Which is the Best Deal?

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This article presents a task in which students explore how to help a shopkeeper decide the best deal for purchasing soaps from three different wholesalers. The students express the options in the language of mathematics to find the cost for a certain number of soaps. They further write linear equations to find the changeover point at which a different shop becomes a better deal. The exploration task lets students "get inside" the mathematics and appreciate how algebra can be used in decision making. This task is suitable for students of classes 8 to 10. It covers the content domains of Arithmetic, Algebraic Expressions, Linear Equations and Coordinate Geometry.

In the current education system, students do not have a positive relationship with Mathematics. There is a lot of fear and anxiety connected to the subject as they have a very procedural relationship with it, and they are not able to connect it with the world around them. When it comes to algebra, the relationship gets further broken. Students who had learnt procedurally are now expected to solve linear equations with one or two variables, which do not make sense to them. This takes them further away from conceptual understanding. The result is that many students drop mathematics as soon as they can.

When middle-schoolers see an expression like $2x + 3$, they think of a polynomial with two terms and one variable. It does not conjure up any image of a function, nor do they think it could be a code for a pattern. When this expression becomes an equation $2x + 3 = 11$, they start thinking about how to manipulate this equation to get the value of x. They miss the **story** of this function.

The situation becomes further challenging when they see x on both sides of the equation $2x + 3 = x + 10$. Some students may know what steps to take to find the value of x. They think of it as a procedure that they have to learn to clear the exam which has no connection to their daily life. The appreciation for algebra is lost for them.

This paper presents an Exploration Task where students help a shopkeeper in deciding from where to source a

Keywords: Arithmetic, Algebraic Expressions, Linear Equations, Coordinate Geometry, Graphs, Line graph, Classroom Engagement

product and how to justify their reasoning for the decision. It is designed for students to work on making and solving linear equations with variables on both sides of the equal to sign. It may take two class sessions of 45 minutes each. This task was implemented in a 1.5 hr online session with teachers and students. The students who participated in the online session were from government schools of Delhi. The presentation of the online session was done using Power Point.

This exploration problem uses the following lesson framework:

1. **Engage:** Group warm-up through:
   - *Ganit Charchaa*, short discussion around a mathematical problem and
   - Mindset message
2. **Explore:** Work on a real-life problem in small groups.
3. **Explain:** Individuals discuss and justify their selection within their group through mathematical reasoning.
4. **Extend:** Work on extensions of the problem which may either be suggested by the facilitator or may surface as questions in the student's mind.
5. **Essence:** Understand the essence of mathematics through individual and group reflections.

Engage

The classroom engagement begins with a *GanitCharchaa* [1], a 5-7 min discussion around a mathematical problem for the whole class. The aim of the discussion is to reinforce that the mathematics classroom is a safe space to express opinions. The participants should be able to see that there are multiple ways of solving a problem, all answers are accepted and participants are encouraged to justify their thinking and to listen respectfully to others' opinions. A *GanitCharchaa* is usually connected to the topic which is to be studied and is a good way to recap the previous lesson or to informally gauge students' level of preparedness for the upcoming lesson.

GanitCharchaa of the relevant topic should be a part of every classroom irrespective of what is planned for the rest of the class. Of course, it need not only be an introductory activity, it should be part of the mathematics classroom culture.

If the sessions are being conducted online, it's best to take the answers in private chat, and then callout participants to explain their reasoning.

GanitCharchaa 1

The participants are shown Figure 1 and asked the following questions:

1. How do you see the Star-Sets grow?
2. If a picture has 28 stars what would be its Star-Set number?

Figure 1: GanitCharchaa about growing Star Sets

In the classroom sessions, when the students were asked about how they saw a pattern grow, they described what they saw rather than just count the stars, and calculate. They described their thinking, and were able to write the mathematical expression for it. They were able to create the equation for which the stars would be 28. The following Figure 2 shows some of the growth patterns that students described.

