lines. Parallel Lines assume a lot of importance when marking out roads or pedestrian crossings, sports courts, athletic tracks and airport runways.

Visually, we are so used to parallel-ness that most of us find it uncomfortable if we see a tube light not aligned parallel to the ceiling, or a tilted picture frame.

While we see them in everyday objects, we must ask, do they have any other importance?

They lie at the centre of many properties involving geometry. Drawing a transversal across pairs of parallel lines creates angles that have special properties.

In this pullout, we explore parallel lines and transversal properties. Solving any geometric problem requires knowledge as well as developing a geometric eye. Hence, in the teaching of geometry, we need to develop certain skills in children that will help to open a geometric eye. How does this geometric eye develop?

Over a period of 30 years, I have met students who were exposed to plenty of activities involving visual posers and who had a chance to play with Jigsaw puzzles, spot the hidden figure problems, spot visual patterns and so on in their primary school years. I have also met several who learnt geometry only in a limited classroom situation. I began to notice that the manner and skill with which these two groups of students approached a problem were quite different. The students who had greater exposure to visual challenges and greater contact with activities involving shapes had a higher ability to visualise and unpack the problem. While one cannot draw absolute conclusions about the kind of experiences that help to develop visual dexterity, I feel quite confident about this: exposing students to visual challenges does have a beneficial effect. Also, an intelligent guess at what one is looking for and having a sound knowledge of the geometric concepts aids the process.

Geometric problems require students to spot a specific object or a relationship. For example, an X (vertically opposite angle); a pair of adjacent supplementary angles; a pair of lines that are perpendicular to each other; or a pair of similar triangles; and all these in a figure which has many crisscrossing lines, angles and triangles. At times, we may have to turn the figure around to spot some of these things. There is a need to focus on relevant information and to ignore the rest of the data. In some ways, one has to turn a blind eye to irrelevant data.

How do we develop this geometric eye? What skills do we need to emphasise? I list a few here.

- Spot hidden geometric shapes.
- Spot right angles and straight angles.
- Spot pairs of equal line segments, perpendicular lines, and parallel lines.
- Spot pairs of shapes that are rotated relative to one another.
- Look at the same shape through different orientations: top-down, left-right.
- Find common line segments or angles in intersecting shapes.
- Spot shapes that are reflections and rotations of each other. Spot symmetries. Spot patterns.
- Hide some features of a diagram and highlight some others, using one's fingers or actual highlighting.

ACTIVITY 1

Objective: Warm up visual exercises of a general nature

Most of the exercises here are self-explanatory. They are all aimed at increasing observational skills. These include applying logic, recognising symmetry, visualising rotations and getting geometric ideas.

Jigsaw Puzzle: Draw an outline of the square, cut the shapes and ask the children to reassemble the pieces in the square.

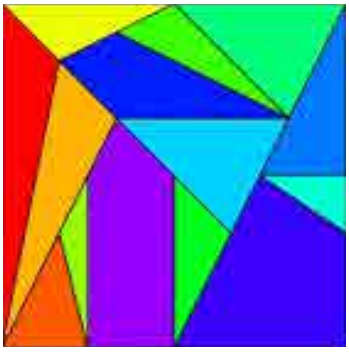


Figure 1

Complete the reflection.

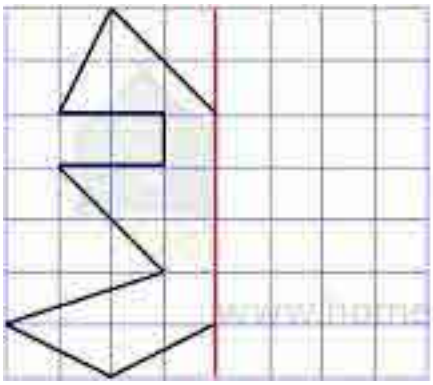


Figure 2

Here are some rotations and reflections of F. Make some rotations and reflections of P.

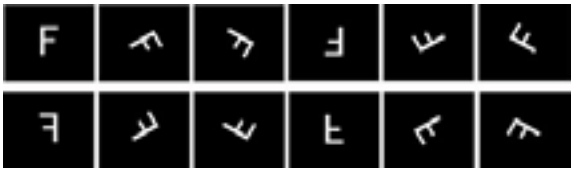


Figure 3

How many rectangles do you see in this figure?

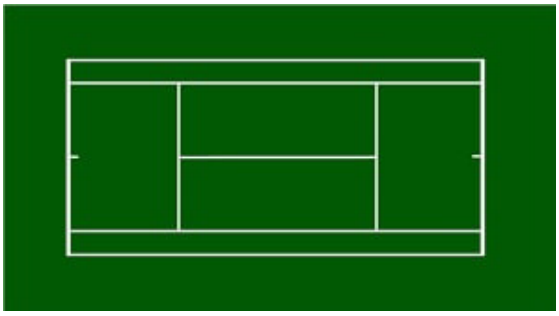


Figure 4

Describe the relative positions of squares A and B.

