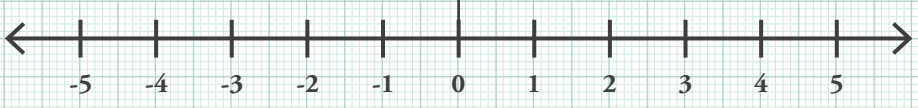
At Right Angles

A Resource for School Mathematics

# QUADRILATERALS

PADMAPRIYA SHIRALI

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# QUADRILATERALS

Quadrilaterals is a topic which need not be 'taught' by a teacher. As a topic it presents ample opportunities for holding discussion based classes. Geometry topics in general need to be approached as explorations. Quadrilaterals is a topic where students can freely construct, measure, and compare to discover the properties of different shapes and establish the relationships between them.

Discussion amongst students is absolutely necessary for formulating ideas and justifying their reasoning. The teacher's role changes to providing meaningful tasks to the students in order to create an exploratory environment, and thereby to spark a discussion leading to understanding and learning.

Constructions can be done in a hands-on manner or in a virtual manner, using dynamic geometry software. Properties of the shapes can be studied and arrived at through observation and experimentation. Through various activities which involve comparison, students can notice the finer points and deepen their understanding.

Most of the activities given here are well known. The focus here is to find ways of generating good discussion and creating opportunities for sharing one's thinking, argumentation, and categorisation.

The crucial work of the teacher in such a class is to observe students as they discuss the terms and to watch out for misconceptions and incorrect usage of vocabulary. By asking clarifying questions like 'why' and 'can you justify', the teacher can encourage students to articulate their reasoning and thereby clear up any possible confusion.

Class organisation plays an important role to have a discussion centered approach. Class and work tables can be organised in such a way that students can work in pairs, or in groups of three or four, to share materials and notes, and to compare and discuss their findings. The group will need to work cooperatively, and a teacher may have to lend a hand if group members are working in isolation or if the group is being dominated by one student.

Each class can end with group presentations and consolidation of learnings from the activity.

There are many ways of initiating this topic; each has its own merits. Hence, the sequence of activities suggested here is not linear, and teachers can opt for what works best for them.

Before starting the activities the teacher:

- Informs the students that quadrilaterals are shapes that have four straight sides.
- Points out the difference between concave and convex quadrilaterals through drawings.
- Pins the names of various shapes with corresponding drawings on a bulletin board for students' reference.

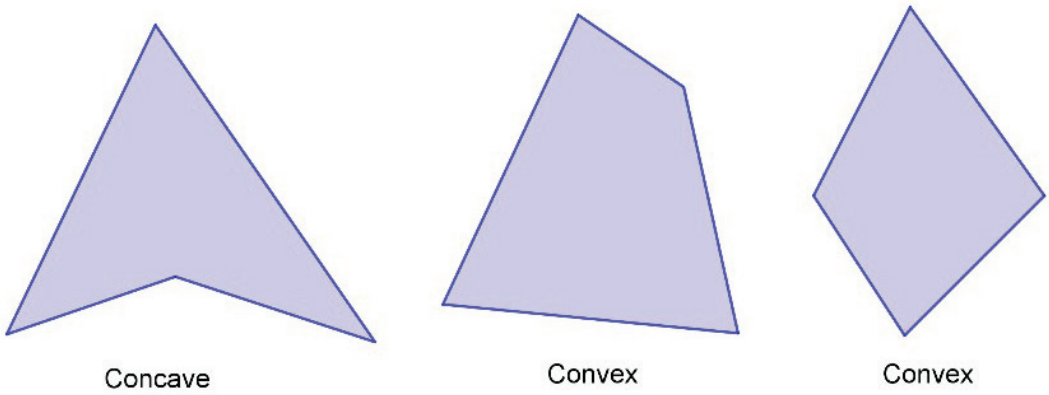


Figure 1

## PRIOR KNOWLEDGE

- How to measure angles; how to use a compass
- Angle categories (right, acute, obtuse, straight, reflex angle, linear pair, supplementary, complementary)
- Triangle categories (equilateral, isosceles, right-angled)
- Sum of the angles of a triangle; sum of the angles of a four-sided figure
- Parallel lines and their properties
- Lines of symmetry of a figure

## GROUP ACTIVITY 1

**Objective:** Tracing quadrilateral shapes on paper and discovering properties

**Materials:** Quadrilateral shapes or stencils (plastic, wooden or cardboard), Ruler, Protractor; one set per student group

Students can use the shapes as stencils to outline them on a paper and cut the shapes.