GanitCharchaa 2

Make a story problem that can be expressed in the form $25x = 300$. Change your story in such a way that it can be extended to express equations of the form $25x + 100 = 300$.

Several different stories came out e.g. One pen costs ₹ 25. If I paid ₹ 300, how many pens did I buy? Extending this further to the second

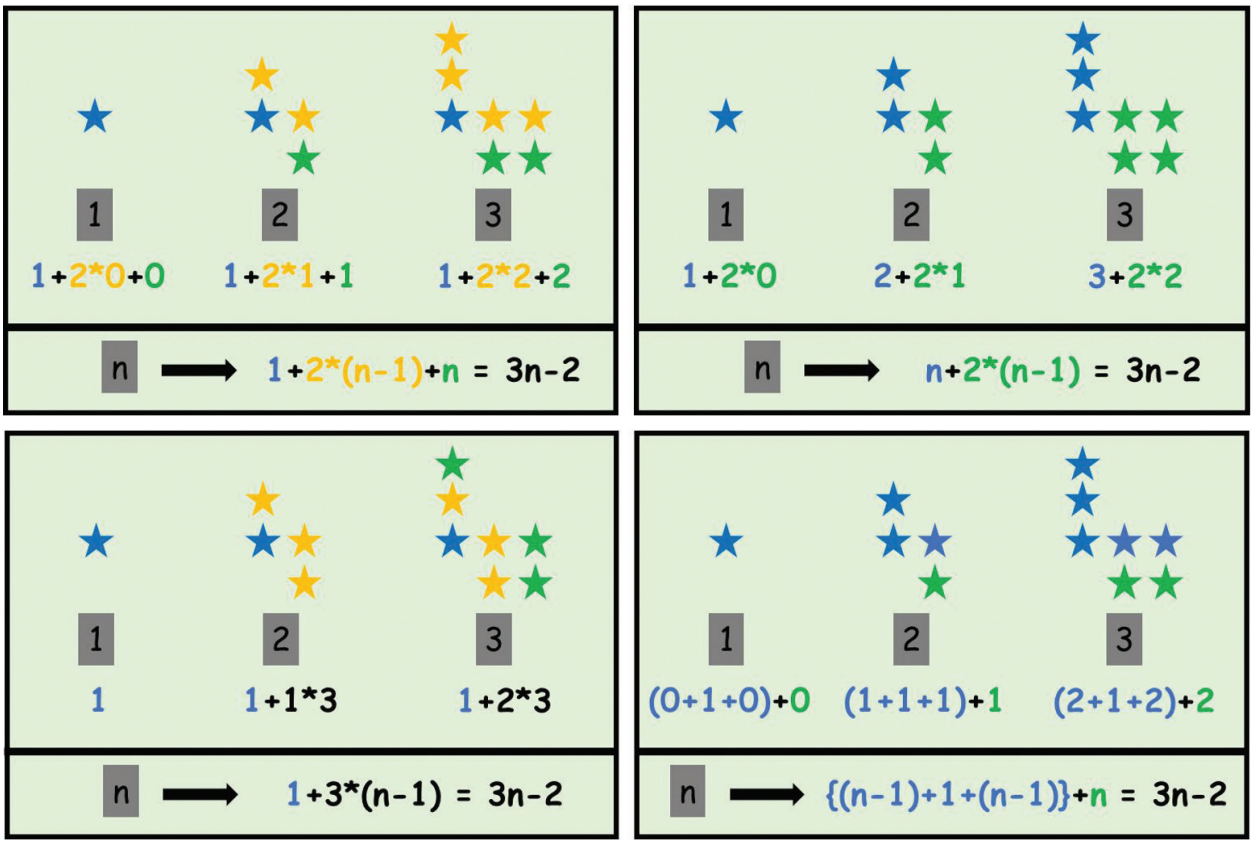

Figure 2: Some of the ways that students see the Star Set pattern growth

question: One pen costs ₹ 25. If I paid ₹ 100 in travelling to the store from home and spent a total of ₹ 300, how many pens did I buy?