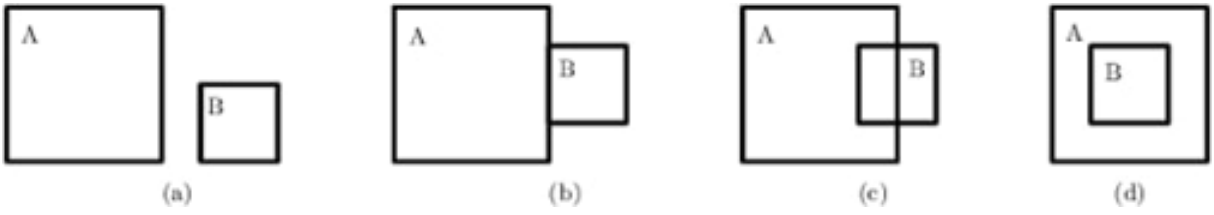


Figure 5

ACTIVITY 2

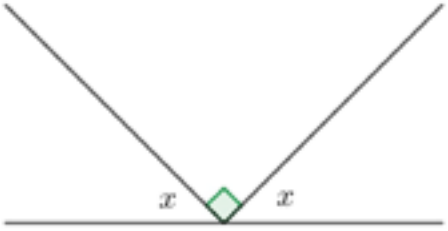
Objective: Warm-up activities to review necessary geometric facts

Complementary angles and supplementary angles property.

Angles on a straight line add up to 180 degrees.

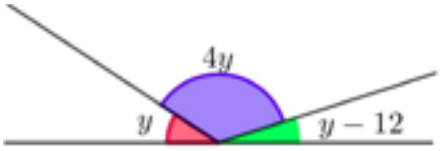
Opposite angles formed by two intersecting lines are equal.

Sum of the angles at a point is 360 degrees.



$x =$

Figure 6



$y =$

Figure 7

If X = 20 degrees and Y is its supplement, what is Y – X?

If L = 50 degrees, M and L are complements, and N is a supplement of M, what is N?

What are a,b,c and d? (See Figure 8.)

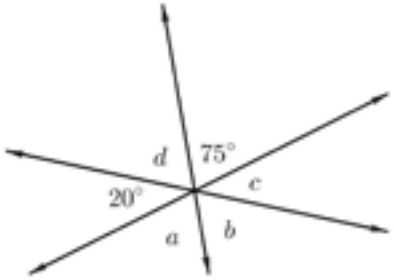


Figure 8

ACTIVITY 3

Objective: Introduction to the concept of parallel lines, definition

Figure 9

Figure 10

Figure 11

Show the students these pictures and ask them to comment on what they notice in them.

What is the common thing that they see in all these pictures?

Ask them whether these lines will ever meet. Help them to come up with a definition for parallel lines in their own words initially. Give them the definition only after that.

Parallel lines are a pair of lines that lie on the same plane, and do not meet however far we extend them beyond both ends.

It is important that the lines should lie on the same plane. A line drawn on a table and a line drawn on the board may never meet but that does not make them parallel.

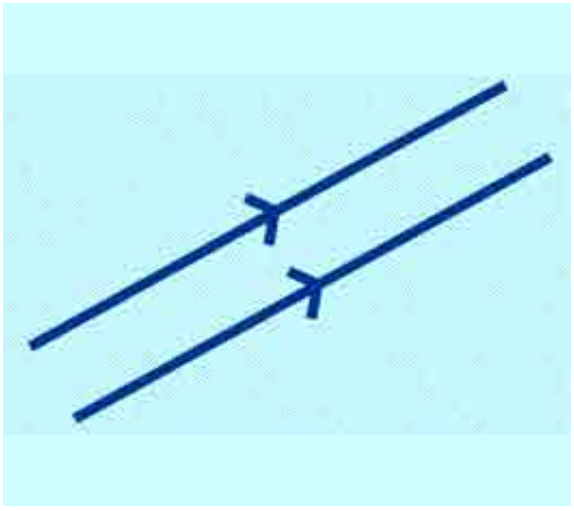


Figure 12

ACTIVITY 4

Objective: Spotting parallel lines

Note to the teacher: Drawings made on the board or on a ground are only approximations of parallel lines.

Let students look at various objects in the class to spot parallel lines in them.

Ex. Notebook edges, blackboard

Point out that when there are two sets of parallel lines, we use double arrows for the second set as shown in the picture.

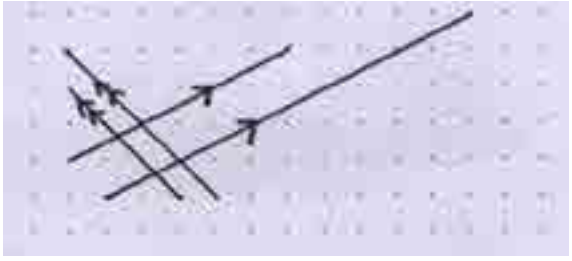


Figure 13

How many sets of parallel lines do you see in this picture?

