At Right Angles
A RESOURCE FOR SCHOOL MATHEMATICS
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Azim Premji University

TRIANGLES

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TRIANGLES

The shapes that are seen most in everyday life are rectangles and circles, but *triangles* are the building blocks in many types of constructions.

Triangles are rarely seen in their pure form in nature, but they are fundamental to our environment, physical as well as virtual. They are also mathematically important. Perhaps it is the simplicity of the triangular form that aids in building complex forms.

Knowledge of triangles and its application was widespread even in ancient times. Triangles are used by astronomers to find distances to stars. They are used by bridge builders, architects, navigators and surveyors. In modern times, they are also put to use in Global Positioning System (GPS) devices.

The triangle is the simplest shape within the set of polygons. Any convex polygon having more than three sides can be dissected into triangles. Hence, understanding the basic properties of triangles allows for a deeper study of polygons with larger numbers of sides.

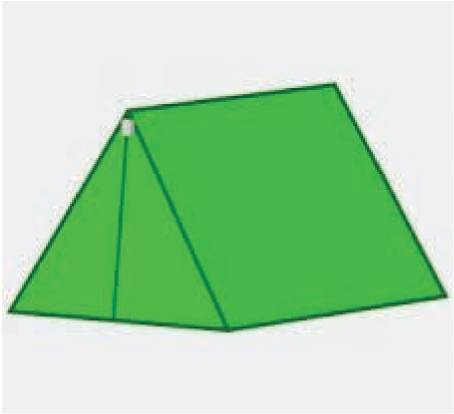
In the world of polygons, triangles stand out as they are the only shape that is rigid; they can stand great pressure, never changing shape. This feature is unique to triangles. Because of this, they are ideal for use in three-dimensional constructions.

The study of triangles can be enriched by incorporating hands-on activities and explorations and by integration with science, e.g., in testing the strength of a triangular structure. Many properties that students will encounter in later years can be discovered through paper-folding exercises and by playing with GeoGebra without getting into formal terminology and formal proof. Similarity of triangles can be introduced through an intuitive understanding of size and proportion.

Experimentation, observation, recording of results followed by discussion will enhance and enrich the learning process. Prior to introduction of the topic, students need to have the skills of using a protractor and a compass. They should also have some knowledge of the different types of angles and line symmetry.

It is good to start the topic with a discussion on triangles and recalling students' knowledge and observations of the shape. The discussion can point out that a triangle is a shape with three sides. Students can talk about the contexts and places where they see triangles. Can they begin to think about why they see triangles in structures like bridges? What is it that they like about triangles? Stimulating interest in a topic can make the learning effective and enjoyable.

Hence the first few activities get the students to explore the uniqueness of triangles.



ACTIVITY 1

Objective: Contrasting a triangle and other polygons to test for rigidity.

Materials: straws, rubber tubes

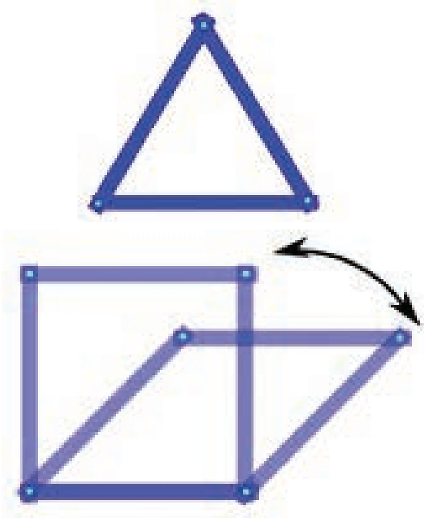
Build a skeletal triangle with straws and rubber tubes or staple pins.

http://teachersofindia.org/en/article/exploring-geometric-shapes-straw-models

Can we change its shape without bending the strips? We cannot. *The triangle is a rigid shape.*

Build any quadrilateral (four-sided shape) with straws and connectors. Can we change its shape? Yes. We can, by pushing and pulling the straws. Attach an extra straw to the shape, thus making two triangles. Can you change its shape now?

Now try this with a five-sided shape. How many more straws do you need to make it rigid?



ACTIVITY 2

Objective: Strength of triangle

Materials: 3 card sheets of same size. One sheet to be shaped as a cuboid, the second to be shaped as a cylinder, and the third to be shaped as a triangular prism.

