The Height of a Tree

MOHAMMAD UMAR

This article is based on an interesting activity done with children, in 2008. At that time, I was working with 'Eklavya'- an Institute for Educational Research and Innovations, Hoshangabad. Being new, I picked up some innovative methods to teach science from my seniors here. Also, I started reading some interesting books based on learning theories and activities. I tried some of the experiments and activities with my students – on some occasions, I was successful, on others not. But learning by doing was very enjoyable for all of us.

Our campus was very beautiful and full of greenery and open to the children from the nearby village Malakhedi who came to play games in this garden every day. A banyan tree stood at the center of this garden. There was also a children's library and we organized several activities in this garden for children from classes 3 to 12. In one such activity, we tried to measure the height of the banyan tree by seven different methods. It was an amazing experience. Today, I am sharing some of my reflections from a daily diary, which was used at that time to document my own learnings and experiences.

It is February and I am teaching the measurement of length in Class 6 of the Government School, Malakhedi. Today, I wanted to do some interesting measurement activity with the children. During my school visits, I had observed that most of the children from the primary classes were using their scale only to draw straight lines, they had difficulties in measuring length with it. In the higher classes, the children were familiar with the common units of measuring length: metre, centimetre and kilometre. The banyan tree was just in front of us. A few children asked to measure the height of this tree. We could not measure the height of this banyan tree by using a scale or inch tape, as it was very high. The challenge was to find the height of this tree - without climbing on it.

So, the question was- how can we measure the height? We discussed some ideas together. Here, in this article, I am trying to share the entire process of this activity.

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First method – By Estimation. The shadow of the big leaves of the tree was blocking the sunshine. In order to make an estimation, we needed some reference point. So, I stood beside the trunk of banyan tree and a student made a mark on the tree trunk near the top of my head. Now, I walked away from the tree and observed it from top to bottom. I also observed the mark showing my height. Here, I was trying to compare these two heights. It seemed to me that the tree was nearly three times my height. My young companions agreed with my impression when I shared it with them.

A boy ran inside the office and brought a scale and inch tape. We measured the height of the mark on the trunk. It was 172 cm.

So, the height of the tree will be 3 times my own height

$= 3 \times 172$ cm.

$= 516$ cm.

$= 5$ m. 16 cm.

That was the very first attempt to say something about the height of the tree, without climbing on it. At this stage we were not able to claim its correctness, as it was based on our estimation only.

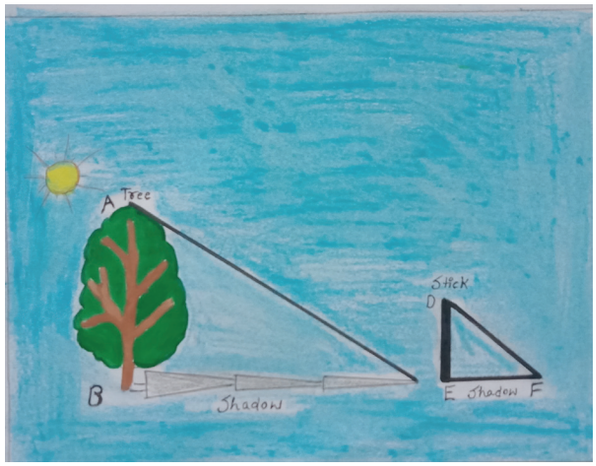
As I said earlier, children from classes 3 to 12 were part of this team. I asked them if they had learned an activity to measure the height of a pole with the help of a stick and its shadow. Two children said that they had, but they were not able to explain any details about the same.

Second Method – Ratio of the lengths of the shadows of a stick and the tree. It was a pleasant winter morning. The sun's rays were casting very beautiful shadows on the green grass. We searched for a straight stick and cut it after measuring off a little more than half a metre.

Now, when we planted this stick vertically in the ground, it stood half a metre above the ground. The stick was casting a straight shadow on the ground.

AB = h metres (height of tree)

DE = height of stick



BC = length of shadow of tree.

EF = Length of shadow of half metre stick.

Here, we could get all three measures (except the height of the tree) easily.

We knew the length of the stick i.e. half metre (= 0.5 m) already. We measured the shadows of the tree and the stick with a measuring tape.

Height of tree = h

Height of Stick = 50 cm

Length of the shadow of tree = 435 cm

Length of the shadow of stick = 33 cm

Δ ABC $\simeq$ Δ DEF (by AA Property)

Hence, AB/DE = BC/EF = AC/DF

(Corresponding sides of similar triangles)

Therefore, AB/DE = BC/EF

Height of tree/height of stick = shadow of tree/shadow of stick

- h/50 cm = 435 cm/33 cm
- $h = 435 \times 50/33$
- $h = 659$ cm
- $h = 6$ m. 59 cm.

By this method, we obtained the height of the tree to be 6 m 59 cm. But we were not very happy, as it was very different from our earlier estimate. My companions and I were a little bit confused, as we

were not able to choose which option was closer to the actual measurement.