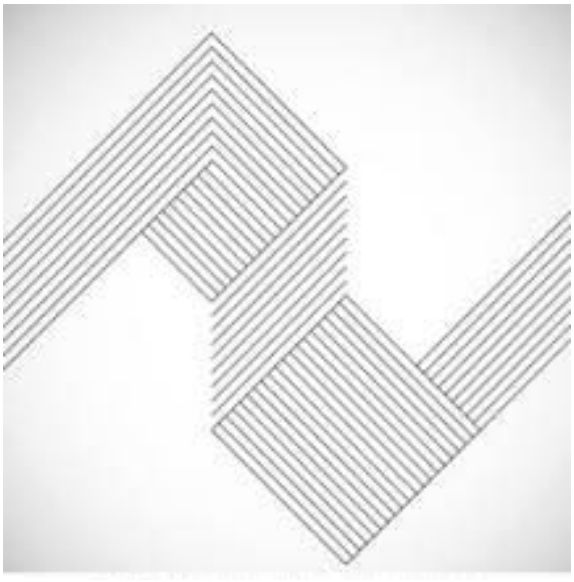


Figure 14

ACTIVITY 5

Objective: Explore parallel lines on dot paper

Materials: Square dot and isometric dot paper

Students can explore creating sets of parallel lines on square dot and isometric dot paper.

Figure 15

Can the students reason out why these pairs of lines will never meet? Let the students give explanations in their own words.

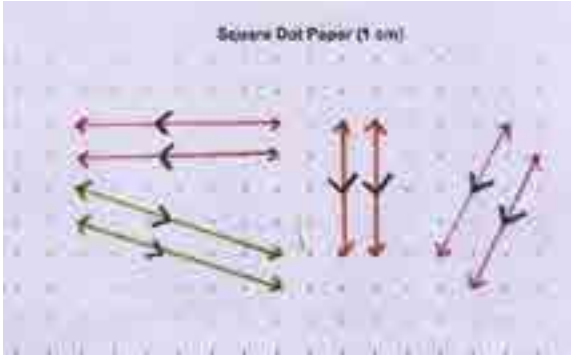


Figure 16

Ensure that the sets of lines are parallel.

It is easy to connect wrong dots and make non-parallel lines.

(Look up AtRiA Jul 2018 TearOut)

Note: It is easier to draw vertical and horizontal lines and the ones inclined at 45° (on rectangular dot sheets), but drawing a line parallel to one which has a different slope (i.e. $m \neq \pm 1$) is slightly harder.

Do students see that lines do not need to be of the same length for them to be parallel?

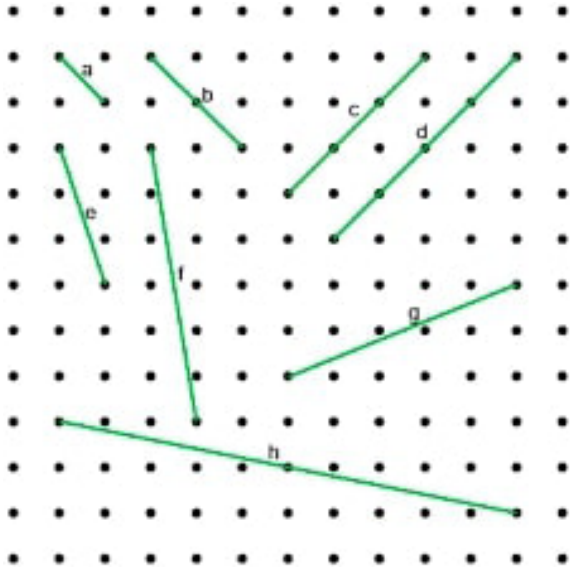


Figure 17

Which sets of lines are parallel in figure 18?

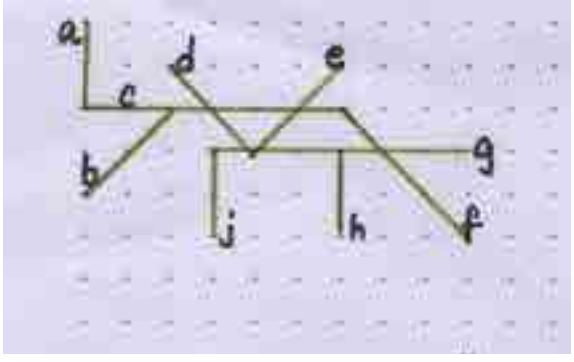


Figure 18

ACTIVITY 6

Objective: To show that when parallel lines differing only in position meet a given line (called the 'transversal'), they all make the same angle with it.

Definition: A straight line drawn across a set of given lines is called a transversal.

Let students use a scale and a set square to draw some parallel lines, as shown in figure 19.

The line x is a transversal.