They experiment with these shapes and discover as many properties as possible and record them on the shapes themselves.

(Square, Rectangle, Rhombus, Parallelogram, Kite, Trapezium, Isosceles trapezium, Irregular quadrilateral)

Figure 2

Some students may choose to measure the lengths and the angles. Some may fold the shapes to discover lines of symmetry. Some may measure the diagonals. Some attributes may not get used, for example, parallel lines.

The activity can be followed by a discussion on what each group has noticed about a particular shape.

Is there any property which is common to all quadrilaterals?

Since it is a preliminary activity, the teacher need not attempt to make an exhaustive list of all properties. At a later point the students can build a table with a complete list of properties based on various attributes.

## GROUP ACTIVITY 2

**Objective:** Building Quadrilaterals and noticing transformations

**Materials:** Straws and thread

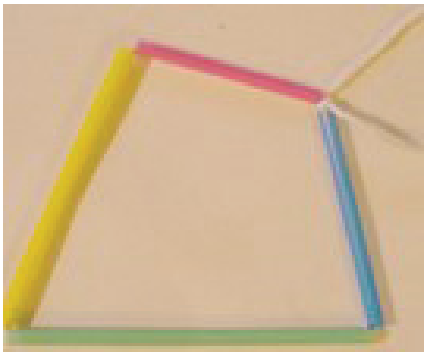

Figure 3

Students can build a quadrilateral shape with straws and thread. They will notice that the shape is not rigid – unlike a triangle.

Let them build a square shape. They can bend the shape and notice the change in shape.

What is the new shape? How have the properties changed in the new shape? What happens to the sides? How have the angles changed? How do the diagonals of the new shape compare with the diagonals of the previous shape? Are they equal?

Let students build a rectangle, bend the shape and observe in a similar manner how the properties change. What is the new shape? How do the angles change? Are the diagonals of the same size in the new shape?

Take the rectangle apart. Use the same four straws to make a quadrilateral which is not a parallelogram. What shape is it now?

Let students build other shapes, play around with them and note down what they notice.

Can a convex quadrilateral be changed to a concave quadrilateral? Why? What happens to the angles?

See also http://teachersofindia.org/en/article/exploring-geometric-shapes-straw-models.

## GROUP ACTIVITY 3

**Objective:** Symmetry of Quadrilaterals

**Materials:** Paper shapes made in group activity 1

Figure 4

Can you fold a square in half so that one half fits exactly over the other?

Does a square have reflection symmetry? Show the line of symmetry with a dotted line. How many such lines are there?

Can you fold a rectangle in half so that one half fits exactly over the other?

Does a rectangle have reflection symmetry? Show it with a dotted line.

Can you fold a parallelogram in half so that one half fits exactly over the other?

Does a parallelogram have reflection symmetry? If yes, show it with a dotted line.

In a similar way, work out lines of symmetry for all the other shapes.

Each group can share its findings and compare it with other groups' findings.

## GROUP ACTIVITY 4

**Objective:** Drawing different types of quadrilaterals on square and isometric dot paper, naming them

**Materials:** Square dot and isometric dot paper

Let each student in a group draw different types of quadrilaterals on dot paper.

They can name their shapes by comparing them with the shapes pinned on the board.

As a challenge, teacher can draw a line of a specific quadrilateral on dot paper and ask the students to complete it as a square, rectangle, kite, or any other shape.

At the end of the activity all students within a group compare their drawings.

Can there be multiple answers? Does orientation affect the shape?