Everybody started thinking again to try and find out some ways to come out from this confusion. Some of the children who were studying in the higher classes were able to contribute well to our discussion.

Third Method – With the help of a Right-angled Isosceles Triangle. Some students who were studying in higher classes told us that they are using trigonometric formulae to get the heights of towers, trees and poles using the formula Tan θ = Perpendicular/Base, where θ was usually 45°. I told them about an activity in which one can measure height by using this principle. Quickly, we cut a right isosceles triangle out of a sheet of cardboard. (One angle 90° and the remaining two 45°.)

Now, I placed this triangle on the ground and tried to find a place from where, the perpendicular side of the triangle can cover the entire tree (from top to bottom). But even after lying prone on the ground, we found it difficult to do this.

So, I decided to observe the tree from a standing position. We again marked the level of my eyes on the trunk of the tree. It was at 158 cm.

Again, I walked away from tree and tried to find an appropriate position. This time, my objective was to cover only the upper part of tree (from the mark on the trunk).

I found one such position. Two children measured this distance, from the tree to my standing position.

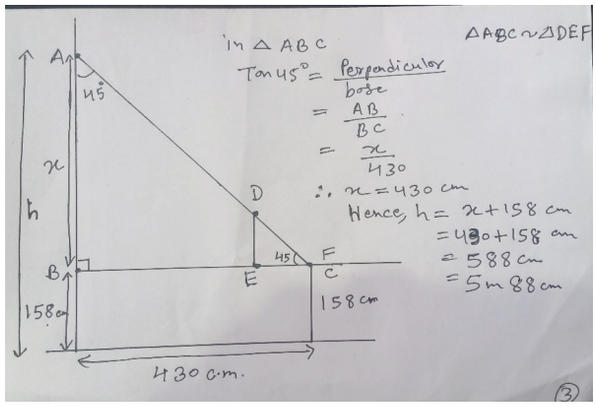
Now, we had to use this information in trigonometric formula.

Height of mark on the trunk (my eye level) = 158 cm

My distance from the tree (where this triangle covered the upper part of tree) = 430 cm

Height of tree = h = (158 + x) cm; here, x is the height of the upper part of the tree.

In Δ ABC, Tan 45° = AB/BC



We know,

Tan 45° = perpendicular/base

= height of upper part of tree/distance from tree to my standing position.

= x cm/430 cm

= x/430

Therefore, 1 = x/430

(by trigonometric ratio Tan 45° = 1)

Which gave x = 430 cm

We know,

Total height of the tree = h

= height from ground to the mark on trunk + height of upper part of tree

= 158 cm + x cm

= 158 cm + 430 cm

= 588 cm

= 5 m. 88 cm.

So, in this way, we got one more measurement for the height of the tree. It was different from the first two heights.

Now, we became more interested in finding different ways to get the height of the tree, rather than the accuracy of the height. I had read about one such activity, where one can measure the height of a tree or a pole with the help of a pen.

I shared this activity with the children. They became very curious about it and wanted to try it immediately.

Fourth Method – By using a pen.

I used a simple ball pen to measure the height of this tree.

First, I walked away from the tree and tried to find a position from where this pen can obscure the entire tree, from top to bottom. I remained in the same position and keeping my hands at the same level, I then rotated this pen by 90° (from vertical to horizontal position). [Imagine mentally that the tree topples over to its side and falls to the ground, so that after it has fallen, it continues to be perpendicular to you; it has neither fallen towards you, nor away from you. Let the pen also make exactly the same movement: let it 'topple over' and fall to its side. For this, we have to rotate the pen by 90° till it is horizontal. In its final position, it neither points towards the tree, nor away from the tree. Now let a child position himself so that he, the tip of the pen, and your eye are all in one line. When this position has been found, the child is effectively at the tip of the fallen tree. So, all we need to do is to measure the distance from the tree to the child, and this will give us the height of the tree.]

Finally, we measured this horizontal distance covered by pen on the ground. It was 465 cm.

Hence, the height of the tree was 4 m 65 cm (according to this measurement).

But we were not happy with this finding, as the height of the tree in this case was very different from all the previous results. There was clearly some mistake, but we were unable to trace it.

A few years later, I was sharing these methods with one of my colleagues who had been working in the field of math education for a while. She told me about a minor but crucial mistake, which we had made.

We had to add the height of my eyes (from ground level) with this result, as I was in standing position and the pen was not placed on the ground.

So, we rectified this calculation (better late than never!) by adding my eye level to the previous measure.

