Higher Secondary

ClassRoom

Folk Method Analysis
(Sixth Method – a local device)

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(Submission from a reader with reference to the article: Mohammad Umar, "The Height of a Tree", AtRiA November 2020, pp. 33-34)

The amount by which different people can bend their bodies forward at the waist is different. Those who are highly flexible may be able to put their heads through their legs and look almost straight up behind themselves as in Figure 1. Those who are less flexible though would probably end up with a position as in Figure 2.

Figure 1

Figure 2

Further differences in posture could be caused by spreading the legs apart by different amounts or if one inadvertently leans a little forward or back (i.e., the legs are not in a perfectly vertical plane w.r.t. the ground). Thus, it would seem that different people attempting this measurement (or even the same person repeating the experiment) could get different results.

Keywords: folk mathematics, analysis, reasoning, similarity

Assumptions

We shall assume that the 'average' person will assume a body posture similar to that of Figure 2 and the same person will be able to replicate the same posture every time. Then, the following parameters (see Figure 3) will remain constant for a particular person:

- $\angle VKE = \angle VFX = 90°$
- E = eye level above the ground in this position
- V = inverted V-tip level above the ground
- EK = horizontal distance between legs and eyes
- By implication, $\angle VEK$ is constant.

It is then claimed that the height OY of an object will be equal to the distance OF.

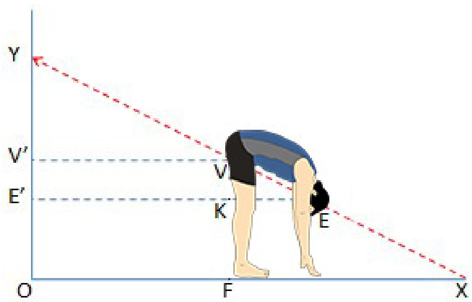

Figure 3

Analysis

Let us now analyse the mathematical implications of this method under the assumptions above. Figure 4 shows objects of 2 different heights ($OY_1$and OY_2) and two different viewer positions (F_1 and F_2) superimposed on each other. Now, if $OY_1 = OF_1$ and $OY_2 = OF_2$, then $\angle Y_1F_1O$ and $\angle Y_2F_2O$ are each 45°. Then $\angle V_1F_1Y_1$ and $V_2F_2Y_2$ must also be 45°.

With a constant posture, $V_1F_1 = V_2F_2$ (as per our assumption). Also, $\angle V_2Y_2F_2 < \angle V_1Y_1\ F_1$ (this can be proved using the sine rule and the fact that $Y_2V_2 > Y_1V_1$). This means that $\angle Y_2V_2F_2 > \angle Y_1V_1F_1$. Hence, the supplementary angles $E_1V_1F_1$ and $E_2V_2F_2$ are different (the former is larger). *However, these two angles in actuality must be the same, as they are completely determined by the posture, which we have assumed to be constant.* Thus, in general, we will **not** get the correct result with our assumption.

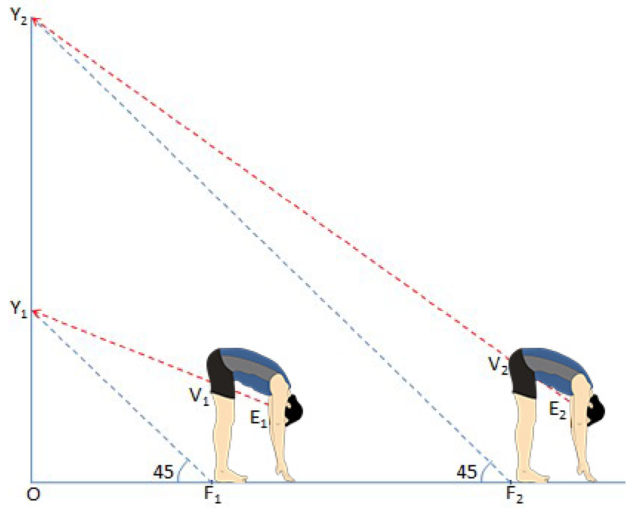

Figure 4

Refining our assumption

If the height to be measured (and therefore OF as well) is much larger than the dimensions of our body, then $\angle YFO$ will almost equal $\angle YEE'(=\angle VEK)$, or equivalently, $\angle VYF$ will be very small (see Figure 5). If we also have $\angle VEK = 45$, then OF (which will be nearly equal to OX if the dimensions of the body are negligible compared to OF) will give a close estimate of the object height OY.

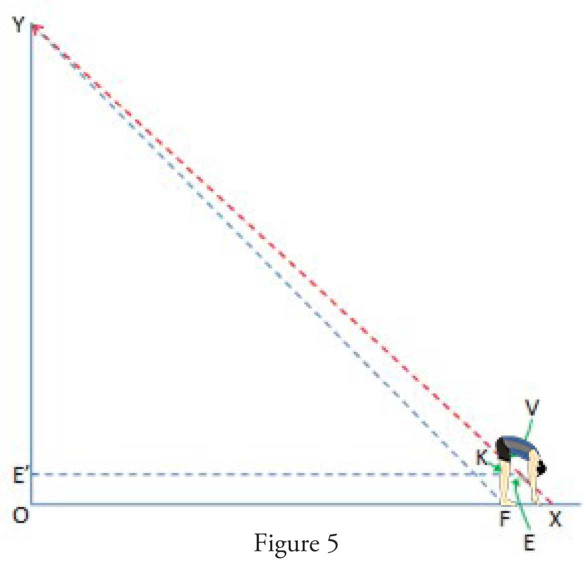

Figure 5

So, one way to get the 'folk method' (i.e., Method 6) to work is to refine our assumptions as follows:

1. One can replicate a body posture such that $\angle VEK \approx 45°$; i.e., $VK \approx EK$.
2. The height to be measured is much larger than the dimensions of the human body.