Notice the angles that the lines make with the given line x. What can be said about the angles?

They make the same angle with the transversal. Only their positions are different.

Figure 19

ACTIVITY 7

Objective: Through a given point, there can be only one straight line parallel to a given straight line.

Materials: Plain paper

Ask students to draw a straight line on the paper. Let them identify and mark a point that is not on the line. Ask them to draw a line parallel to the given line using a scale and a set square as in the previous activity.

Where will they place the scale?

Discuss the previous situation of activity 5 and see if they can adapt the same idea in this case.

Now ask them if they can make a different parallel line to the first line going through the same point.

What do they notice?

The teacher can help them to state that through a given point there can be only one straight line parallel to a given straight line.

How many lines can be drawn through the same point that are not parallel to the given straight line?

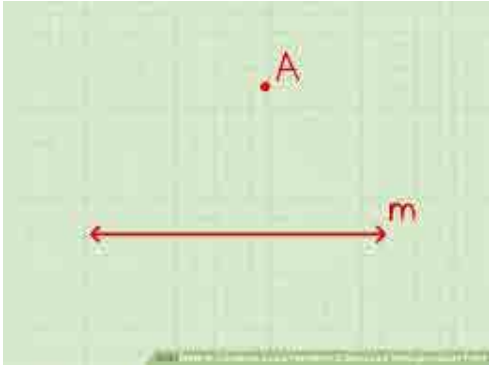


Figure 20

ACTIVITY 8

Objective: To highlight the notion of transversal and the angles that it creates.
Materials: Rigid parallel frame in one colour and one long strip in a different colour
Vocabulary: Interior angles

A rigid frame *(note the pasted thin sticks in the picture to create a rigid frame)* and a strip serve as handy material for studying parallel lines and transversal properties.

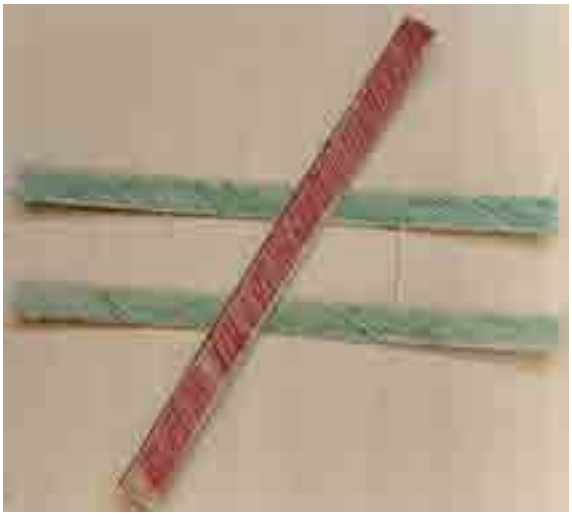


Figure 21

(If the model is not satisfactory the students can place a scale on paper and draw lines along both sides, then draw any transversal and measure the angles formed.)

Place the transversal initially in one position so that you can measure the different angles of the top intersection accurately. (Hold the transversal strip firmly by clipping or stapling to the parallel strips.)

Ask the question: "If you know the measure of one angle (of the top intersection), can you figure out the other angles?"

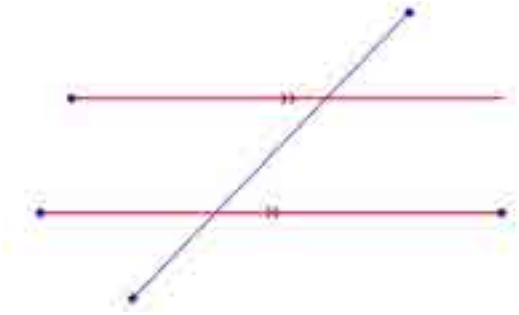


Figure 22

Are the students able to use their knowledge of vertically opposite angles and a linear pair?

Let the students now measure one angle of the bottom intersection and deduce the other angles.

They can now measure the angles using a protractor to verify.

Students should make a drawing of this and note down the information. What do they notice? Let them record the angles.

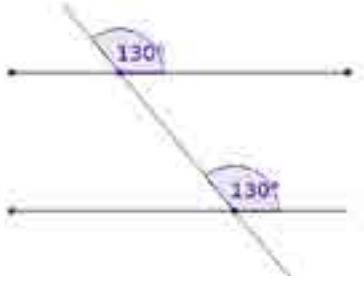


Figure 23

Ask them to color all angles of the same size in one color. How many colors did they use?

Do they notice any pattern? How many different angles are there?

Students can place the transversal in a new position and observe the angles that have formed.

Let the students make a second drawing, note down the information and color as earlier.

Do they see the same pattern again? Let them number the angles in an anti-clockwise manner as shown.

Which sets of angles are equal?