Hence,

$$\begin{aligned}\text{The height of tree} &= 465 \text{ cm} + 158 \text{ cm} \\ &= 623 \text{ cm} \\ &= 6 \text{ m } 23 \text{ cm}.\end{aligned}$$

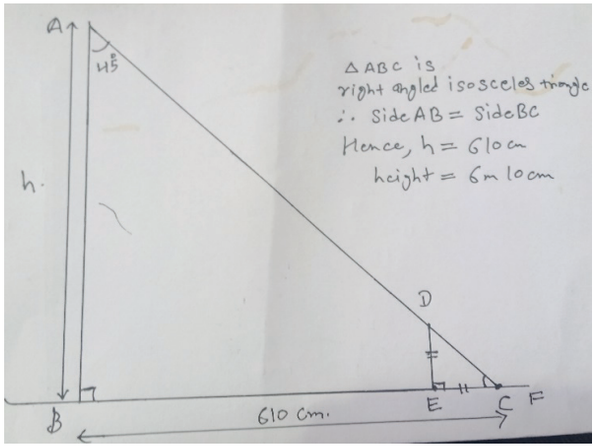
Fifth Method – Using our eyes and hands.

One of my colleagues, who had been working in science education for many years, was watching our efforts silently. He asked about all the methods, which we had already used and then suggested one more method.

'*There is no material required for this method. We have to use our hands and eyes only*'- he said. Children were very happy, as they sensed one more method to measure the height of tree.

This method is mathematically very similar to the third one, where we used the right angled isosceles triangle made from cardboard. But, here, we don't need any triangle. Only our hands and eyes are enough.

In this method, I lay down, a few steps away from the tree with my back on the ground and my feet toward the tree (I could see it easily).



Now, I folded my right arm to make a 90° angle at the elbow. I tried to cover the tree from top to bottom with the vertical part of my right hand (please see picture).

It was a little difficult to get the appropriate location to do so, but after changing positions a few times, I succeeded. Here, my right hand worked as a right-angled isosceles triangle. I checked the distances from shoulder to elbow and from elbow to ring figure of my right hand, they were the same. That was why this whole arrangement provided a natural isosceles right triangle with angles 90°, 45° and 45°.

Therefore, the distance from my head to the tree was equal to the height of the tree. Now, I measured this distance very easily with the help of the tape. It was 610 cm. So, the height of the tree by this method would be = 6 m. 10 cm.

This result gave us a lot of satisfaction, as it was very close to the estimate, which we had made at the beginning.

The children, who were actively involved in all these activities, asked me again for any new method. I remembered that I had read about a tribal community which could estimate heights of trees easily. Their method was very interesting.

Sixth Method – A local device: By folding the body and observing the tree from the folded position. In this small community, some of the senior members could estimate the height of any tree. To do this they turned their backs to the tree, bent their bodies at the waist without bending their knees and observed the tip of the tree through the inverted V formed by their own legs (see picture).

I had read about this method in a book, but had no practical experience of the same.

I invite our readers to justify the mathematics behind this folk method.

When I had shared this method with the children, some of them started laughing, while others immediately started trying it out. I also decided to give this a try.

I moved into position and started walking away from the tree in this folded position. Initially, the

trunk of the tree was visible from between my legs. I started walking slowly away from the tree. I was simultaneously searching for an appropriate location from where I could see the entire height of the tree through my legs, i.e., at such a point, the tip of the tree should just touch the vertex of the inverted V, which was formed by my legs.

Suddenly, I was able to see the banyan tree, from top to bottom. One child placed a stone, just to remember this position. Later, we measured the distance from the tree to this stone. The distance between tree and stone was 624 cm

According to the book, the height of tree = 6 m. 24 cm. which was close to the values which we had obtained by the other methods.

At this stage, we had done 6 different methods to measure the height of a tree. Now, it was difficult to think about any new method, other than climbing on it, just like a monkey and measuring it with the tape.

Seventh Method – By a long broom attached to a bamboo. We were planning to wind-up. Suddenly, Sakeena, a girl studying in class 6 suggested that we could use a long broom (used to clean the roofs and walls) and measure it directly. That was a very good idea. Children wanted to confirm all these answers too.

Gopal, a class 5 student, rushed to the office and came back with a very long broom, made longer by attaching it to a bamboo stick.

'*Sir, now we can measure its height*' said Gopal. The other children started laughing, but Sakeena was happy, as we were going to implement the idea given by her. We placed this very long broom, just parallel to the main trunk of the tree. It was a little longer than the height of the tree.

Now, we placed this broom (attached with long bamboo) on the ground and measured its length with the help of the tape. It gave one more estimate of the height of the banyan tree.

Here, I am not going to tell you the height of the tree, which we got by the seventh method as I want to give you an opportunity to learn for yourself. It would be good, if you can do all these activities with your students and let them learn by themselves.

Here, finding the correct answer is not the important thing. As a mathematics teacher my interest is in finding different ways to get the answer. I am sure your students will also enjoy this activity.

MOHAMMAD UMAR is a Resource Person with Azim Premji Foundation, Rajsamand, working with material development and capacity building of teachers. He has studied the sciences, sociology and journalism as well as theatre and he has taught school level mathematics and science and also served as pedagogy and curriculum specialist and as a research associate at Eklavya, Hoshangabad. He is interested in reading and writing, with published articles in several educational journals. All his published articles are available at https://azimpremjifoundation.academia.edu/MohammedUmarSheikhMansoori. He may be contacted at mohammed.umar@azimpremjifoundation.org