Problem Corner | Upper Primary

Middle School Problems on Polygons

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Polygons are closed 2-D figures whose sides are straight line segments.

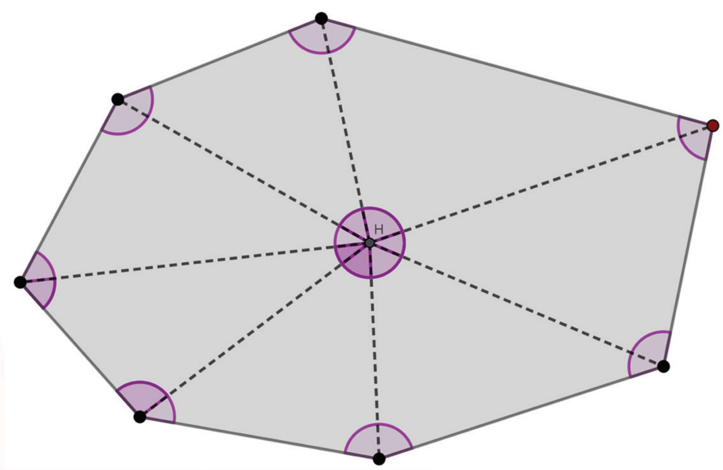

Figure 1

Figure 1 is an example of a heptagon - a polygon with 7 sides.

Here are some generalisations about polygons.

- A polygon must have at least 3 sides.
- A polygon has as many vertices as sides.
- The sum of the interior angles of a polygon of n sides is given by $(n-2) \times 180°$. (To understand this formula, look at the heptagon - in which H is any point in the interior. If you add up the angles of the 7 triangles you see in the heptagon, you would see

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that the sum is $7 \times 180°$. To get the sum of the interior angles of the polygon, you would have to subtract the angles at the centre, which sum up to $360°$ or $2 \times 180°$. So the sum of the interior angles of the heptagon is $5 \times 180°$ or $(7-2) \times 180°$.)

- The sum of the exterior angles of any polygon is always $360°$. (An exterior angle is the angle formed when a side is extended at a vertex. See if you can reason why the exterior angles add up to $360°$.)
- If all the sides and angles of a polygon are equal, we call it a regular polygon. An equilateral triangle and a square are regular polygons of three and four sides, respectively. Each angle of a regular polygon is $\frac{(n-2) \times 180}{n}$ degrees.

Problem X-1-M-.1

It is possible to have a triangle whose angles are in the ratio 1:2:3 or a quadrilateral with angles in the ratio 1:2:3:4. It is not possible to have a pentagon with angles in the ratio 1:2:3:4:5 or an octagon with angles in the ratio 1:2:3:4:5:6:7:8. Why?

Problem X-1-M-.2

Find expressions in terms of n for the least and greatest angles of a polygon of n sides with angles in the ratio 1:2:3 . . . . :n.

The remaining problems are all concerned with regular polygons.

Problem X-1-M-.3

Regular polygons A and B have number of sides in the ratio 1 : 2, and interior angles in the ratio 3 : 4. Find the number of sides of the regular polygons.

Problem X-1-M-.4

Regular polygon A has 3 more sides than regular polygon B, while the interior angle of the former is $6°$ more than that of the latter. Find the number of sides of the polygons.

Problem X-1-M-.5

Regular polygons A, B and C have m, n and $2mn$ sides, respectively. If the sum of an interior angle of polygon A and an interior angle of polygon B equals an interior angle of polygon C, find the values of m and n.

Pedagogical Note: These problems are an excellent means to combine a student's knowledge of arithmetic (ratio, fractions), geometry and algebra. By getting them to draw their solutions, they will be able to visualise their meaning. Drill and practice become easier with such problems.

Solutions to the Problems

Solution to Problem X-1-M-.1

The sum of the interior angles of a pentagon equals (5– 2) $\times$ 180° = 540°.This has to be divided into 5 parts in the ratio 1:2:3:4:5. Let these angles be k, 2 k, 3 k, 4 k and 5 k.

Adding, we get $k + 2k + 3k + 4k + 5k = 15k = 540°$. This gives $k = 36°$. But then one of the angles is $5k$ which is 180° which is not possible. Try proving this for an octagon now!

Solution to Problem X-1-M-.2

So the angles can be taken to be k, $2k$, $3k$,....nk. Then

$$\frac{kn(n+1)}{2} = (n-2) \times 180°$$

Or, $k = \dfrac{(n-2)\,360°}{n(n+1)}$. This is the least angle, while the greatest angle, which is n times this, is $\dfrac{(n-2)\,360°}{(n+1)}$.

Pedagogical Note: It is a good idea for students to try getting the angles and drawing different polygons with different values of n.

Solution to Problem X-1-M-.3

	Polygon A	Polygon B
Sides	n	$2n$
Each interior angle	3ϑ $\dfrac{(n-2) \times 180}{n}$	4ϑ $\dfrac{(2n-2) \times 180}{2n}$

Equating both expressions for ϑ we get:

$$\frac{(n-2) \times 180}{3n} = \frac{(n-1) \times 180}{4n}$$

Solving this we get $n = 5$. So, the number of sides of the polygons are 5 and 10.

Pedagogical Note: Confusing statements in word problems become much easier when students use tables.

Solution to Problem X-1-M-.4

	Polygon A	Polygon B
Sides	$n + 3$	n
Each interior angle	$\vartheta + 6$ $\dfrac{(n+1) \times 180}{n+3}$	ϑ $\dfrac{(n-2) \times 180}{n}$

Equating both expressions for ϑ we get,

$$\frac{(n+1) \times 180}{n+3} - 6 = \frac{(n-2) \times 180}{n}$$

From this we get $n^2 + 3n - 180 = 0$.

Solving the quadratic equation we get the only positive value of $n = 12$.

So, the number of sides of regular polygons A and B are 15 and 12, respectively.

Pedagogical Note: This problem requires the solution of a quadratic equation. While this is beyond the middle school math syllabus, students can understand the meaning of this expression by substituting different values of n and they may even find the solution by trial and error.

Solution to Problem X-1-M-.5

	Polygon A	Polygon B	Polygon C
Sides	m	n	$2mn$
Each interior angle	ϑ_1 $\dfrac{(m-2) \times 180}{m}$	ϑ_2 $\dfrac{(n-2) \times 180}{n}$	ϑ_3 $\dfrac{(2mn-2) \times 180}{2mn}$

Since $\vartheta_1 + \vartheta_2 = \vartheta_3$

We form the equation

$$\frac{(m-2) \times 180}{m} + \frac{(n-2) \times 180}{n} = \frac{(mn-1) \times 180}{mn}$$

This simplifies to $mn - 2m - 2n + 1 = 0$. This expression is not factorisable. Adding 3 to both sides, we get $mn - 2m - 2n + 4 = 3$, which can be factorised as $(m-2)(n-2) = 3$. The only factorisation of 3 in positive integers is 1×3 and so we could say

$$m - 2 = 1 \text{ or } m = 3 \text{ and } n - 2 = 3 \text{ or } n = 5.$$

The number of sides of polygons A, B and C are then 3, 5 and 30.

Pedagogical Note: An interesting way to solve an equation and a great way for students to see the uses of factorisation.

A. RAMACHANDRAN has had a longstanding interest in the teaching of mathematics and science. He studied physical science and mathematics at the undergraduate level and shifted to life science at the postgraduate level. He taught science, mathematics and geography to middle school students at Rishi Valley School for two decades. His other interests include the English language and Indian music. He may be contacted at archandran.53@gmail.com.