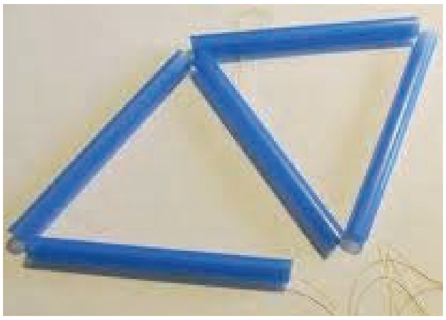
Students can assess the strength of each shape by the number of books each structure can hold without collapsing.

Follow this up with a discussion on the strength of triangle. Both equilateral and isosceles triangles are stronger due to their symmetric shape as compared to scalene triangles.

ACTIVITY 3

Objective: Show that the sum of any two sides must be greater than the third side to form a triangle

Materials: 10 to 12 straws of multiple lengths (4,6,8,10,12,etc.) and twine thread or rubber tube



Students work in pairs. Each pair picks up three straws at random and tries to build a triangle with the straws.

They enter the measurements in a table drawn on the board and note down whether they were able to build a triangle with those measurements.

Was it possible for all student pairs to build triangles?

If necessary, teachers could pick up straws which do not form a triangle to demonstrate that for some combinations, it is not possible to form a triangle.

Why is it that for some combinations of lengths we are not able to form a triangle?

Students can create a table for the triangle measures and study the table.

To explore the relationship of sum of two sides to the third side

AB	BC	AC	AB + BC	AB + AC	BC + AC

How does AB + BC compare with AC? In a similar way compare other pairs of sides.

Does any pattern or conclusion emerge?

The teacher poses leading questions to help students realise that the sum of any two sides needs to be greater than the third side for them to form a triangle.

As a follow up to this, students can repeat the activity to verify the result.

- 'Can a triangle be made with lengths 12 cm, 2 cm and 8 cm?' Why or why not?
- 'Can a triangle be made with lengths 10 cm, 5 cm and 5 cm?' Why or why not?

ACTIVITY 4

Objective: Angles of a triangle add up to 180 degrees

Materials: Straws of different sizes, protractor, triangle cut-outs

Students can verify the property by folding the three corners of a triangle on the dotted lines as shown. The folding should be done so that the base is the longest side (we must not have an obtuse angle at the base).

The midpoints of two sides (D and E) are located first, and the fold lines are then made using the perpendiculars from D and E to the base BC.

The three angles will fit on the base, forming 180 degrees.

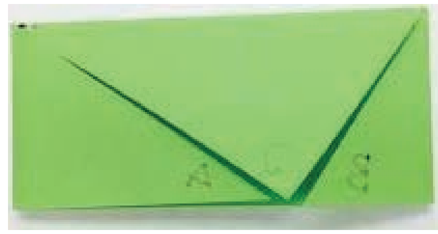
The students should try this folding activity with all the types of triangles to verify the result.

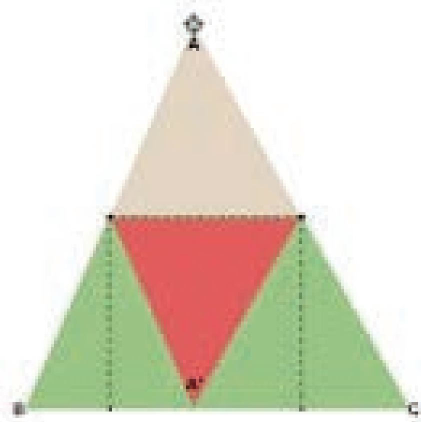
Fix three straws to form a triangle. Measure the interior angles and record them in a table.

Repeat for various types of triangles.

S. No	Angle 1	Angle 2	Angle 3	Angle 1 + Angle 2 + Angle 3

What do you infer from the table?





ACTIVITY 5

Objective: Exterior angle is the sum of the two interior opposite angles

Materials: Set of triangles of varied types (right-angled; right-angled isosceles; acute-angled isosceles; obtuse-angled isosceles; equilateral; scalene; acute-angled; obtuse-angled)

Note: This activity follows from the earlier and can be treated as a derivation of the earlier result.

Each student draws the outline of a triangle. They label and measure the three angles.

They then extend any one side and measure the exterior angle formed by the extended side. The information is recorded in a table on the board.

How does the exterior angle relate to the adjacent interior angles?

The teacher poses leading questions to help students notice that the exterior angle equals the sum of the two interior opposite angles.

