Winning Ways

JAMES METZ

The following practical application of exponents easily leads to a consideration of special cases when the exponent, the base, or both are zero.

Yunus, Laurie and Jim play a simple game of tossing a ball into a basket. The first person to put the ball in the basket wins the game. They decide to play a tournament with the champion the first one to win 3 games. What is the total number of different ways in which Laurie can win?

Since Laurie wins 3 games, Yunus and Jim each win either 0, 1 or 2 games (3 options for each of them), so the total is 3 times 3 or 9 ways in which Laurie can win. See Figure 1.

Yunus	Jim
0	0
0	1
0	2
1	0
1	1
1	2
2	0
2	1
2	2

Figure 1. The 9 ways for Laurie to be champion by winning 3 games

If the champion must win 4 games, then there are 4 times 4 or 16 ways for Laurie to win. In general, for a championship in which Laurie must win n games, there are n^2 ways for Laurie to win.

If Nadia joins the tournament, there will be n^3 ways, since each of the 3 losers can win 0, 1, 2, ... $n - 1$ games. In general, if there are p players and Laurie must win n games to be champion, there are

Keywords: Experiment, observation, documentation, exponents, indeterminate forms

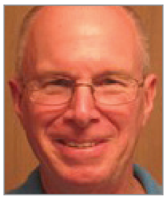
n^{p-1} ways. For example, if 3 people play and 8 victories are needed for the champion, there are $64 = 8^2$ ways for Laurie to win. (There are also 64 ways if 4 people play and 4 victories are needed for the champion.)

What if Laurie plays alone and decides she must win 3 games to be champion? Then there is only one way that she can win. Also, by our formula, there are 3^0 ways. So we have that $3^0 = 1$, and we have found a nice practical way to show why it is a good idea to define $n^0 = 1$ for $n \neq 0$.

It makes no sense for Laurie to play alone and be a champion if the game is not defined by a fixed number of wins, so it makes sense that 0^0 is indeterminate.

There is no way for 2 or more people to have a tournament with 0 victories required for a champion, so $0^{p-1} = 0$, that is $0^k = 0$ for k = 1, 2, 3, ...

Putting exponents in a practical context may help make sense of special cases.



JAMES METZ is a retired mathematics instructor. He volunteers for Teachers Across Borders, Southern Africa for one month each year and enjoys doing mathematics. He may be contacted at metz@hawaii.edu