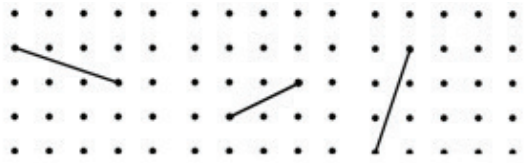

Figure 5

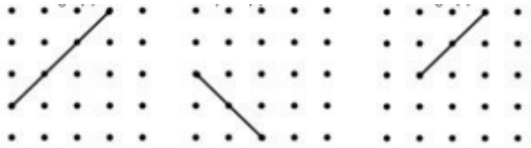

Figure 6

## GROUP ACTIVITY 5

**Objective:** Classifying quadrilaterals on the basis of one attribute

**Materials:** Cutouts of different quadrilateral shapes of multiple sizes

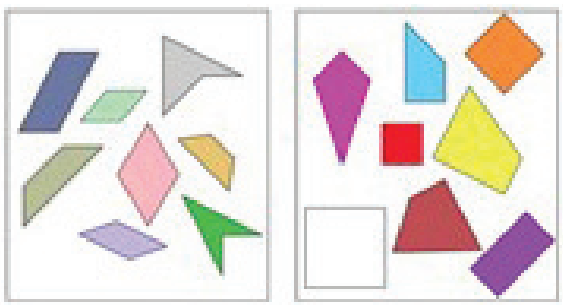

Figure 7

What is the basis for the way the shapes in the drawing have been sorted?

Each group can select an assortment of various shapes and sort them using some attribute.

It could be based on number of equal angles (all four equal, opposite angles equal, etc.), number of equal sides (all four equal, opposite sides equal, etc.), presence of parallel lines, lines of symmetry, etc.

Other groups need to figure out the attribute based on which the shapes have been sorted.

## GROUP ACTIVITY 6

**Objective:** Building a table of attributes for different quadrilaterals

**Materials:** Cutouts of different quadrilateral shapes of multiple sizes

Let students fill the properties they have noticed based on each attribute given, in the table.
Here is an example of a table that is constructed based on diagonal properties.

| | Square | Rectangle | Rhombus | Kite | Trapezium | Isosceles trapezium | Parallelogram |
|---|---|---|---|---|---|---|---|
| Equality of diagonals | | | | | | | |
| One diagonal bisects the other | | | | | | | |
| Both diagonals bisect each other | | | | | | | |
| Diagonals meet at right angles | | | | | | | |
| One diagonal bisects the angles at the vertices | | | | | | | |
| Both diagonals bisect the angles at the vertices | | | | | | | |

Tables can be made comprehensive by including attributes like sides, angles, symmetry, etc.

## GROUP ACTIVITY 7

**Objective:** Representing quadrilaterals as sets in a Venn diagram

**Prerequisite:** Familiarity with Venn diagrams

**Materials:** Name cards of different quadrilaterals

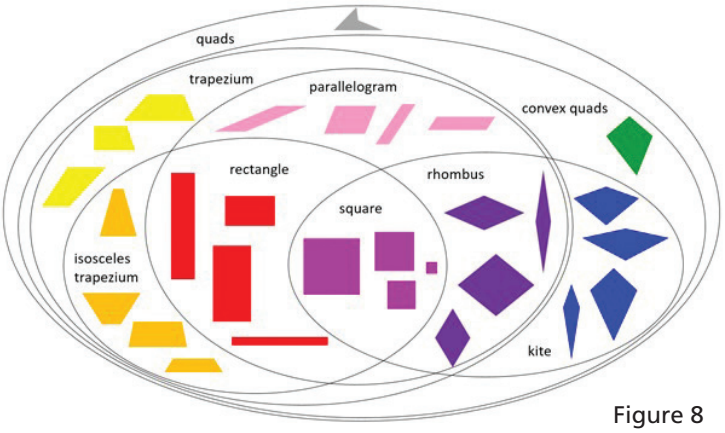


Figure 8

Let each group work out a Venn diagram to represent the relationships of different kinds of quadrilaterals and place the name cards in the appropriate circles. It is not an easy task and students will have to do a fair amount of rearranging the cards before they come up with a satisfactory solution.

If the students are finding it difficult to get started, teachers can ask a few leading questions. What shape set is a part of the parallelogram set?