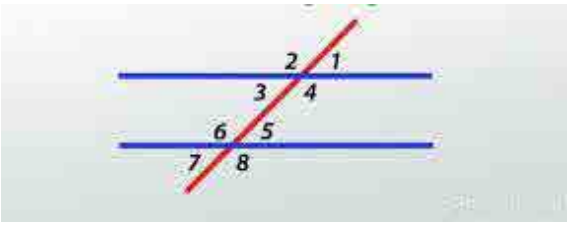


Figure 24

They can repeat the process once more to verify if the pattern repeats.

Students can describe the property (when a transversal intersects two parallel lines ….) in their own words and the teacher can give a formal definition later.

What do they notice about the interior angles? What about exterior angles?

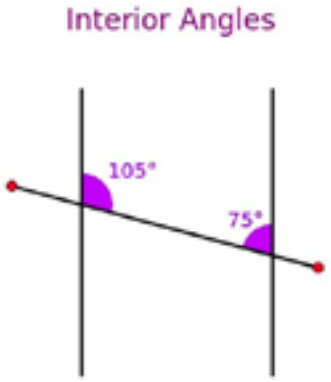


Figure 25

Raise the question: Is there a situation when all the 8 angles are equal?

A special transversal: Students should be able to see that a transversal can form a 90 degrees angle with a set of parallel lines in some situations.

Note that when two pairs of parallel lines intersect each line acts as a transversal for the other set.

Figure 26

ACTIVITY 9

Objective: To verify if two given lines are parallel to each other

Materials: Tracing paper

Students can be given sets of parallel and non-parallel lines with transversals to verify if these sets are parallel.

They use the tracing paper to copy the angle of the top intersection and match it with the lower intersection to verify if it is the same.

Note: This act of tracing and verifying is a slow activity and aids in remembering corresponding angles.

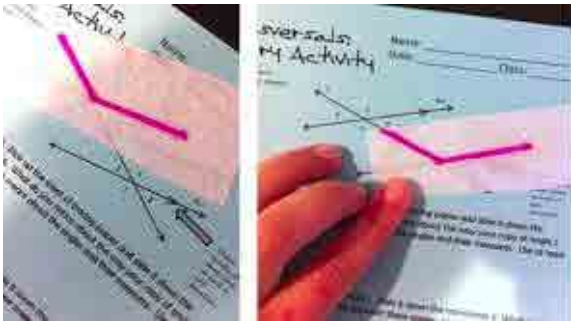


Figure 27

Source: https://www.mathgiraffe.com/blog/transversals-parallel-lines

ACTIVITY 10

Objective: To help students to recognise corresponding angles ('F angles')

Note: Students should be taught to identify various angles in a graded way and not all at one time. In this activity, the focus is on corresponding angles.

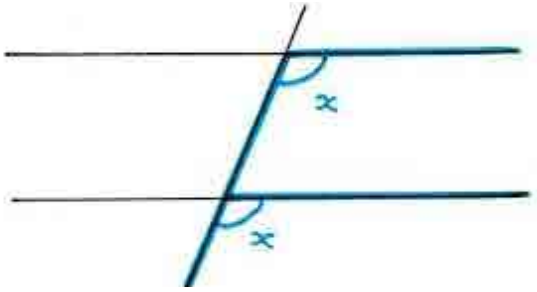


Figure 28

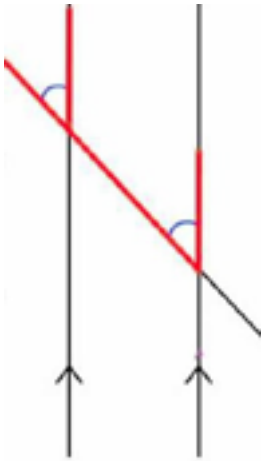


Figure 29

The blue lines and red lines in Figures 28 and 29 are parallel. Any pair of parallel lines makes an F shape with a line that crosses them.

The marked angles are called 'F angles.' Notice the 'F shape' in them.

The F angles are equal. Reflect F and rotate F to recognise F in different orientations.

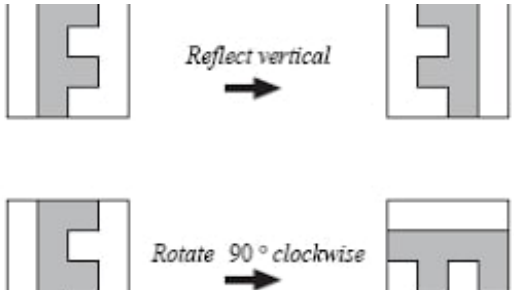


Figure 31

Here we show different orientations of F to be able to recognise them in various situations.

In the diagram given below, the students should find different F's to list all the corresponding angles.

