Miscellaneous problems for the MIDDLE SCHOOL

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Here are some problems collected from various sources. They can be termed 'word problems' – some of them can be termed 'story problems' as they present an episode leading to a question.

Problem–IX–3–M–1. Three travelers meet at an oasis one evening and quickly become friends. As the day draws to a close, one of them (say traveler A) says, "Friends, I haven't brought any food with me. If you have brought some, could you share with me? I can pay for it." Traveler B says he has 3 loaves of bread while traveler C says he has 5 loaves with him. They then pool the loaves and share them equally. Next morning traveler A gets up early, while the other two are still asleep. He leaves 8 silver coins and departs. When the others wake up and see the coins, traveler B remarks, "Since I had 3 loaves and you had 5, I shall take 3 coins and you can have 5."

Do you think that is a fair deal? If not, what is a fair division of the coins?

Problem–IX–3–M–2. A farmer goes to market with a basket of eggs for sale. His first customer buys half the eggs in the basket and half an egg. His next customer buys half the eggs left and half an egg. Then a third customer buys half of what is left now and half an egg. If 3 eggs are now left in the basket, how many were there at the start?

Problem–IX–3–M–3. A bakery sells cookies of three types priced at Rs.3, Rs.2 and Rs.0.50 per piece. A person buys 20 cookies (including all types) and is charged Rs.20. How many does he buy of each type?

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Problem–IX–3–M–4. A census-taker knocks at a door, which is opened by a lady. On being questioned, she says, "I have three children. The product of their ages is 36. The sum of their ages is this door number." After a pause, the other says, "Sorry. I am unable to work out their ages." The lady then says, "My eldest child plays the piano." The other then says, "Oh, good. Now I know their ages."

What are the ages of the three children? (The term 'age' refers to the number of years completed by the person, i.e., it is an integer.)

Problem–IX–3–M–5. Boss calls his peon and says, "Take this cheque to the bank and encash it. On the way back, please take 20 paise for a cup of tea and bring me the rest of the amount." The cashier at the bank, in a fit of absentmindedness, interchanges the figures given in the cheque under rupees and paise, and hands over the amount. After spending 20 p. the peon hands over the rest to Boss. On checking the amount, Boss exclaims, "Very strange. This is twice the amount I had indicated."

What was the amount mentioned in the cheque?

Pedagogical Note

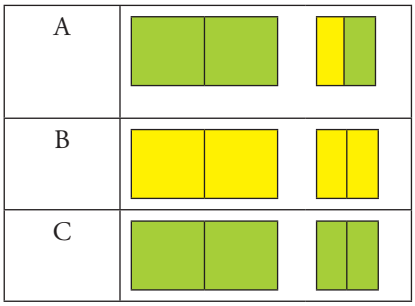
These word problems can become very interesting for students if they are encouraged to model the situation. Teachers may also provide the representations as a stimulus to jump start their thinking. This process of problem solving can help take away their aversion to word problems.

Solutions to Problem–IX–3–M–1.

8 loaves

B's contribution C's contribution

Each person's share

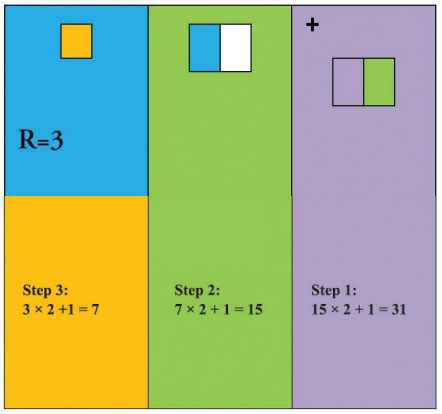


A gets $\frac{1}{3}$ of the bread loaf from B and $\frac{7}{3}$ from C. So, of the 8 coins B should get 1 part which is 1 coin and C 7 parts, that is, 7 coins.

Alternative Solution: 8 loaves were shared by three persons equally. So each consumed $\frac{8}{3}$ loaves. Traveler B brought 3 loaves, out of which he consumed $\frac{8}{3}$ and so his contribution towards traveler A's share is $3-\frac{8}{3}=\frac{1}{3}$. Traveler C brought 5 loaves, out of which he consumed $\frac{8}{3}$ and so his contribution towards traveler A's share is $5-\frac{8}{3}=\frac{7}{3}$. So it would be fair if B takes 1 coin and C takes 7 coins.