Mindset Message: In our society, there are many fears and biases set against mathematics. Our society is willing to accept that some people get math while many don't. Such statements send a message to our children that math is difficult, and it's okay to not know it. It is very important for us to consistently work on breaking these biases. And our classrooms are the best place for us to start working on it.

Mindset messages [see 2] are the underlying themes of the classroom that help set the classroom culture and are intended to break the biases against mathematics that could be in the mind of students. This is also the space where we celebrate the mathematicians of today to show our students that there is still work going on in the field of mathematics. One such message is given in Figure 3.

In this class, the students teased out the problem and did not go directly to the solution; they were able to find the beauty and relevance of mathematics by this patient exploration and the mindset message certainly spoke to them.

The message celebrates Mariam Mirzakhani and the mindset message through her quote sets a tone for the session that exploring mathematics and understanding it needs consistent work and patience. There is no shortcut to it.

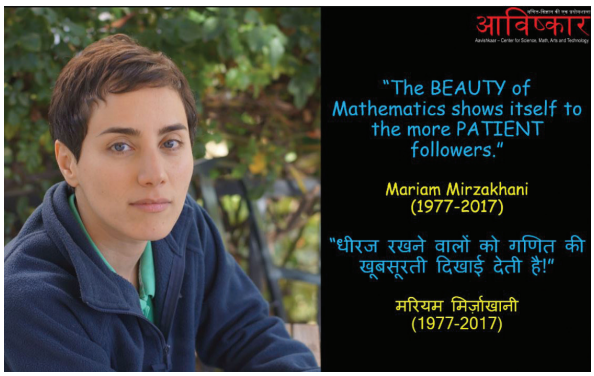

Figure 3: Mindset Message

Explore

The exploration problem presented here is aimed at delving deeper into the concept of linear expressions. This problem is set in a small village of Himachal. Aanchal runs a general store in the small village of Kandbari in Kangra Valley. She is running out of soaps at the store and needs to stock up some. She made some phone calls and collected information about the cost involved in soap purchase.

Since her village is rather small, she plans to stock 25 to 35 soaps of a particular brand. She called 3 different wholesalers in nearby towns and found out the price of the same brand of soap. Everyone gave the price of a single soap, but they all only sell the soap in packs of 5. Based on the information that she has collected, the class has to help her decide the best deal. Every group also needs to justify their thinking.

Based on Table 1, the students make initial predictions on which dealer they think is giving the best deal. After collecting responses, they work on the problem and fill out the table below. The problem is divided into four parts for students to work on. There are some key guiding questions written for teachers to help students in their thinking process.

1. In the first question students need to answer: Aanchal plans to buy 25 to 35 soaps. Which wholesaler is giving the cheapest deal? To help them answer this question and help them organize their thoughts, the following Table 2 may be provided.
2. Write a general expression for finding the cost of any number of soaps from a given wholesaler (including the travel cost).
3. Plot the general cost expression for each shop on a single graph.
4. Is there a particular number of soaps for which it will not matter which shop you bought the soap from? If yes, what is that number? How will you find out? Did you see this in the graph you drew? Justify your answer for whatever choice you make.

The students will work in their respective groups for this task and explain their reasoning in the larger group. If doing it in the same physical space, it would be best for every group to do a poster presentation of their work. In an online setting, they should be encouraged to make a slide presentation. Here are some guiding questions to ask the students in their small groups to help encourage their thinking.

