CRYPTARITHMS

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Here is an interesting type of problem which can exercise your thinking and reasoning skills.

Question. Given that each letter represents a unique digit in the range of 0 to 9, find the values of A, B, D, E, F, G, H, J and K from the computation given below:

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  ABJ
+ADEF
EGHJK        .......(1)
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The + operator carries its usual arithmetic meaning.

In such problems, the convention is that ABJ represents a three-digit positive integer, ADEF a four-digit positive integer, and EGHJK a five-digit positive integer.

In questions such as these:

i. There is a one-to-one mapping between each letter (variable) and a particular digit, i.e., each letter corresponds to exactly one digit and each digit corresponds to exactly one letter.
ii. There are no leading zeroes (the zeroes in 003 are leading zeroes and the zeroes in 300 are trailing zeroes).

Equation (1) might be written as:

C1	*C2*	*C3*	*C4*	
		A	B	J
+	A	D	E	F
E	G	H	J	K

Table 1

Here, *C1, C2, C3* and *C4* represent carry digits.

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If we add any two digits in the range of 0 to 9, then the maximum carry digit will be 1 and the minimum carry digit will be 0. For example, 2 + 3 = 05 (carry digit is 0), and 8 + 9 = 17 (carry digit is 1). The smallest sum is 0 + 0 = 0 and the largest sum is 9 + 9 = 18.

Since the sum has 5 digits and there are no other digits in the first column (from the left), we can safely conclude that *C1* is 1 and that therefore, the value of E is 1.

For E to be 1, the value of A will necessarily be 9 and the value of *C2* will necessarily be 1. If A = 9, *C2* = 1 and E = 1, then the value of G will be 0.

With these conclusions, we may now rewrite the table as:

1	*1*	*C3*	*C4*	
		9	B	J
+	9	D	1	F
1	0	H	J	K

Table 2

Now we move from conclusions to conjecture:

If J = 2, and F = 3, then the value of K will be 5 (and the value of *C4* will be 0).

Now B + 1 should be equal to J (assumed to be equal to 2; second column from the right in *Table 2*). The value of J will be 2 only if B = E = 1 or B = E = 6. We know that the value of E is 1 and no two letters can have the same value. Hence, let us try with new values for J and F.

Let J = 3, and F = 4. K = J + F, therefore, the value of K will then be 7.

It is easy to infer from the second column (from the right) of *Table 2*, that the value of B will then be 2. On substituting the values in *Table 2*, we obtain:

1	*1*	*C3*	*C4*	
		9	2	3
+	9	D	1	4
1	0	H	3	7

Table 3

We have now arrived at E= 1, A = 9, G = 0, J= 3, F = 4, K = 7, B =2.

The value of D cannot be 7 (because K = 7). The value of D cannot be 8 (because then H would be 7 and we know that the value of K is 7).

If D = 6, then the value of H will be 5. We may now write:

1	*1*	*0*	*0*	
		9	2	3
+	9	6	1	4
1	0	5	3	7

Table 4

Thus, E = 1, A = 9, G = 0, J= 3, F = 4, K = 7, B = 2, D = 6 and H = 5.

It is important to note that certain cryptarithms might have a unique solution and certain cryptarithms may have more than one solution. To arrive at other solutions, we go back to *Table 2*, since we cannot change the values of E, G and A.

The combination J = 5 and F = 6 is not possible, because in that case K would be 1. K cannot be 1, because the value of E is 1.

Let J = 6, and F = 2. K = J + F, therefore, the value of K will be 8 and the value of C4 will be 0. With J = 6, and E = 1, B will have to be 5 and the value of *C3* will be 0.

On substituting the values of J, F, K, *C4*, B and *C3* in *Table 2*, we obtain:

1	*1*	*0*	*0*	
		9	5	6
+	9	D	1	2
1	0	H	6	8

Table 5

If D is 6, then H becomes 5 (and *C2* becomes 1). But this gives us H = B.

If D is 7, then H becomes 6 (and *C2* becomes 1). But this gives us H = J.

If D is 4, then H becomes 3 (and *C2* becomes 1). Then we arrive at a second valid solution set:

1	*1*	*0*	*0*	
		9	5	6
+	9	4	1	2
1	0	3	6	8

Table 6

E = 1, A = 9, G = 0, J = 6, F = 2, K = 8, B = 5, D = 4 and H = 3.

Now, suppose J = 6 and F = 7, then equation (i) might be written as:

1	*1*	*0*	*1*	
		9	B	6
+	9	D	1	7
1	0	H	6	3

Table 7

Thus the value of B would be 4.

The value of D cannot be 6 (because J = 6). The value of D cannot be 7 (because F = 7). So, let the value of D be 8. But if D is 8, then the value of H would be 7 (which is not possible because F = 7).

Problems such as these are invaluable in building reasoning skills for students. They begin to understand the role played by constraints and boundary conditions. They have opportunities to practise mathematical communication by building strong arguments and justifying their conclusions. They learn to listen to others and accept that there is more than one correct solution. Best of all, they learn to frame similar problems for their peers. And they do this all in the spirit of play.

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