These two assumptions make Method 6 mathematically equivalent to Method 3 described in the article. In particular, under assumption 2, Method 3 will also work well even without accounting for the eye-level above ground since it will be negligible.

Alternatively, instead of the second assumption, we could claim $OX = OY$ (the object height), where point X is obtained by extending line VE to ground level. Assumption 1 and the claim that $OX = OY$ then make Method 6 mathematically equivalent to Method 5.

Finally, we can even relax the requirement for replicating the same posture with $\angle VEK \approx 45°$ for every measurement by noting that triangles VEK and YXO are similar, and VK, EK and OX are measurable. Then we can determine the object height OY by using the similarity relation $OY : OX = VK : EK$. The same applies to Methods 3 and 5 as well.

Example calculations

Let's assume that the height measured through Method 6 was 6 m (it actually gave 6.24 m, but we round it off for convenience) and that this was indeed the correct height of the tree. The relevant body dimensions (EK, KF in Figure 6) could realistically be assumed to be roughly 0.5 m. Then, in Figure 6, if $OY = OF = 6\text{m}$, we'll get $EE' = 6.5\text{m}$ and $YE' = 5.5\text{m}$. This means $\angle YEE' = \angle VEK = \tan^{-1}\frac{5.5}{6.5} \approx 40°$ which is a different posture from our assumption of $\angle VEK = 45°$. One possibility of ending up with such a posture when $EK = 0.5\text{m}$ is that $VK = EK \cdot \tan 40° = 0.5 \cdot \tan 40° \approx 0.42\text{m}$. Then for such a person to achieve $\angle VEK = 45°$, the posture would have to be adjusted (e.g., by bending the back and neck differently) such that $EK = VK = 0.42\text{m}$. The table below compares the results we get for various tree height estimates with these postures for $\angle VEK = 40°$ and $45°$. For clarity, a sample calculation has been worked out in the "Calculation Details" section at the end.

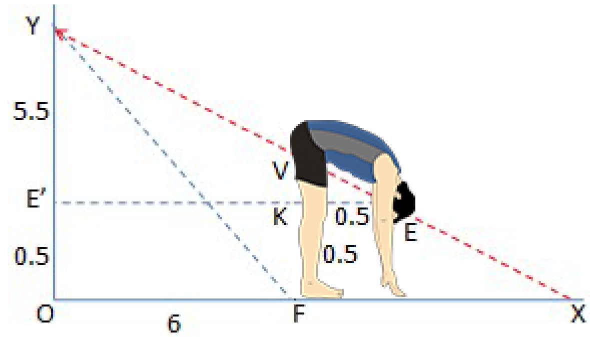

Figure 6

As expected, in both cases, $\angle YFO$ approaches $\angle VEK$ as the object height increases. Though the estimate for a tree of 6 m height turns out to be better when $\angle VEK = 40°$, using this posture means that the error magnitude keeps increasing with object height while relative error approaches the value $\frac{\tan 45}{\tan 40} - 1 \approx 19\%$. On the other hand, when $\angle VEK = 45°$, the absolute error remains constant (and in fact can be eliminated if we use $OY = OX$) while the relative error keeps reducing as we move further for higher objects. Thus, having $\angle VEK = 45°$ would be the more desirable posture for making height measurements with Method 6. As indicated while stating assumption 1, $\angle VEK = 45°$ can be achieved in practice by bending in such a way that we get $VK = EK$.

Calculation details (true object height OY = 60 m, $\angle VEK$ = 40°)

In Figure 6, if we use the posture with $\angle VEK = 40°$, then as calculated above, we'll have $VK = 0.42\text{m}$. Note that $\angle VXF = \angle VEK$. Since $VF = 0.5 + 0.42 = 0.92$, we get in triangle VXF, $FX = \frac{VF}{\tan\angle VXF} = \frac{0.92}{\tan 40°} \approx 1.1\text{m}$. Also, in triangle YXO with $OY = 60\text{m}$, we get $OX = \frac{OY}{\tan\angle YXO} = \frac{60}{\tan 40°} \approx 71.5\text{m}$. Then, $OF = OX - FX = 71.5 - 1.1 = 70.4\text{m}$ will be the estimated object height. Finally, in triangle YFO, $\angle YFO = \tan^{-1}\frac{YO}{FO} = \tan^{-1}\frac{60}{70.4} \approx 40.44°$.

Other results listed in the table are calculated similarly. In particular, for the case when $\angle VEK = 45°$, we have assumed that the posture is such that $EK = 0.42 = VK$.

True Object Height (m)	$\angle VEK = 40°$			$\angle VEK = 45°$		
	Height Estimate (m)	Error (%)	$\angle YFO$	Height Estimate (m)	Error (%)	$\angle YFO$
6	6.05	+0.8	44.76	5.08	-15.3	49.75
60	70.4	+17.3	40.44	59.08	-1.5	45.44
100	118	+18	40.28	99.08	-0.9	45.26