Magic Squares Using Semi-Primes

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Wikipedia [1] defines a semi-prime as *a natural number that is the product of two prime numbers.* The definition allows the two primes in the product to be equal to each other, so the semi-primes include the squares of prime numbers. Displayed below are the first forty semi-primes.

4	6	9	10	14	15	21	22	25	26
33	34	35	38	39	46	49	51	55	57
58	62	65	69	74	77	82	85	86	87
91	93	94	95	106	111	115	118	119	121

Many number theoretic questions of interest can be asked about the semi-primes, for example: what is the longest sequence of consecutive numbers, all of which are semiprimes? It is not possible to have a sequence of four consecutive numbers, all of which are semi-primes, for the simple reason that any sequence of four consecutive numbers must contain a multiple of 4, and the only multiple of 4 which is a semi-prime is 4 itself (and it does not contain any semi-prime adjacent to it).

However, instances of three consecutive numbers, all of which are semi-primes, are easy to find. Table 1 reveals that the first such instance is 33, 34, 35, and the next one is 85, 86, 87. Do there exist infinitely many such instances? It is difficult to say.

More instances of consecutive semi-primes

Here are all instances of three consecutive numbers under 1000, all of which are semi-primes, listed in the form of a matrix (each row gives the three numbers).

Keywords: Magic square, semi-prime

Everyone knows what a magic square is. Sometimes, it is fun to try to make a magic square in which all the numbers belong to some specified subset of the natural numbers. For example, we may want all the numbers to be primes; or we may want all the numbers to be squares; and so on. In this short note, we explore the possibility of constructing a magic square entirely using semi-primes. This seems quite difficult! However, if we relax the conditions slightly, we are able to make progress.

$$\begin{pmatrix} 33 & 34 & 35 \\ 85 & 86 & 87 \\ 93 & 94 & 95 \\ 121 & 122 & 123 \\ 141 & 142 & 143 \\ 201 & 202 & 203 \\ 213 & 214 & 215 \\ 217 & 218 & 219 \\ 301 & 302 & 303 \\ 393 & 394 & 395 \\ 445 & 446 & 447 \\ 633 & 634 & 635 \\ 697 & 698 & 699 \\ 841 & 842 & 843 \\ 921 & 922 & 923 \end{pmatrix}$$

We display below the results of two such attempts. It is rather curious that many of the triples listed above can be seen in these arrays.

A partial magic square of order 6

Here is a partial magic square of order 6, all of whose entries are semi-primes; all six of the columns have the same total (1732), but only the first four rows have that total (the other two row sums are 1996 and 1468, respectively).

634	218	217	219	301	143
445	142	303	302	94	446
201	447	87	86	697	214
202	141	95	698	393	203
215	85	635	394	34	633
35	699	395	33	213	93

Another attempt at a magic square of order 6

Here is the result of another such attempt. This time we have permitted ourselves the use of a few numbers which are not semi-primes (specifically, the triple 117,118,119), but we do obtain a complete magic square of order six, with magic constant 1442. Here it is:

634	85	301	87	118	217
202	201	447	203	86	303
215	33	214	446	393	141
213	395	219	218	95	302
143	635	119	394	117	34
35	93	142	94	633	445

References

1. Wikipedia, “semi-prime”, https://en.wikipedia.org/wiki/semi-prime

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