How will you show that? Is any other set part of parallelogram?

Each group can present their drawing to the other groups for raising questions and clarifying doubts.

Are the representations of all the groups similar or different? If some diagrams are different, can the students explain the logic behind it?

As an extension, the teacher can give copies of a Venn diagram based on any one attribute. Students try to find an attribute which will correspond to the given drawing.

What attribute has been used to create these sets?

What is the property common to all of the figures in the left circle? What is the property common to all of the figures in the right circle?

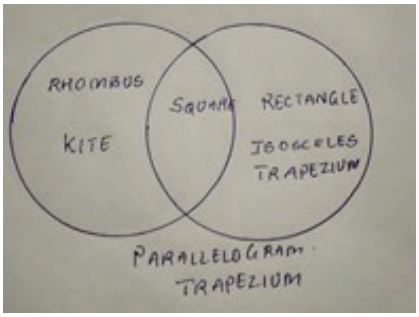


Figure 9

The teacher can give a hint if necessary.

## GROUP ACTIVITY 8

**Objective:** Classifying quadrilaterals with a tree diagram

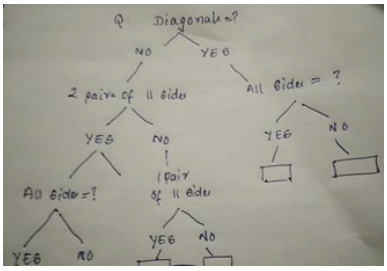


Figure 10

Students can be taught ways of drawing tree charts with Yes /No questions. They must understand that all possible shapes need to be covered.

Teachers can give hints to the kind of questions they can begin with to draw the tree. 'Does the shape have equal angles?' 'Does the shape have a pair of parallel sides?'

## GROUP ACTIVITY 9

**Objective:** Guessing the shape

**Materials:** Different quadrilateral shapes

One group selects a shape. The other group must deduce the shape by asking questions.

The other groups are allowed to ask questions that will require only a Yes/No answer.

For example: 'Are both pairs of opposite sides parallel?' 'Are all the angles equal?' 'Is any angle more than 180 degrees?'

Group which asks the minimum number of questions to identify the shape wins.

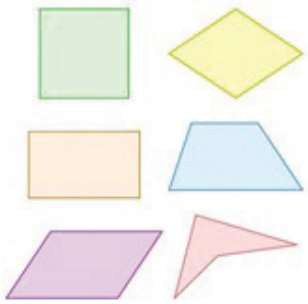

Figure 11

## GROUP ACTIVITY 10

**Objective:** Match properties of quadrilaterals with their names

**Materials:** Cards describing the properties and corresponding name cards

Example: The properties can be written as follows:

The shape has two pairs of parallel sides

The shape has four right angles

The length of each side is the same

The shape is irregular

The shape has two lines of symmetry

The shape is a parallelogram with a right angle

## GROUP ACTIVITY 11

**Objective:** Building quadrilaterals with triangles

**Materials:** Equilateral triangle, isosceles triangles (acute, obtuse, right-angled) and right angled triangles (isosceles, scalene)

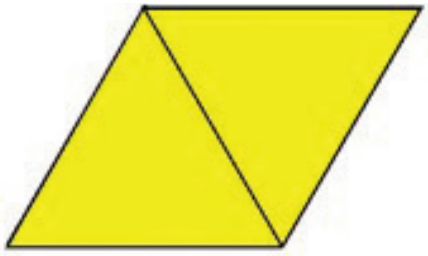

Figure 12

Let students try building the various quadrilaterals using different types of triangles

Can you create a rectangle using equilateral triangles? Using four identical isosceles triangles? Using two pairs of identical isosceles triangles? Using right-angled triangles?

Can you build a rhombus with two isosceles triangles? With two equilateral triangles?

Can you build a square with two equilateral triangles? Why not?

Can you create any other quadrilaterals in a similar manner?

## GROUP ACTIVITY 12

**Objective:** Making different quadrilaterals on a circle

**Materials:** Compass

Let students construct a large circle with 8 equally spaced dots.