Guiding questions for Part 1:

A. What will be the cost of soaps from each shop in case she decided to buy 25 soaps? How did you calculate?

B. Which is the best deal in this case (25 soaps)?

C. What will be the cost of soaps from each shop in case she decided to buy 35 soaps? How did you calculate?

D. Which is the best deal in this case (35 soaps)?[1]

#	विक्रेता Dealer	आना-जाना (रु) Travel (Rs)	दाम/साबुन (रु) Cost per soap (Rs)
1	तुलसीराम-मानचंद, पालमपुर Tulsiram-Manchand, Palampur	200	20
2	जनरल सेल्स एजेंसी, धर्मशाला General Sales Agency, Dharamsala	500	10
3	गणेश थोक विक्रेता, बीड़ Ganesh Wholesale, Bir	320	16

Table 1: Information collected by Aanchal from three different wholesalers.

#	विक्रेता Dealer	आना-जाना (रु) Travel (Rs)	कुल दाम Total Cost		
			1 साबुन (रु) 1 soap (Rs)	25 साबुन 25 soaps	35 साबुन 35 soaps
1	तुलसीराम-मानचंद, पालमपुर Tulsiram-Manchand, Palampur	200	20		
2	जनरल सेल्स एजेंसी, धर्मशाला General Sales Agency, Dharamsala	500	10		
3	गणेश थोक विक्रेता, बीड़ Ganesh Wholesale, Bir	320	16		

Table 2: Table with wholesaler data and additional columns provided to help students organize their thoughts.

[1] **Note for Teachers:** Ask the students to write the arithmetic expression for questions A and C rather than write the answer directly. It will help them see the pattern and write the general expression.

Guiding questions for Part 2 and Part 3:

A. What will be the price for purchasing "x" soaps from each shop?

   Note for Teachers: Ask students to review the expression written in Part 1 for responding to this question. During the online session, some students had difficulty in this question and needed to be guided to observe their response to part 1 of the question. It seemed to happen due to their fear of *"x"* and the way they have interacted with algebra.

B. Is there a condition when it will not matter where she buys the soaps from?

C. If yes, find the value of *"x"* for which it will not matter where she buys soaps from? How would you find it? Justify your answer.

D. If not, how would you justify it? Show your work.[2]

Assuming x = number of soaps, the students should come up with the following three expressions:

Purchase cost from Palampur = $200 + 20x$

Purchase cost from Dharamsala = $500 + 10x$

Purchase cost from Bir = $320 + 16x$

The students should be able to write the equation equating the cost of two shops to find the value of x for Part C. And then calculate the price at the third wholesaler for that value of x. Some students check for all three sets of equations to justify their answer. It's important to encourage them to justify their answer in the way they see fit.

Guiding question for Part 4 before plotting the graph:

A. What will the graphs of these three equations look like? Can you predict for each wholesaler where the line would intersect with the y-axis? (They should see the fixed travel cost as the y-intercept.)

B. Where will the three lines intersect? [3]

C. Describe a situation where the line does not pass through the common intersection point in the three situations mentioned in the problem.

The students should be able to make a graph as shown in Figure 4.