Does it hold if another side is extended? Students measure and verify the result.

Triangle 1	Angle A	Angle B	A + B	Ext. angle C
	Angle B	Angle C	B + C	Ext. Angle A
	Angle A	Angle C	A + C	Ext. Angle B

Can the students justify the reason for this relationship?

Reference: https://azimpremjiuniversity.edu.in/SitePages/resources-ara-issue-no-7-july-2020-triangles-to-tetrahedrons.aspx

ACTIVITY 6

Objective: Exploring a set of triangles in terms of sides

Materials: Types of triangles in multiple sizes (equilateral, scalene, isosceles). Chart showing types of triangles and corresponding names to be put up on the board.

Given a set of triangles, the students sort them into different types on the basis of sides and learn to name them.

After identifying the three types of triangles (based on sides), students measure their angles and record the angle measurements.

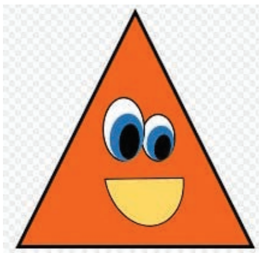
What do you notice about the angles of an equilateral triangle?

What can you say about the angles of an isosceles triangle?

What can you say about the angles of a scalene triangle?

Type of triangle	Statement
	All angles are......
	Two angles are.....
	All angles are.....

Teachers help the students to arrive at accurate definitions by posing leading questions.



Does the size of an equilateral triangle affect its angles?

What happens to an equilateral triangle if one of its sides is replaced with a longer one? A shorter one?

Do the students notice the relationship of the equal angles to the equal sides in the case of an isosceles triangle?

If an isosceles triangle gets sharper at the vertex, how does this affect the base angles?

If the sides remain the same and the angle between them is widened, how does it affect the base?

ACTIVITY 7

Objective: Exploring a set of triangles in terms of angles

Materials: Types of triangles of different angles (acute-angled, obtuse-angled, right-angled). A chart showing different types of triangles (based on angles) and corresponding names

Given a set of triangles, the students sort them into different types on the basis of angles and learn how to name them.

Do the students notice that in the case of an acute-angled triangle, all three angles are acute?

Why cannot an obtuse-angled triangle have three obtuse angles? Can it have two obtuse angles?

Can there be two right angles in a triangle? Why or why not? If one angle of a triangle is a right angle, what can we say about the other two angles?

GAME: FILL A PROPERTY GRID

(https://nrich. maths. org/2927)

Materials: Set of cards (12 to 15 cards) which describe properties of triangles involving sides, angles, lines of symmetry and so on, as shown in the table.

Prepare a 4 x 4 grid in which a student places selected property cards in the first column and the first row.

The challenge for the other student is to draw a triangle in the corresponding box, so that the triangle has both the properties at the top of the column and at the start of the row.

There may be some combinations that are not possible! If the student cannot draw a shape, he/she can 'pass' and see whether the other can draw it. Whoever draws the last shape wins.

Discuss: If a square is a special rectangle, is an equilateral triangle a special isosceles triangle?

Note: Teachers can create a table similar to the above, using angles versus sides.

Reference: https://www.cut-the-knot.org/triangle/Triangles.shtml

	All angles different	**Contains a right angle**	**Contains a right angle but has no line of symmetry**
Has 2 equal sides			
Has all 3 sides equal			
Has no sides equal			

ACTIVITY 8

Objective: The longest side of a triangle is opposite the largest angle of the triangle

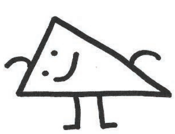
Materials: Different scalene triangles

Students label the vertices. Students measure sides and angles of these triangles and note them down in a table format.

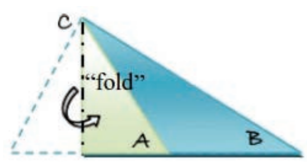
Side measure	Opposite to	Angle Measure
AB	angle	
BC	angle	
AC	angle	

The longest side is opposite to the angle.......

The shortest side is opposite to the angle.........



This can also be done through paper-folding.



To compare two sides of the triangle: fold one side onto the other, i.e., fold along the bisector of the included angle.

To compare two angles of the triangle: fold the triangle so that one vertex falls upon the other, i.e., fold along the perpendicular bisector of the common side.