How many differently shaped quadrilaterals can be made by joining the dots on the circle?

Each group can share their work with the other groups, and the class can work out the total number of differently shaped quadrilaterals that are possible.

Figure 13

## GROUP ACTIVITY 13

**Objective:** Learning to justify

**Materials:** Cards with statements

Here are a few samples of statements:

Quadrilaterals will always be rectangles.

A rhombus can sometimes be a trapezoid.

A trapezium is a parallelogram.

An equilateral parallelogram is equiangular.

No quadrilateral has 3 lines of symmetry.

Teacher distributes these cards to different groups, and each group has to say whether the statement is true, sometimes true or false.

Most importantly, they should justify their answer.

## INDIVIDUAL ACTIVITY 14

**Objective:** Practice writing and understanding the symbols of a drawing

**Materials:** Quadrilaterals of all types with appropriate markings

Let the students record the information as sentences. Here are a few samples:

A trapezium has ____ pair of parallel sides, ____ pairs of equal sides, and _____ right angles.

A parallelogram has _____ pairs of parallel sides, _____ equal sides, and ____ pairs of congruent angles.

As the sides are equal and the angles are not _____ the shape is a _____.

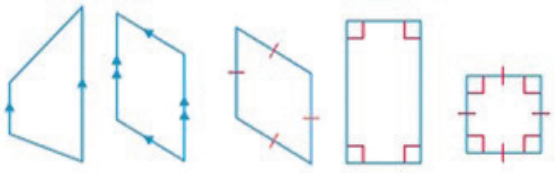

Figure 14

In a convex quadrilateral like a square or a rectangle, the two diagonals lie _____.

## GROUP ACTIVITY 15

**Objective:** Quiz

**Materials:** Cards with questions

Here are a few sample questions:

When isn't a rectangle a rhombus?

Can a figure be a square, a rectangle and a parallelogram at the same time? Why?

The diagonals of a four-sided shape bisect each other. What shape can it be?

When all the sides are different in a quadrilateral, are all the angles always different as well?

"A quadrilateral which has two right angles is always a trapezium." Is this so?

Can a quadrilateral have three obtuse angles? Can it have four acute angles?

If a quadrilateral has two right angles, what can the other two angles be?

What are the similarities between a rectangle and a rhombus? What are the differences?

Each group is required to pick up a card, discuss the question amongst themselves, and present their answer to the class.

## GROUP ACTIVITY 16

**Objective:** To strengthen visualisation

**Materials:** Pose questions which require visualisation

Ex. Draw two more lines to make this shape a kite.

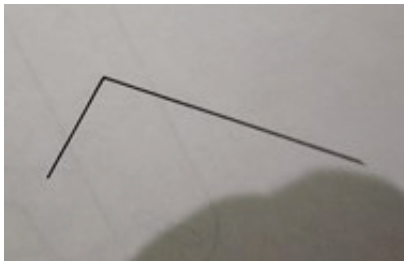

Figure 15

How many parallelograms do you see in this figure?

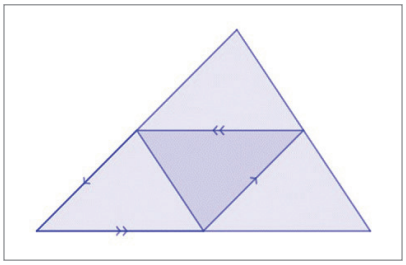

Figure 16

How many isosceles trapeziums do you see in this figure?

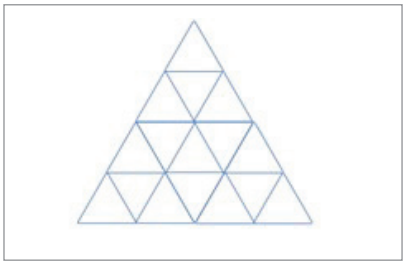

Figure 17

## GROUP ACTIVITY 17

**Objective:** Playing with quadrilaterals

**Materials:** Tangrams

Students can try to create different quadrilaterals using Tangram shapes. Teacher can raise some questions.

Can you use two Tangram pieces to make a square?