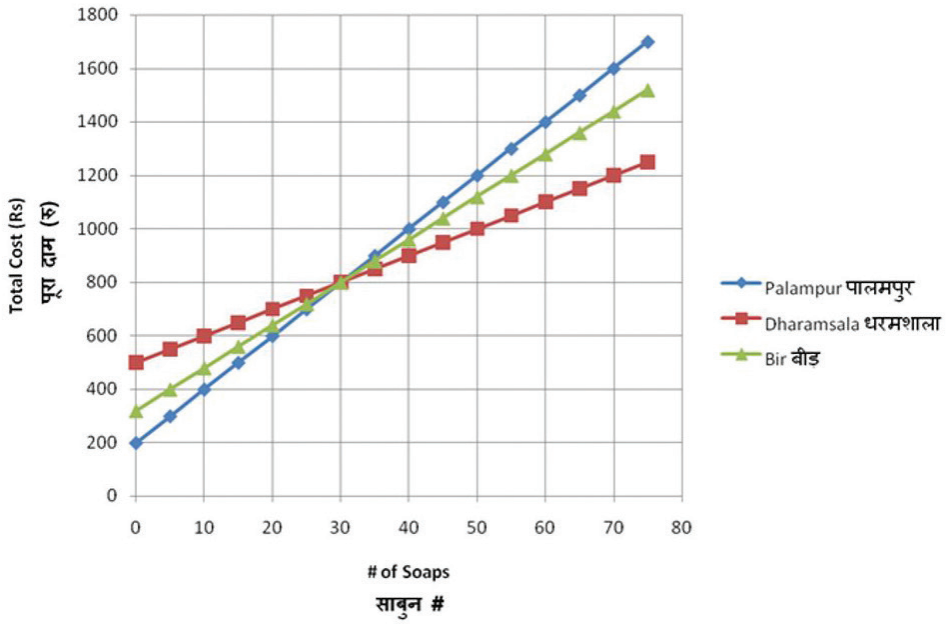

Figure 4: The cost vs # of soaps plot for the three wholesalers showing the point of intersection

[2] **Note for Teachers:** During the online sessions, it was important to check for understanding of questions with the students. Some students thought that it would never be possible. Teachers encouraged students to justify their answer.

[3] **Note for Teachers:** The data in this example is chosen so that the three lines are concurrent, this may not always be the case.

Explain

When students returned to the large group, they shared their responses to each question and gave a justification for it. For most of the students, the prediction of best deal without any calculation was different from what they arrived at after calculations. They were surprised by the fact that best deal for different number of soaps was different. The best deal depended on the number of soaps that were being purchased.

To find the number of soaps for which it wouldn't matter which shop Aanchal went to, several students used the trial-and-error method initially. Using the concept of equality of expressions was not intuitive to them.

For example, when students were asked to do a quick prediction of the best deal, students chose different options for different reasons.

Option 1 – Travel cost is very high compared to cost of soap, so that will decide the final price.

Option 2 – The cost of soap is least, so the high cost of travel won't matter.

Option 3 – Option 1 has highest soap cost while Option 2 has highest travel cost. Option 3 has the best of both worlds.

Filling out Table 2 made students realize that best deal will be different for different number of soaps. It was more like an "Aha!" moment as they had to do a prediction first.

Extension

Here are some possible extensions to the problem that students can work on individually or in small groups. [Note: These extensions were given in one of the online sessions when we were conducting the session with a group of teachers. The extension has not been tried with students.]

1. Aanchal wants to make a profit of Rs 400 by selling the soaps, she also knows that people will not pay more than Rs 30 for a soap. How many soaps does she need to sell to make this money? Which shop would you recommend her to buy the soap from? Why?

2. Immediately after her call to the shops, Ganesh Wholesaler calls back. He is giving one free face-mask with a pack of 5 soaps. It's Corona time, and Aanchal also sells face-masks. It costs her Rs 10 to buy a face-mask. Now which deal is the cheapest? Why? Describe your reasoning.

Essence

Appreciating its value in decision making: Many students had predicted that the Bir shop would be the best deal as the cost of the shop was in the middle and so was the travel cost. They were surprised that the best deal depended on how many soaps were being purchased.

Students were able to connect the problem to the real-life situation of shopkeepers. After completing the task one participant reacted, "Now I see why wholesalers go to faraway places to buy while the retailer buys from the local wholesaler."

There was a lot more appreciation for the fixed travel cost in any purchase. Students were able to connect it to their life. Some students questioned the value of going far away for an item if it can be purchased nearby.

The students were surprised that there was a value for which all expressions resulted in the same cost. Initially the students solved the problem of finding that value by trial-and-error. It was also insightful for them that linear equations can be used to solve for such situations and help in decision making. Making the justification visible through the use of graphs was revealing to the students.

Acknowledgement

The author is thankful to Tarun Agrawal of Aavishkaar for his support in creating this session. She would also like to thank the Trikon team of Teach For India, Delhi Fellows for implementing this lesson with their students and for sharing their constructive feedback.

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