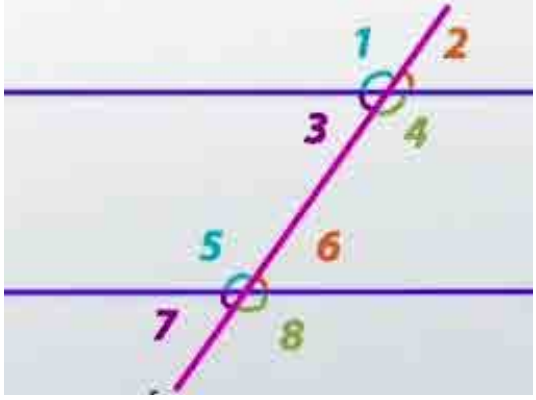


Figure 32

Angle 1 = Angle 5, Angle 2 = Angle 6, Angle 3 = Angle 7, Angle 4 = Angle 8

Let students practise many problems that require them to identify F (corresponding) angles.

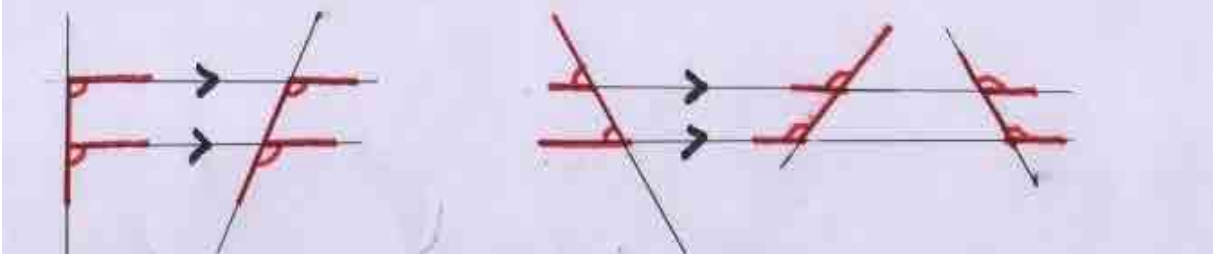


Figure 30

ACTIVITY 11

Objective: To help students to recognise alternate angles ('Z angles')

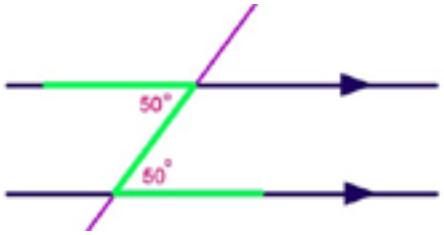


Figure 33

Let them draw different orientations of the parallel lines to spot reflection and rotation of Z angle.

Figure 34

Note: In this activity, the focus is on alternate angles. Since students are already familiar with vertically opposite angles, we may not need more reinforcement for X angles.

The Z shape is made by two parallel lines and a transversal, as shown in the pictures.

Here again, students should be able to spot rotations of the Z shape. In the top figure, we see different orientations of Z.

Why are the Z angles equal to each other?

Notice what happens when we extend two of the lines. The lines now make an X angle.

Students know that the vertically opposite angles in X are equal.

Can they also see the F (corresponding angles) here?

From these facts, can we see that the two angles of the Z are equal?

Let students practise many problems that require them to identify Z (alternate) angles.

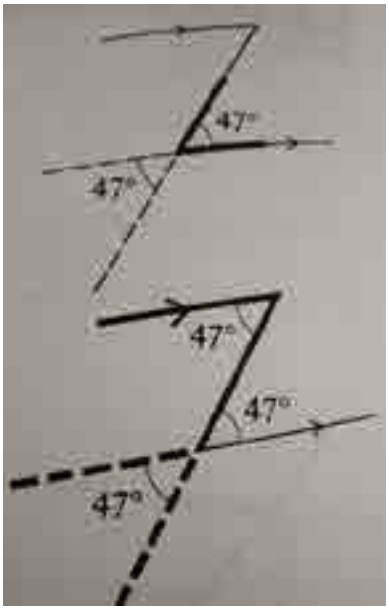


Figure 35

Show them the difference between alternate exterior angles and alternate interior angles.

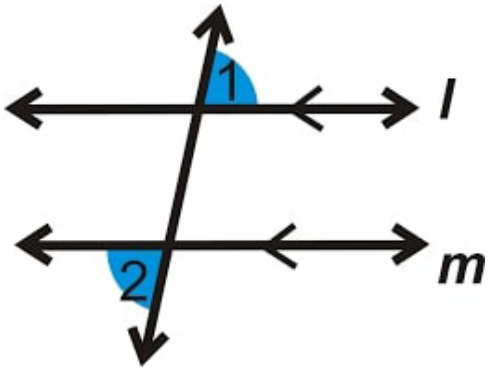


Figure 36

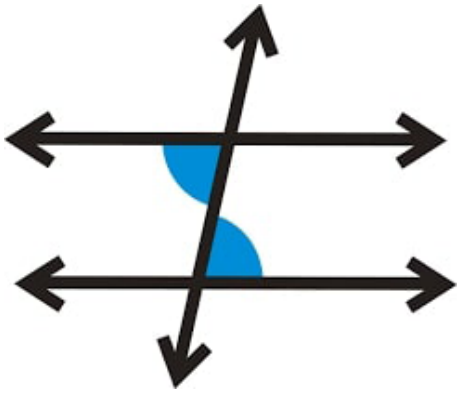


Figure 37

They can use a transparency to verify that the alternate angles match.