ACTIVITY 9

Objective: Determine the type of triangle: right-angled, acute-angled or obtuse-angled triangles

Materials: Square grids from 1×1 to 12×12, with the reverse side plain, without grid lines

Let students initially use the plain side of the grids to put together triangular shapes using edges of the grids.

They will be able to make some acute-angled, some obtuse-angled and some right-angled triangles.

Students can then turn the shapes over to uncover the grids and record the information of each such triangle in a table format.

What do they notice? What is the relation when the triangle is acute-angled?

What is the relation when the triangle is obtuse-angled?

What is the relation when the triangle is right-angled?

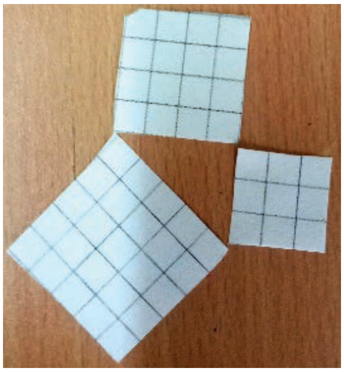
What happens when the triangle is isosceles?

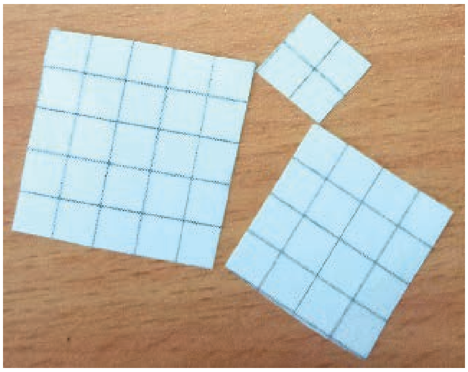
What happens when the triangle is equilateral?

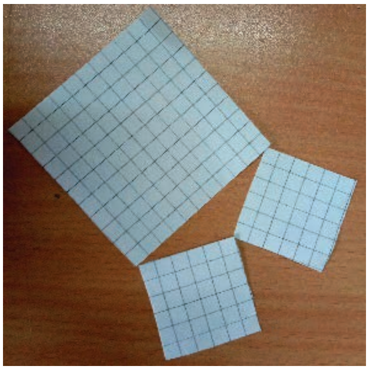
Is it possible to determine the type of triangle by looking only at the lengths of the sides?

If the sides of a triangle have lengths 4, 5 and 8, what kind of a triangle would it be?

If the sides of a triangle have lengths 6, 5 and 7, what kind of a triangle would it be?





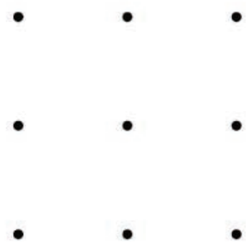


S. No.	Type of triangle	Length of sides	Areas of the squares	Relation $a^2 + b^2 > c^2$	Relation $a^2 + b^2 < c^2$	Relation $a^2 + b^2 = c^2$

ACTIVITY 10

Objective: Triangles on square dot paper

Materials: Square dot paper



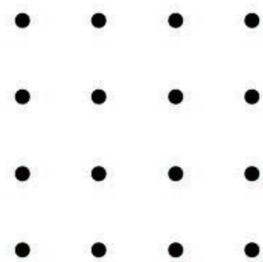
The teacher can show the students how to use row and column number to name the points.

Students can outline a 3×3 grid and explore different triangles that can be drawn inside the grid.

Each vertex of the triangle must be at a grid point of the grid.

How many different triangles are possible?

Is it possible to make an equilateral triangle?



Is it possible to make a right-angled isosceles triangle?

Draw all possible distinct triangles on a 4×4 dot grid. Have you found them all? How do you know?

Further extension:

Outline an 8×8 grid. Let one vertex of a triangle be at the point (4, 4).

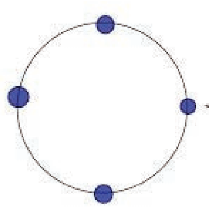
How many different isosceles triangles can be drawn with this point as a vertex?

ACTIVITY 11

Objective: Triangles on a circle

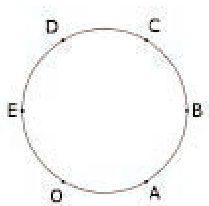
Materials: Circle with 4 equidistant points, with 6 equidistant points, with 8 equidistant points

Students can draw circles and mark the necessary number of points.



How many different triangles can you make on a circle that has 4 equally spaced points?