Solutions to Problem–IX–3–M–2.

Working backwards from the final answer, we reverse all the operations. So, instead of halving, we double and instead of dividing or subtracting, we multiply or add.



Alternative Solution: If the original number of eggs in the basket is taken as x, we can form the following equation:

$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}x-\frac{1}{2}\right)-\frac{1}{2}\right)-\frac{1}{2}=3$, yielding $x = 31$.

Solutions to Problem–IX–3–M–3.

Here, the 3 types of cookies should add up to 20 cookies and the cost of the 20 cookies should also sum to ₹20

Number of cookies	p	q	r	$p + q + r = 20$
Cost	$a = 3p$	$b = 2q$	$c = ½\ r$	$a + b + c = 20$

Guess and check now:

Case 1: If c is ₹ 10, then $r = 20$ (not possible with the given condition)

Case 2: If c = ₹ 9, we have $r = 18$; then $p + q = 2$

p	q	$a + b$
1	1	$3 + 2 = 5$; and $a + b + c < 20$

Case 3: If c = ₹ 8, we have $r = 16$; then $p + q = 4$

p	q	$a + b$
1	3	$3 + 6 = 9$; and $a + b + c < 20$
3	1	$9 + 2 = 11$ and $a + b + c < 20$
2	2	$6 + 4 = 10$ and $a + b + c < 20$

Case 4: If c is ₹ 7, we have $r = 14$; then $p + q = 6$

p	q	$a + b$
1	5	$3 + 10 = 13$; and $a + b + c = 20$ (meets the condition)
5	1	$15 + 2 = 17$ and $a + b + c > 20$
4	2	$12 + 4 = 16$ and $a + b + c > 20$
2	4	$6 + 8 = 14$ and $a + b + c > 20$
3	3	$9 + 6 = 15$ and $a + b + c > 20$

Alternative Solution: If we take the number of cookies of the first type to be x and the number of cookies of the second type to be y, then we can form the following equation:

$$3x + 2y + \tfrac{1}{2}(20 - x - y) = 20.$$

Upon simplification this yields $5x + 3y = 20$, with the following possible integer solutions:

$x = 1$, $y = 5$, and $x = 4$, $y = 0$.

Rejecting the second solution as we are told that all three types were bought, we have (1,5,14) as the solution set.

You can try similar strategies for the remaining problems:

Solution to Problem–IX–3–M–4.

If the product of three numbers is 36, we have the following possible sets of factors:

(1,1,36), (1,2,18), (1,3,12), (1,4,9), (1,6,6), (2,2,9), (2,3,6) and (3,3,4). The sums of the numbers in the above sets are 38, 21, 16, 14, 13, 13, 11 and 10. Of these, all except 13 are unique. So the door number must be 13, else the census-taker would have obtained the ages of the children at that stage. The last statement of the lady tells us that there is *an* eldest child, not two. This leads to the solution (2,2,9).

Solution to Problem–IX–3–M–5.

Assuming that the cheque was made out for ₹ X and Y paise, the expected amount was 100X + Y paise. The amount issued by the cashier was ₹ Y and X paise, i.e., 100Y + X paise. After 20 paise was spent from this it became 100Y + X – 20 paise. This is twice the amount indicated in the cheque. So we can form the equation 2(100X + Y) = 100Y + X – 20.

On simplification this yields 199X = 98Y – 20. We need to obtain integer values of X and Y that satisfy this equation. As this appears to be a formidable task, we try a different approach.

When we assume that the cheque was made out for ₹ X and Y paise, it gives rise to two situations: (i) Y < 50 (ii) Y ≥ 50.

In case (i) we get the following relations: Y = 2X and X – 20 = 2Y. (That is, there is no carryover from paise to rupees in doubling the amount of the cheque.) Solving these we obtain $\left(X = {}^{-20}/_{3}, Y = {}^{-40}/_{3}\right)$ which we reject though it satisfies the equation obtained earlier: 199X = 98Y – 20.

In case (ii) there is a carryover and so we get the relations Y = 2X + 1 and X – 20 = 2Y – 100. Solving these we get (X = 26, Y = 53). As expected, this solution set too satisfies the equation obtained initially. The cheque was made out for ₹ 26.53.

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