Rotate the transparency by 180° to get the alternate interior angles to match.

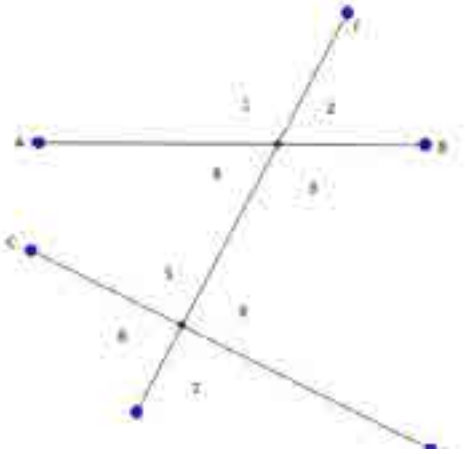


Figure 38

Let students draw a pair of non-parallel lines with a line cutting across and measure the angles. How does it compare with the earlier findings?

Will the F angles be equal here? What about Z angles?

GAME 1

Objective: Vocabulary practice

Players: 2 to 4

Vocabulary: Corresponding angles, alternate angles, alternate interior angles, alternate exterior angles, linear pair, vertically opposite angles

Make a drawing on the ground and let students play hopscotch, responding to the orders given.

Student 1 gives the orders, and student 2 has to place his/her two feet in the appropriate places. (Figure 39)

Student 2 is initially at the start position outside the drawing.

Order 1: Corresponding (Student 2 jumps onto a corresponding pair by placing one foot in one angle and the other in its corresponding angle)

Order 2: Linear pair (Student 2 now hops onto a linear pair)

In case of a mistake, he/she loses and it is the turn of Student 1 to play.

The pace of the game needs to be fast, i.e., the orders have to come without any delay so that the responses are also performed quickly.

Figure 39

ACTIVITY 12

Objective: To explore whether line *k* is parallel to line *m* when line *k* is parallel to line *n* and line *n* is parallel to line *m*

Materials: Dot paper

Will two straight lines that are parallel to a third line be parallel to each other?

Let students draw a straight line and draw two parallel lines to the given straight line.

Let them now check if the second and third lines are parallel to each other.

How are the students checking?

Are they using F angles? or Z angles?

Straight lines that are each parallel to a common straight line are parallel to one another.

Can two intersecting straight lines ever be parallel to a third straight line?

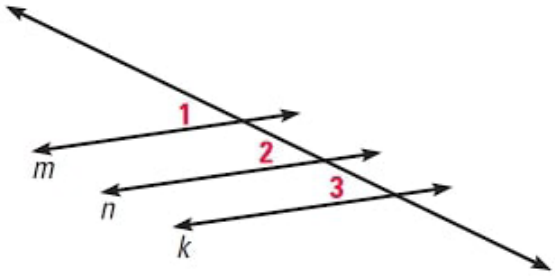


Figure 40

ACTIVITY 13

Objective: Problem solving

Students should now be able to use their understanding of the properties of parallel lines and a transversal to solve problems. They can look for F and Z angles to spot corresponding and alternate angles.

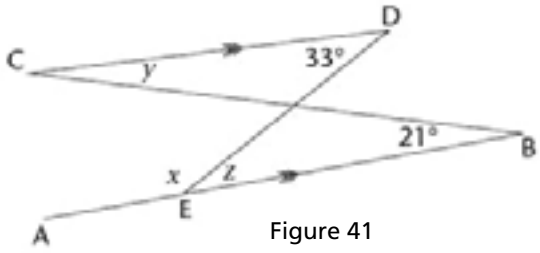


Figure 41

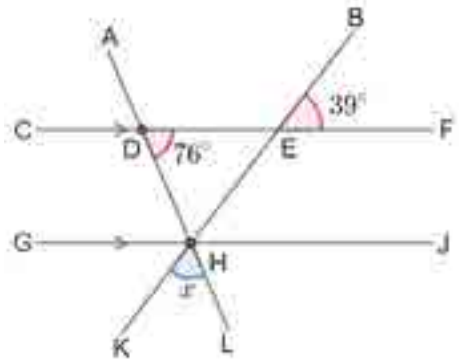


Figure 42

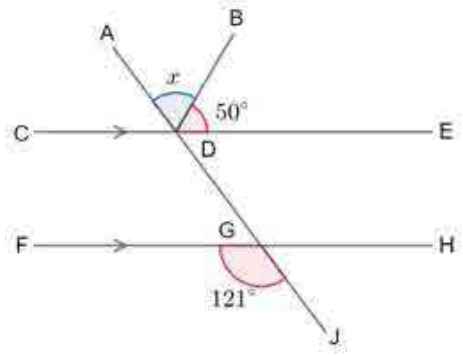


Figure 43

ACTIVITY 14

Objective: Parallel lines in polygons, designs, and 3D shapes

Materials: Various quadrilateral and Polygon stencils

Students can study the various shapes and verify if their opposite sides are parallel. They can use stencils of these shapes to draw their outlines, and check whether pairs of opposite sides are parallel to each other.