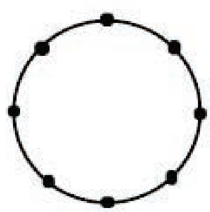
What are the angles of such a triangle?



How many different triangles can you make on a circle that has 6 equally spaced points?

Can you be sure that you have found all of them?

Students can use coloured pencils to differentiate each type of triangle and measure its angles.



What are the angles of each triangle?

Do the students make use of ideas of symmetry to spot identical triangles?

How many different triangles can you make on a circle that has 9 equally spaced points?

How many triangles have you found? What are the angles of each triangle?

How many different triangles can you draw on an eight-point circle?

Can you find the angles of each triangle?

ACTIVITY 12

Objective: Exploring cubes and visualising triangles

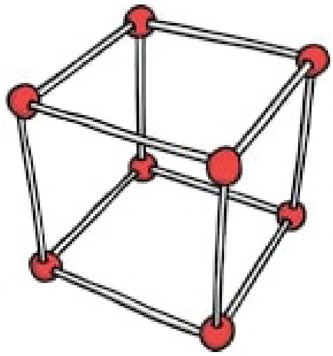
Materials: Straws and balls made of modelling clay

Let students make a skeletal cube using straws and clay balls.

Triangles are formed by joining the vertices of a skeletal cube.

How many different types of triangles are there?

Challenge: Can they find the total number of triangles that can be made?



ACTIVITY 13

Objective: Equilateral triangle folds

Materials: Equilateral triangle papers of different sizes

Students can fold an equilateral triangle in various ways to observe the shapes that emerge.

If an equilateral triangle is folded on its line of symmetry, what shape is formed?

Will the shape be the same if it is folded on another line of symmetry?

Will the shape be different if the size of the equilateral triangle is bigger?

Can the shape that you get by folding on a line of symmetry ever be an equilateral triangle?

Can you fold an equilateral triangle paper in such a manner that you get 4 smaller equilateral triangles?

ACTIVITY 14

Objective: Isosceles triangle folds

Materials: Isosceles triangle papers of different sizes (two types of acute-angled isosceles, right-angled isosceles, obtuse-angled isosceles)

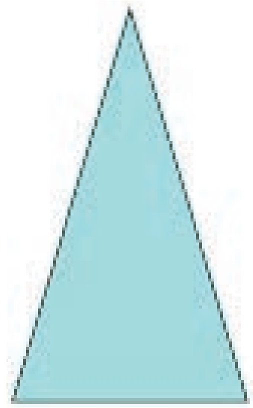
Students can fold an isosceles triangle on its line of symmetry.

What kind of triangle do you get when you do this?

Will the folded shape be different if the angle at the vertex is wider?

Can the resulting shape be ever an isosceles triangle?

Note: Students are allowed to fold along any line, not just the line of symmetry, and multiple folds are allowed.



ACTIVITY 15

Objective: Square paper folding and triangles

Fold a square paper so that you get 4 equal triangles. What type of triangles do you notice?

Note down the angle measures. Justify why the angles would be the same for all folded square papers.

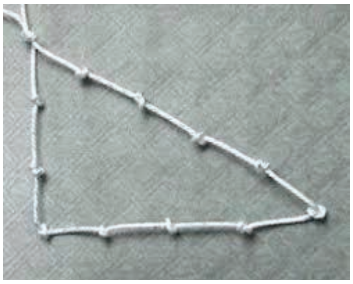
ACTIVITY 16

Objective: Rope with 12 equal sections

The ancient Egyptians used to make right-angled triangles using a rope with twelve equal sections divided by knots.

What other triangles could you make if you had a rope like this?

Will they have a right angle?



ACTIVITY 17

Objective: Construction of a triangle with perforated strips

Materials: Thick card strips of 3 different lengths which form a triangle. Ex. strips of lengths 10 cm, 7 cm and 5 cm, with holes at their ends.

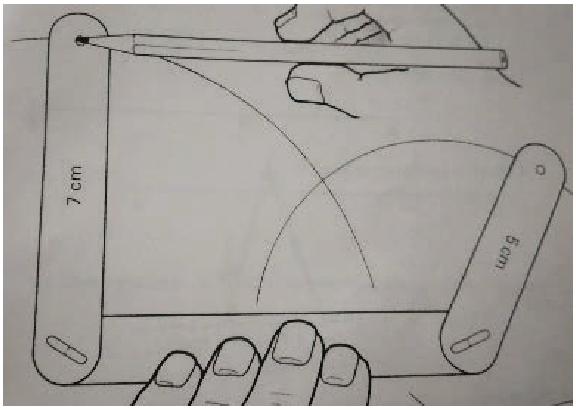
Secure the three strips together to form a triangle by using staple pins.