Can you use two Tangram pieces to make a trapezium?

Can you use three Tangram pieces to make a rectangle?

Can you use three Tangram pieces to make a square?

Can you use three Tangram pieces to make a parallelogram?

Can you use four Tangram pieces to make a square? Is there more than one way?

Various other combinations can be tried.

## GROUP ACTIVITY 18

**Objective:** Quadrilateral playing cards

**Materials:** Set of cards with shapes, names and two properties for each shape.

Figure 18

As a reinforcement exercise, students can create a set of playing cards with names of quadrilaterals, quadrilateral shapes and cards with properties of the shapes.

The winner will be the first who puts together the shape card, name card and two property cards.

Source: https://nrich.maths.org/2924

## GROUP ACTIVITY 19

**Objective:** Tiling patterns with Quadrilaterals

**Materials:** Multiple (12 to 15) sets of each shape

Students can explore how quadrilaterals are used to make tiling patterns.

The teacher can raise questions while they are doing the activity.

In a tiling pattern, the tiles must fit together exactly at each vertex of the pattern.

Do the students see that the angles at each vertex must add up to 360 degrees?

Can four identical squares fit together in more than one way? Why do four squares fit together?

Can four squares of different sizes fit together?

In how many ways can four identical rectangles fit together? What can you say about the angles at the vertex?

Can four identical rhombi fit together? What do you notice about the angles at the vertex?

Students can explore kites, parallelograms,

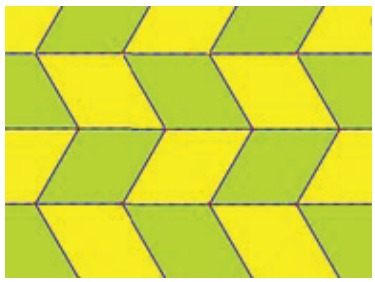

Figure 19

isosceles trapeziums, trapeziums, concave quadrilaterals and irregular quadrilaterals in a similar way.

Let them combine two types of shapes and explore the possibilities.

Can two squares and two rhombi fit together around a point? When is that possible?

Are there combinations that are not possible?

## GROUP ACTIVITY 20

**Objective:** Constructing Quadrilaterals

**Materials:** Compass, protractor and ruler

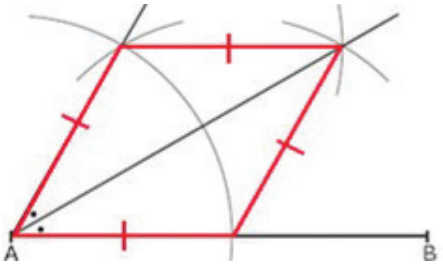


Figure 20

Ask 'How many pieces of data can be found in a quadrilateral?' Four sides, four angles and two diagonals.

Points for discussion

What piece of data is needed to make a square? Would it be possible to construct a square if you knew the diagonal length?

What pieces of data are needed to make a rectangle? Would any other data help to make a rectangle?

Would it be enough to know the length of the side of a rhombus to construct it? What else is needed? Is there any other way?

Let the students explore data needed for other shapes.

To construct a general quadrilateral, data about five parts is needed. For example, the data could be: four sides and one diagonal, three sides and two diagonals, three sides and two included angles, two adjacent sides and three angles, and so on.

Let students construct various quadrilaterals that satisfy certain conditions. Methods of construction can be brainstormed with students and demonstrated on the board.

Padmapriya Shirali

Padmapriya Shirali is part of the Community Math Centre based in Sahyadri School (Pune) and Rishi Valley (AP), where she has worked since 1983, teaching a variety of subjects – mathematics, computer applications, geography, economics, environmental studies and Telugu. For the past few years she has been involved in teacher outreach work. At present she is working with the SCERT (AP) on curricular reform and primary level math textbooks. In the 1990s, she worked closely with the late Shri P K Srinivasan, famed mathematics educator from Chennai. She was part of the team that created the multigrade elementary learning programme of the Rishi Valley Rural Centre, known as 'School in a Box' Padmapriya may be contacted at padmapriya.shirali@gmail.com