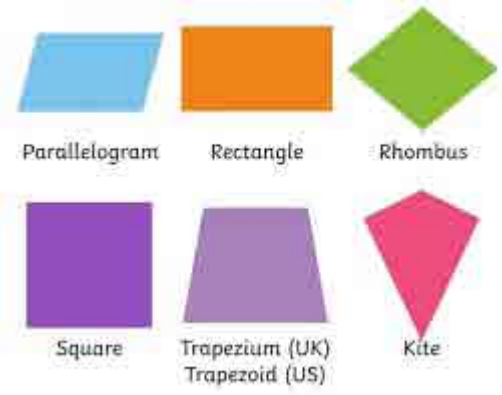


Figure 44

Students can study the various polygons and mark the parallel sides of the various polygons.

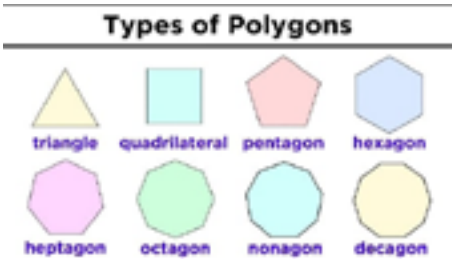


Figure 45

Can a triangle have parallel sides? Why? What happens if a kite has parallel sides?

Let the students draw regular polygons with different numbers of sides. What do they notice?

Do all regular polygons have pairs of parallel sides?

Let the students discover that polygons with an even number of sides have pairs of parallel sides while the rest don't.

They can also draw the diagonals of regular polygons to check whether they are parallel.

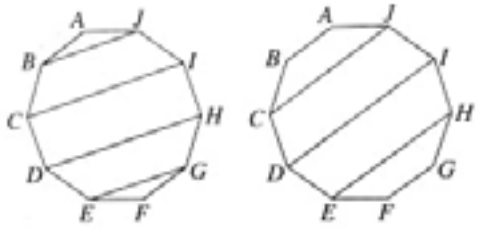


Figure 46

Now let the students draw regular polygons with different numbers of sides, and let them also draw their diagonals. What is the smallest number of sides required for a regular polygon to have parallel diagonals?

In regular polygons with an even number of sides, is there a shape which has a diagonal with no other diagonal parallel to it? Are there any sides which are parallel to it?

Is there a polygon which has two sets of parallel diagonals?

They can also study various 3D objects, drawn on isometric paper, and name the parallel lines.

Note the definition that parallel lines lie on the same plane.

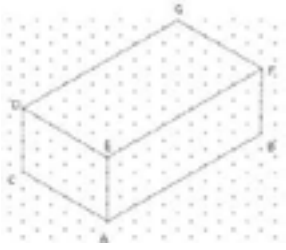


Figure 47

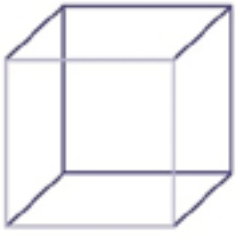


Figure 48

Figure 49

ACTIVITY 15

Objective: Art designs with parallel lines

Materials: Dot paper and some Rangoli drawings, border drawings and celtic drawings.

Many rangoli designs, celtic designs and border designs make use of parallel lines. Students can study and learn to make such designs.

Figure 50

Figure 51

Figure 52

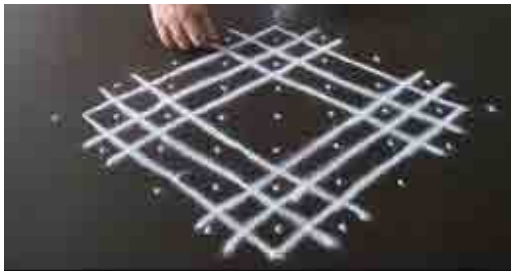


Figure 53

ACTIVITY 16

Objective: To spot places according to information

Materials: Design of a locality map

Group Project: There are many projects that require students to make use of their understanding of parallel lines and transversal to create a map based on some guidelines.

Design a map of a locality with three roads that are parallel to each other and one road that intersects all the three roads.

Name your roads ex. M G Road, Ramanujam Road.

Figure 54

Now give some clues. Ex.

The school and the hospital lie in congruent alternate exterior angles.

The medical shop and the hospital lie in congruent corresponding angles.

The gas station and the bus stop are at vertically opposite locations.

The hospital and the bookshop are on the same side of the transversal at exterior locations.

Students work in groups to create a map which fits the given clues. Are their maps identical?

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