Open the triangle at the joint of 10 cm and 5 cm strips. Hold the Middle strip (10 cm) firmly down or pin it to the paper. Use a pencil point in the hole of one strip (7cm) to draw an arc, by rotating the strip.

Use a pencil point in the hole of the other strip (5 cm) to draw another arc, by rotating the strip.

The two curves cross each other at one point. Mark the point.

Notice that this is the same point where the 7cm and 5cm strips were joined at first.



ACTIVITY 18

Objective: Construction of a triangle with compass and ruler

Teachers can contrast the compass-and-ruler construction with the earlier activity.

The length of the strip is now replaced by the length measured out by the compass.

Do the students realise that the arcs they are drawing are parts of a circle which is at a fixed length from the given point?

While students restrict themselves to drawing small arcs and finding the point of intersection, it will be good if they can clearly articulate the logic of the construction process.

The first arc is a collection of points at a distance of 4 cm from one end of the line.

The second arc is a collection of points at a distance of 5 cm from the other end of the line.

The point where they intersect is at a distance of 4 cm from one end and 5 cm from the other end.

Now ask: Can you construct a triangle if three angles are given? Will your triangle look the same as your friend's triangle?

Further constructions

- Discussion on the construction of a triangle using a protractor.
- Discussion on the construction of an equilateral triangle.
- Discussion on the construction of an isosceles triangle.
- Discussion on the construction of a right-angled isosceles triangle.

Note

The following activities are for students to explore and discover some properties of triangles through paper folding. Students are not required to learn terminology or formal knowledge of these properties which are generally taken up in higher classes.

ACTIVITY 19

Objective: Discovering properties through folding triangles – 1

Materials: Different triangle shapes (all 8 types)

Reference: www.arvindguptatoys.com (for videos on geometry and paper-folding activities).

Students can fold each side of the triangle upon itself, vertex on vertex. What kind of lines are formed when we do this?

Do the three lines intersect at a point? Do they intersect in this manner for all triangles?

Note: For obtuse-angled triangles, the point of intersection would lie outside the triangle. To observe this, one can draw the triangle in a circle.

What is the distance from the point of intersection to each of the three vertices?

What is the length from the point of intersection to each side? (This will be the perpendicular distance and can be achieved through paper folding.)

What do you notice?

ACTIVITY 20

Objective: Discovering properties through folding triangles – 2

Materials: Different triangle shapes (all 8 types)

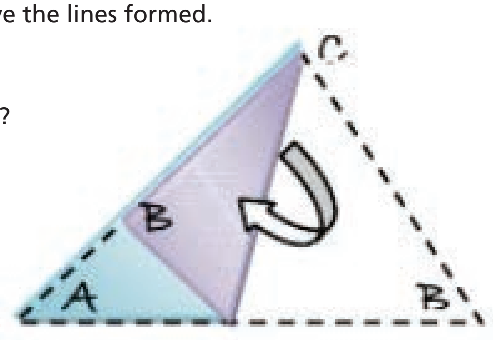
Students can fold each angle of a triangle onto itself and observe the lines formed.

Do the three lines intersect at one point?

What is the length from the point of intersection to each vertex?

What is the length from the point of intersection to each side?

What do you notice?

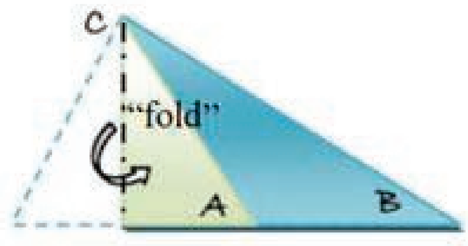


ACTIVITY 21

Objective: Discovering properties through folding triangles – 3

Fold the sides so that the fold goes through the opposite vertex.

Do the three lines intersect at one point?



Note: The obtuse-angled case needs to be done differently since the point of intersection lies outside the triangle. This is equivalent to dropping a perpendicular from a point. Here the reference 'Unfolding' (http://www.teachersofindia.org/en/article/un-folding) can help.

What is the length from the point of intersection to each vertex?

What is the length from the point of intersection to each side?

What do you notice?

ACTIVITY 22

Objective: Discovering properties through folding triangles – 4

Mark the midpoint of each side. Make 3 folds to join each vertex to the midpoint of its base.

Do the three lines intersect at one point?

Is there anything special about this point?

What is the distance from the point of intersection to each vertex?

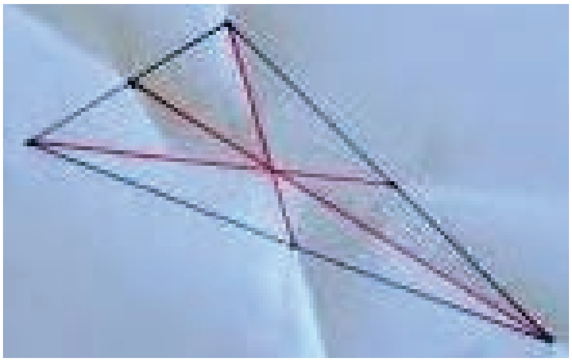
What is the distance from the point of intersection to each side?

How does the distance from the intersection point to a vertex compare to the distance from the corresponding base?

Try the same activity with another vertex and base.

Try the activity with the third vertex and base.

Do you notice some pattern?



Experiment: This point is also the 'balancing point' of a triangle. Draw such a triangle on a piece of cardboard, cut it out, and locate this point. If done accurately, the triangle will balance on the tip of a pencil, or hang perfectly level from a piece of string that is attached to it.

ACTIVITY 23

Objective: Perimeter relationships

Students draw a triangle and mark the midpoint of each side of the triangle.

Join the three points together to make another triangle.

Ask: How does the perimeter of the new triangle connect with the perimeter of the original triangle?

What do you notice about the other triangles that have formed?

ACTIVITY 24

Objective: Similar triangles

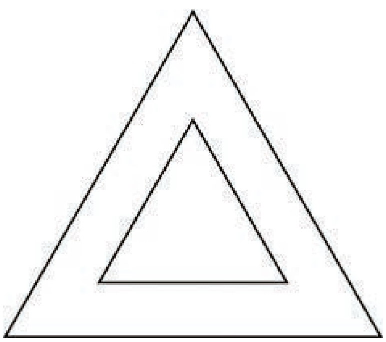
Students can be shown some pictures of similar triangles.

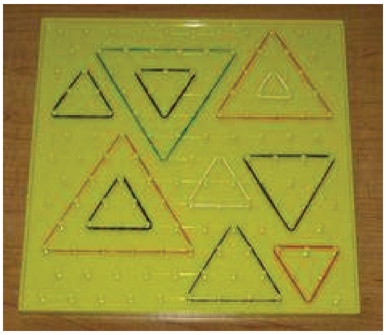
In what way are these triangles 'the same'? In what way are they 'different'?

What can you say about the angles of these triangles?

If one side of a triangle is extended, will they have the same appearance?

'Similar' means that the triangles have exactly the same shape, but possibly not the same size.



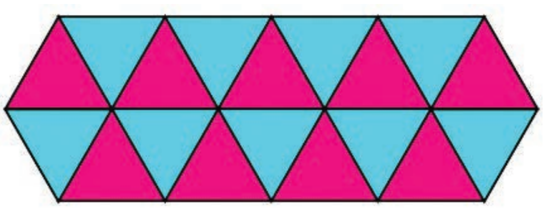


ACTIVITY 25

Objective: Tiling triangles

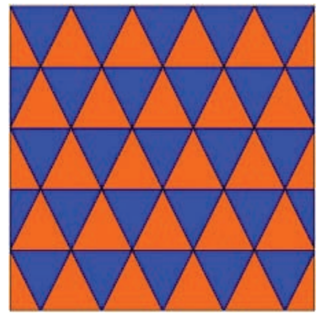
Note: All triangles tile. Teachers must take care to point out this to students by experimenting with different types of triangles.

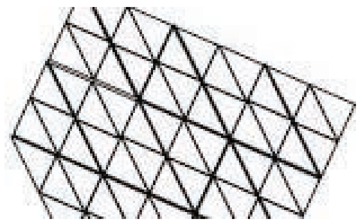
In how many ways can two equilateral triangles be put together if the sides have to match?



In how many ways can two isosceles triangles be put together if the sides have to match?

Do scalene triangles tessellate?





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Paper folding pictures: https://www.scribd.com/document/366908902/paper-folding-discovery

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